SYDNEY GRAMMAR SCHOOL

2011 Half-Yearly Examination

FORM V
MATHEMATICS 2 UNIT

Wednesday 11th May 2011

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators may be used.
- All necessary working should be shown in every question.
- Start each question on a new leaflet.

Structure of the paper

- Total marks — 90
- All six questions may be attempted.
- All six questions are of equal value.

Collection

- Write your name, class and master clearly on each leaflet.
- Hand in the six questions in a single well-ordered pile.
- Hand in a leaflet for each question, even if it has not been attempted.
- If you use a second leaflet for a question, place it inside the first.
- Write your name on the question paper and place it inside your leaflet for Question One.

5P: SJG 5Q: TCW

Checklist

- Writing leaflets: 6 per boy.
- Candidature — 31 boys

Examiner

SJG

QUESTION ONE (15 marks) Start a new leaflet.

(a) Write as a fraction in lowest terms:

(i) $0{\cdot}12$

(ii) $17{\cdot}5\%$

(b) Express $\dfrac{6}{\sqrt{3}}$ with a rational denominator and simplify.

(c) Evaluate $\sin 33^\circ 45'$ correct to two decimal places.

(d) Simplify $4\sqrt{2}+3-\sqrt{2}$.

(e) Factorise:

(i) $x^2-5x-36$

(ii) a^3+8

(f) Determine whether the point $(3,-2)$ lies on the line $2x+3y-1=0$.

(g) Solve:

(i) $x^2-4=0$

(ii) $|x|=7$

(h) Simplify $\dfrac{8ab}{5}\div\dfrac{4ab}{15}$.

(i) Solve the inequation $7x-9\leq 26$. Graph the solution on a number line.

Exam continues next page ...

QUESTION TWO (15 marks) Start a new leaflet.

(a) Simplify $3\sqrt{45}$.

(b) (i) On a set of axes, draw a ray representing the angle $\theta = 210^\circ$.

(ii) Find $\cos 210^\circ$.

(c) How far and in what direction has the parabola $y = x^2$ been translated to produce $y = (x+4)^2$?

(d) (i) Solve the inequation $|x+3| \leq 5$.

(ii) Graph your solution on a number line.

(e) If $P(x) = x^2 - 3x + 2$, find and simplify:

(i) $P(3)$

(ii) $P(a) + 3$

(iii) $P(a+3)$

(f) Simplify $\dfrac{1}{x} - \dfrac{1}{x+1}$.

(g) Expand and simplify $\left(\sqrt{3}+1\right)^2$.

(h) Find the acute angle θ to the nearest degree if:

(i) $\tan\theta = 1{\cdot}4$

(ii) $\operatorname{cosec}\theta = 1{\cdot}3$

Exam continues overleaf ...

QUESTION THREE (15 marks) Start a new leaflet.

(a) Factorise $3b^3 - 3c^3$.

(b) Find the natural domain of each function:

(i) $f(x) = \dfrac{1}{x+1}$

(ii) $f(x) = \sqrt{7-x}$

(c) Consider the parabola $y = x^2 + 2x - 8$.

(i) Find the y-intercept.

(ii) Find the x-intercepts.

(iii) Find the coordinates of the vertex.

(iv) Clearly sketch the parabola, showing all intercepts and the vertex.

(v) Hence solve $x^2 + 2x - 8 > 0$.

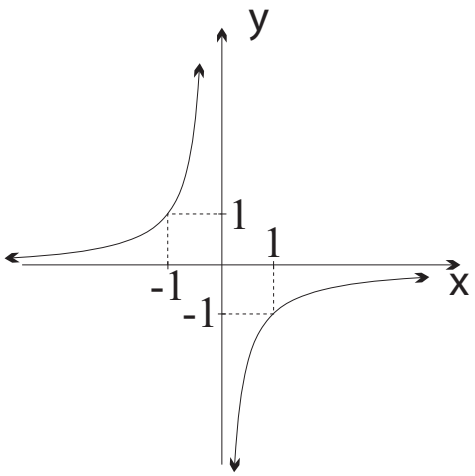
(d) Simplify:

(i) $\sqrt{27} + \sqrt{75} - \sqrt{48}$

(ii) $\left(2\sqrt{6} + 3\right)\left(2\sqrt{6} - 3\right)$

(e) Solve $\sin x = \dfrac{1}{\sqrt{2}}$, for $0° \leq x \leq 360°$.

(f)



The graph above shows the curve $y = -\dfrac{1}{x}$. What is the equation of the vertical asymptote?

Exam continues next page ...

QUESTION FOUR (15 marks) Start a new leaflet.

(a) Two positive numbers differ by 5 and their squares add to 233. Form an equation and solve it to find the two numbers.

(b) A man standing on a rooftop looks down at a car parked some distance away. If the angle of depression of his line of sight is 18° and he is 16 metres above the ground, how far away from the base of the building is the car? Give your answer to the nearest centimetre.

(c) Factorise $3x^2 - 10x - 8$.

(d) Express the following fractions with a rational denominator:

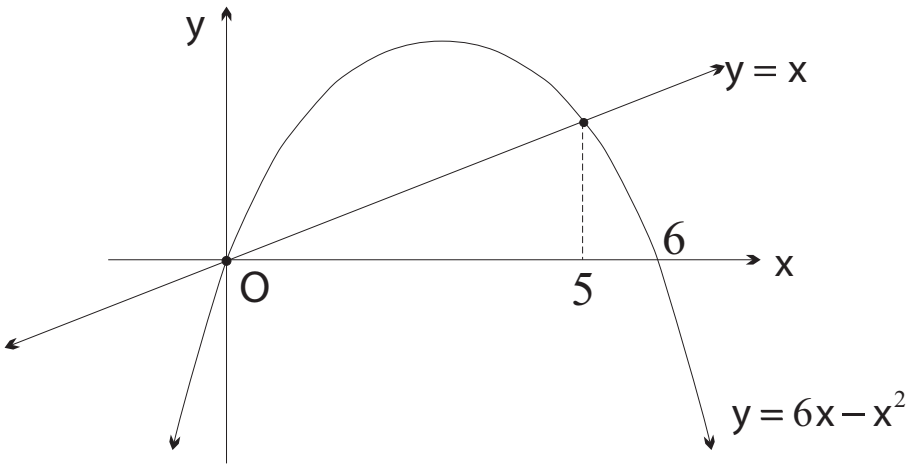
(i) $\dfrac{\sqrt{2}}{\sqrt{3}}$

(ii) $\dfrac{1}{\sqrt{5}-\sqrt{3}}$

(iii) $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

(e) Given the function $f(x) = x^3 - x$, find $f(-x)$ and hence determine whether the symmetry of $f(x)$ is even, odd or neither.

(f)



Using the graph above, or otherwise, solve the inequation $6x - x^2 < x$.

(g) If a ranger can see his camp on a bearing of 127°T, what is the bearing of the ranger from the camp?

Exam continues overleaf ...

QUESTION FIVE (15 marks) Start a new leaflet.

(a) Showing all working, write $1\cdot\dot{6}0\dot{3}$ as a fraction in lowest terms.

(b) Simplify $\dfrac{x+1}{x^2-6x+5} - \dfrac{x-4}{x^2+4x-5} + \dfrac{1}{x^2-25}$.

(c) If $\cos\theta = \dfrac{\sqrt{5}}{3}$ and $\tan\theta < 0$, find the exact value of $\sin\theta$.

(d) (i) Find the centre and radius of the circle $x^2 - 4x + y^2 + 2y = 4$.

(ii) Using a graph, or otherwise, determine the range of $x^2 - 4x + y^2 + 2y = 4$.

(e) (i) On a single set of axes, sketch $y = |x-3|$ and $y = |x+1|$.

(ii) Hence, or otherwise, solve $|x-3| = |x+1|$.

(f) Prove the identity $\cos\theta + \tan\theta\sin\theta = \sec\theta$.

Exam continues next page ...

QUESTION SIX (15 marks) Start a new leaflet.

(a) Sketch the graph of the function $y = \dfrac{1}{x+2} - 1$, clearly showing any asymptotes and intercepts with the axes.

(b) Shade the region where $x^2 + y^2 \leq 16$, $x > -1$ and $y < 2$.

(c) A ship at sea sees a lighthouse on a bearing of 320°T. After the ship sails into port, which lies 6 kilometres due west of its original position, the lighthouse is still visible but on a bearing of 048°T. How far is the port from the lighthouse? Leave your solution correct to 2 decimal places.

(d) Solve $\cot 2\alpha = -1$, for $0° \leq \alpha \leq 360°$, giving your solutions correct to one decimal place.

(e) (i) Sketch $y = -\sqrt{16 - x^2}$.

(ii) Sketch $y = 4 - x^2$, for $-4 \leq x \leq 4$, on the same number plane as part (i).

(iii) Solve $4 - x^2 = -\sqrt{16 - x^2}$, leaving your solutions as exact values.

(iv) Hence, or otherwise, solve $4 - x^2 + \sqrt{16 - x^2} \geq 0$.

END OF EXAMINATION

Form V HY : Mathematics SSG

Q1. a) i) $0.12 = \frac{3}{25}$ ✓

ii) $17.5\% = \frac{7}{40}$ ✓

b) $\frac{6}{\sqrt{3}} = 2\sqrt{3}$ ✓

c) $\sin 33°45' = 0.56$ (to 2 d.p.) ✓

d) $4\sqrt{2} + 3 - \sqrt{2} = 3 + 3\sqrt{2}$ ✓

e) i) $x^2 - 5x - 36 = (x-9)(x+4)$ ✓

ii) $a^3 + 8 = (a+2)(a^2 - 2a + 4)$ ✓

f) $2 \times 3 + 3 \times (-2) - 1 \neq 0$

$\therefore (3, -2)$ is not on the line $2x + 3y - 1 = 0$ ✓

g) i) $x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x = 2$ ✓ or $x = -2$ ✓

ii) $|x| = 7$

$\therefore x = 7$ ✓ or $x = -7$ ✓

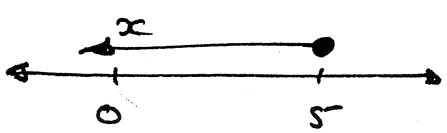
h) $\frac{8ab}{5} \div \frac{4ab}{15} = \frac{8ab}{5} \times \frac{15}{4ab}$

$= 6$ ✓

i) $7x - 9 \leq 26$

$7x \leq 35$

$x \leq 5$ ✓



✓

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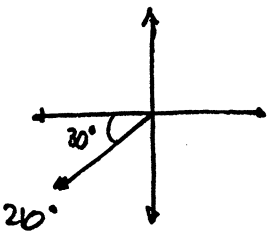
Q2. a) $3\sqrt{45} = 9\sqrt{5}$ ✓

b) i) 

✓

ii) $\cos 210° = -\cos 30°$
$= -\frac{\sqrt{3}}{2}$ ✓

c) shifted left 4 units ✓

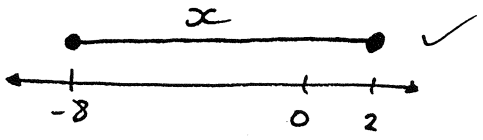
d) i) $|x+3| \leq 5$

$x+3 \leq 5$ and $-x-3 \leq 5$

$x \leq 2$ and $-x \leq 8$ ✓ one answer only

$x \geq -8$

$\therefore -8 \leq x \leq 2$ ✓ both solutions and interval

ii) 

✓

e) i) $P(3) = 9 - 9 + 2$
$= 2$ ✓

ii) $P(a)+3 = a^2 - 3a + 2 + 3$
$= a^2 - 3a + 5$ ✓

iii) $P(a+3) = (a+3)^2 - 3(a+3) + 2$ ✓
$= a^2 + 6a + 9 - 3a - 9 + 2$
$= a^2 + 3a + 2$ ✓

f) $\frac{1}{x} - \frac{1}{x+1} = \frac{x+1-x}{x(x+1)}$

$= \frac{1}{x(x+1)}$ ✓

g) $(\sqrt{3}+1)^2 = 3 + 2\sqrt{3} + 1$
$= 4 + 2\sqrt{3}$ ✓

h) i) $\tan^{-1}(1.4) = 54°$ (to nearest degree) ✓

ii) $\text{cosec}^{-1}(1.3) = \frac{1}{\sin^{-1}(1.3)}$

$= 50°$ (to nearest degree) ✓

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Q3. a) $3b^3 - 3c^3 = 3(b-c)(b^2+bc+c^2)$ ✓

b) i) $x+1 \neq 0$ ii) $7-x \geq 0$

$\therefore x \neq -1$ ✓ $x \leq 7$ ✓

c) i) y-intercept: $y = -8$ ✓

ii) x-intercepts: $x^2+2x-8=0$

$(x+4)(x-2)=0$ ✓

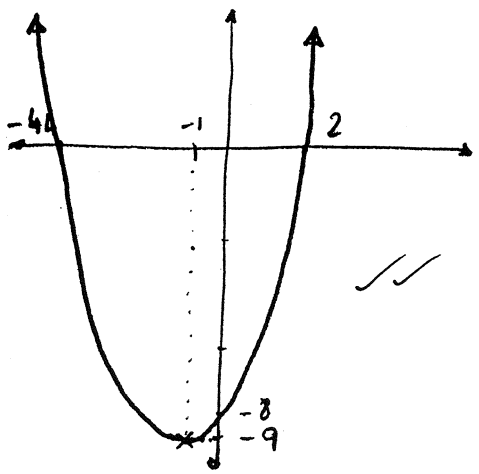
$\therefore x=-4$ and $x=2$ ✓

iii) vertex: $x = \frac{-b}{2a}$ $\therefore y = (-1)^2 + 2\times(-1) - 8$

$= -1$ $= -9$

$\therefore$ vertex is $(-1,-9)$ ✓

iv)



✓✓

v) $x^2+2x-8>0$

$\therefore x<-4$ or $x>2$ ✓

d) i) $\sqrt{27}+\sqrt{75}-\sqrt{48} = 3\sqrt{3}+5\sqrt{3}-4\sqrt{3}$

$= 4\sqrt{3}$ ✓

ii) $(2\sqrt{6}+3)(2\sqrt{6}-3) = 24-9$

$= 15$ ✓

e) $\sin x = \frac{1}{\sqrt{2}}$ $\therefore x = 45°$ ✓ or $x = 135°$ ✓

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f) $x=0$ ✓

Q4. a) Let the smaller number be x.

$x^2 + (x+5)^2 = 233$ ✓

$x^2 + x^2 + 10x + 25 = 233$

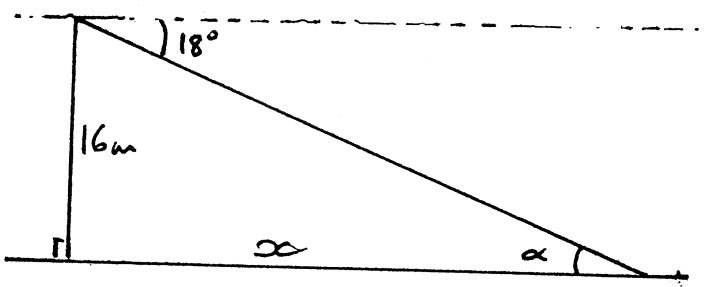
$2x^2 + 10x - 208 = 0$

$(x+13)(2x-16) = 0$ ✓

$\therefore x = 8$ or $x = -13$ (not a solution to the problem)

Hence the two numbers are 8 and 13. ✓

b)



$\alpha = 18°$ (alternate angles on parallel lines)

$\tan 18° = \frac{16}{x}$ ✓

$x = \frac{16}{\tan 18°}$

$= 49.24\text{m}$ ✓

c) $3x^2 - 10x - 8 = 3x^2 - 12x + 2x - 8$ ✓

$= 3x(x-4) + 2(x-4)$

$= (3x+2)(x-4)$ ✓

d) i) $\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}$ ✓ ii) $\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{5-3}$

$= \frac{\sqrt{5}+\sqrt{3}}{2}$ ✓

iii) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$

$= 3 + 2\sqrt{6} + 2$ ✓

$= 5 + 2\sqrt{6}$ ✓

e) $f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$ ✓ $\therefore f(x) = -f(-x)$ and $f(x)$ is <u>odd</u>. ✓

f) from the graph: $6x - x^2 < x$ when $x < 0$ or $x > 5$. ✓

g) Bearing of ranger from the camp: $127 + 180 = 307°\text{T}$ ✓

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Q5. a) $x = 1.\dot{6}0\dot{3}$

$1000x = 1603.\dot{6}0\dot{3}$ ✓

$\therefore 999x = 1602$

$x = \frac{178}{111}$ ✓

b) $\frac{x+1}{x^2-6x+5} - \frac{x-4}{x^2+4x-5} + \frac{1}{x^2-25} = \frac{x+1}{(x-5)(x-1)} - \frac{x-4}{(x+5)(x-1)} + \frac{1}{(x+5)(x-5)}$ ✓

$= \frac{(x+1)(x+5) - (x-5)(x-4) + (x-1)}{(x+5)(x-5)(x-1)}$ ✓

$= \frac{x^2+6x+5 - x^2+9x-20+x-1}{(x+5)(x-5)(x-1)}$

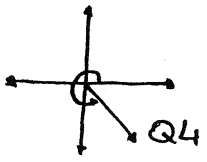
$= \frac{16x-16}{(x+5)(x-5)(x-1)}$

$= \frac{16}{(x+5)(x-5)}$ ✓

c) $\cos\theta = \frac{\sqrt{5}}{3} \quad \therefore \cos^2\theta = \frac{5}{9}$

$\therefore \sin^2\theta = \frac{4}{9}$ ✓

$\sin\theta = \pm\frac{2}{3}$



$\tan\theta < 0$

$\therefore \sin\theta < 0$

$\therefore \sin\theta = -\frac{2}{3}$ ✓

d) i) $x^2 - 4x + y^2 + 2y = 4$

$(x-2)^2 + (y+1)^2 = 9$ $\therefore$ circle of centre $(2, -1)$ ✓

and radius $= 3$ ✓

ii) range is centre $\pm$ radius

$\therefore -4 \leq y \leq 2$ ✓

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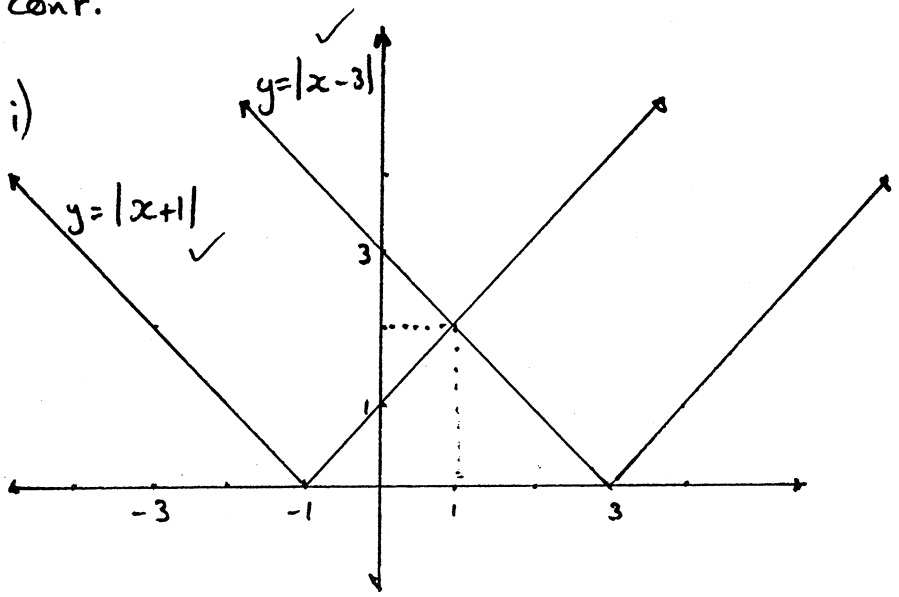
Q5. cont.

e) i)



ii) intersection: $|x-3| = |x+1|$

when $x = 1$ ✓

f) LHS: $\cos\theta + \tan\theta \sin\theta = \cos\theta + \dfrac{\sin^2\theta}{\cos\theta}$ ✓

$$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \text{RHS}$$ ✓

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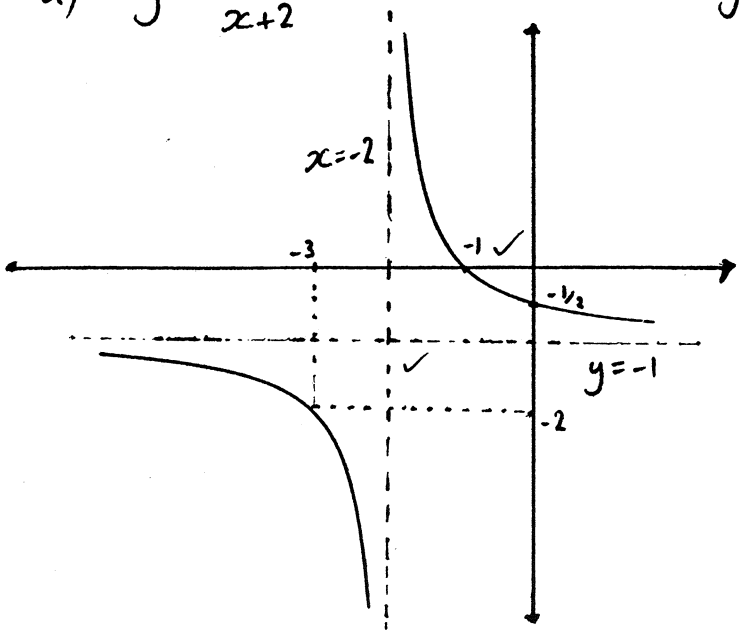
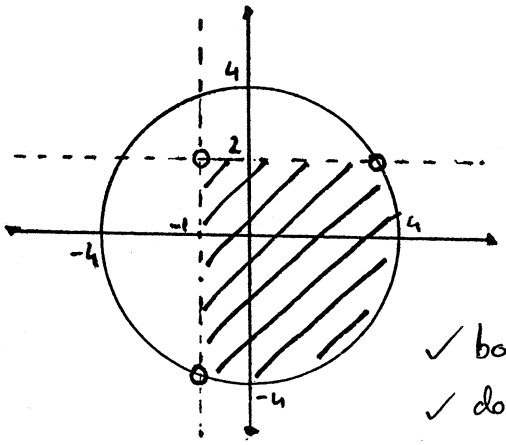
Q6. a) $y = \frac{1}{x+2} - 1$ is the curve $y = \frac{1}{x}$ shifted left 2 units and down 1 unit.



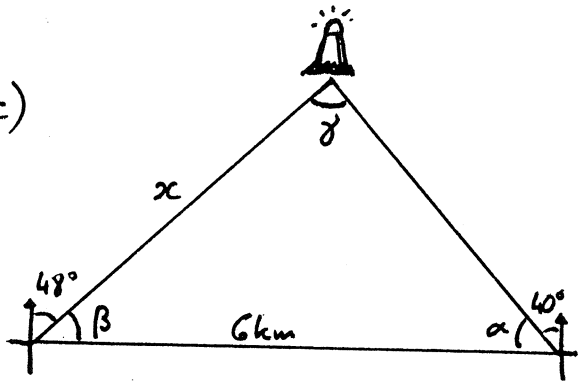
✓ hyperbola

b)



✓ boundaries
✓ dotted/full
✓ lines/curves/shading

c)



$\alpha = 50°$ (adjacent angles)

$\beta = 42°$ (adjacent angles)

$\therefore \gamma = 88°$ (angle sum of triangle

$\therefore \frac{x}{\sin\alpha} = \frac{6}{\sin\gamma}$ ✓ $\therefore x = \frac{6 \sin 58°}{\sin 88°}$

$= 4.60$ km ✓ (2 d.p.)

d) $\cot 2\alpha = -1$: let $u = 2\alpha$

$\therefore \cot u = -1$, $0° \leq u \leq 720°$

$\tan u = -1$ (Q2 and Q4) ✓

$u = 135°$	$\alpha = 67.5°$
or $u = 315°$	$\therefore$ or $\alpha = 157.5°$
or $u = 495°$	or $\alpha = 247.5°$
or $u = 675°$	or $\alpha = 337.5°$

✓ 4 solutions

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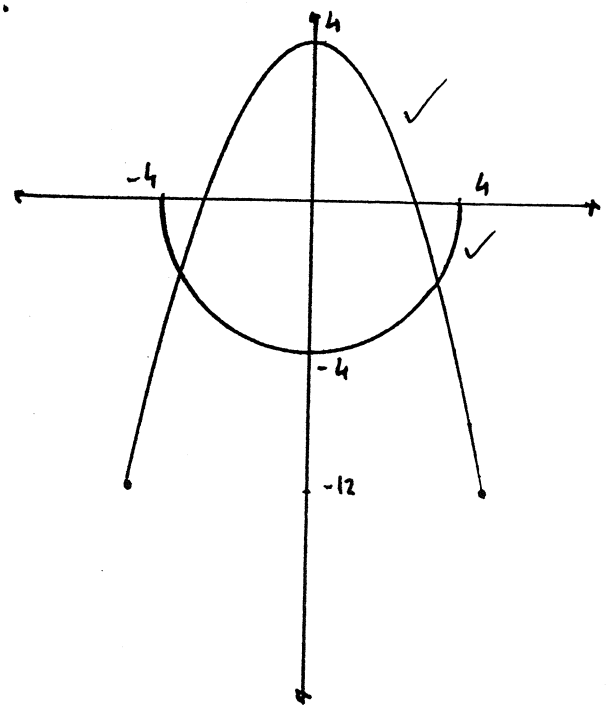
Q6. cont.

e) i)

ii)



iii) $(4-x^2) = -\sqrt{16-x^2}$

$(4-x^2)^2 = 16-x^2$

$16-8x^2+x^4 = 16-x^2$

$x^4-7x^2 = 0$

$x^2(x^2-7) = 0$

$\therefore \quad x^2 = 0 \quad$ or $\quad x^2 = 7$

$x = 0 \qquad x = \pm\sqrt{7}$ ✓

not a solution ✓ (see graph) — two solutions only

iv) from graph: $-\sqrt{7} \leq x \leq \sqrt{7}$ ✓

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