



2012 Half-Yearly Examination

FORM V

MATHEMATICS 2 UNIT

Wednesday 16th May 2012

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 85 Marks

- All questions may be attempted.

Section I — 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II — 75 Marks

- Questions 11–15 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your name and master on this question paper and submit it with your answers.

5P: SG

5Q: BDD

5R: RCF

Checklist

- SGS booklets — 5 per boy
- Multiple choice answer sheet
- Candidature — 49 boys

Examiner

RCF

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is the midpoint of the interval joining $A(-2, 5)$ and $B(4, -7)$?

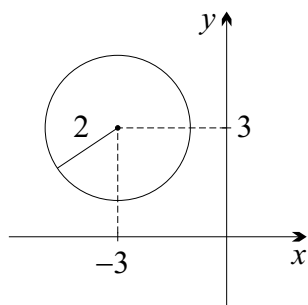
- (A) $(1, 1)$
- (B) $(1, -1)$
- (C) $(-1, 1)$
- (D) $(-1, -1)$

QUESTION TWO

Which of the following is the exact value of $\sin 45^\circ$?

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) $\frac{1}{\sqrt{3}}$

QUESTION THREE



Which of the following is the equation of the circle graphed above ?

- (A) $(x - 3)^2 + (y + 3)^2 = 4$
- (B) $(x + 3)^2 + (y - 3)^2 = 2$
- (C) $(x + 3)^2 + (y - 3)^2 = 4$
- (D) $(x - 3)^2 + (y + 3)^2 = 2$

QUESTION FOUR

Which of the following lines is perpendicular to the line $y = 2x + 4$?

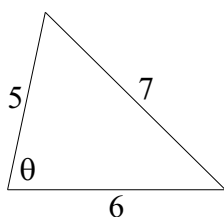
- (A) $y = 2x - 1$
- (B) $y = \frac{1}{2}x + 4$
- (C) $y = -2x - 1$
- (D) $y = -\frac{x}{2} - 4$

QUESTION FIVE

Which of the following is the correct expansion of $(5 - 2x)(5 + x)$?

- (A) $25 - 3x - 2x^2$
- (B) $2x^2 - 5x + 25$
- (C) $-2x^2 - 5x + 25$
- (D) $25 + 5x - 2x^2$

QUESTION SIX



In the diagram above, which of the following is a correct expression for $\cos \theta$?

- (A) $\frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$
- (B) $\frac{5^2 + 7^2 - 6^2}{2 \times 7 \times 6}$
- (C) $\frac{5^2 + 6^2 + 7^2}{2 \times 5 \times 6}$
- (D) $\frac{5^2 + 6^2 - 7^2}{2 \times 7 \times 5}$

QUESTION SEVEN

Which of the following is a general form of the line with gradient 2 and y -intercept 4?

- (A) $2y + x - 4 = 0$
- (B) $y - 2x - 4 = 0$
- (C) $2x - y - 4 = 0$
- (D) $2y - x + 4 = 0$

QUESTION EIGHT

Which of the following is the correct factorisation of $8x^3 - y^3$?

- (A) $(2x - y)(4x^2 - 2xy + y^2)$
- (B) $(2x + y)(4x^2 - 2xy + y^2)$
- (C) $(2x - y)(4x^2 + xy + y^2)$
- (D) $(2x - y)(4x^2 + 2xy + y^2)$

QUESTION NINE

Which of the following is equivalent to $\cos^2 \theta$?

- (A) $1 + \sin^2 \theta$
- (B) $1 - \sin^2 \theta$
- (C) $1 + \tan^2 \theta$
- (D) $\sin^2 \theta - 1$

QUESTION TEN

Which of the following functions has even symmetry?

- (A) $y = \sin x$
- (B) $y = x^2 - 2x$
- (C) $y = \frac{1}{x + 1}$
- (D) $y = \sqrt{16 - x^2}$

_____ End of Section I _____

Exam continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet.

(a) Fully simplify the following:

(i) $2x^2 + 3x - x^2$

(ii) $\sqrt{8} + \sqrt{2}$

(iii) $(a - 3)^2$

(b) Factorise:

(i) $x^2 - 4$

(ii) $x^2 - 4x$

(iii) $x^2 - 4x - 5$

(c) (i) Sketch the graph of $y = \sin x$, for $0^\circ \leq x \leq 360^\circ$.

(ii) State the range of $y = \sin x$.

(iii) Use your graph to solve $\sin x = 0$, for $0^\circ \leq x \leq 360^\circ$.

(d) Rationalise the denominator of $\frac{2 + \sqrt{3}}{\sqrt{2}}$.

(e) Consider the points $A(6, 3)$ and $B(9, 4)$.

(i) Find the distance AB .

(ii) Find the gradient of the line AB .

(iii) Hence find the equation of the line AB . Give your answer in gradient-intercept form.

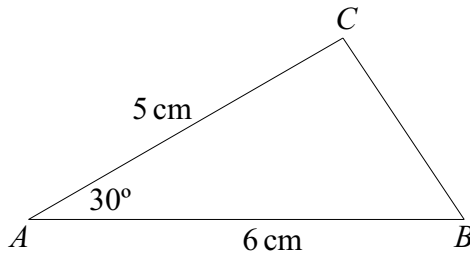
QUESTION TWELVE (15 marks) Use a separate writing booklet.

(a) Solve the following equations.

(i) $(x - 3)(x + 2) = 0$

(ii) $|x - 1| = 5$

(b)



Find the area of triangle ABC shown above.

(c) If $f(x) = \frac{1 - x}{x + 1}$, find $f(-3)$.

(d) Consider the parabola with equation $y = x^2 - 3x - 10$.

(i) Find the y -intercept.

(ii) Find the x -intercepts.

(iii) Find the equation of the axis of symmetry.

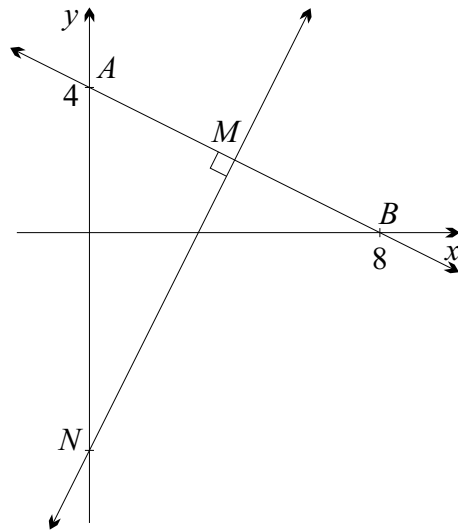
(iv) Find the co-ordinates of the vertex.

(v) Sketch the graph of $y = x^2 - 3x - 10$, clearly marking all the above features.

(vi) Hence solve the inequation $x^2 - 3x - 10 \leq 0$.

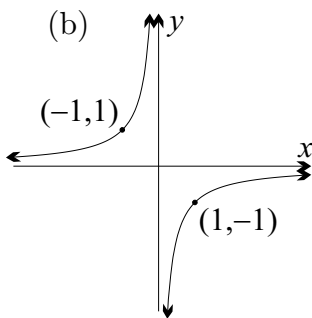
QUESTION THIRTEEN (15 marks) Use a separate writing booklet.

(a)

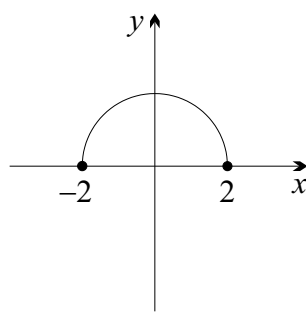


- (i) Show that the gradient of AB is $-\frac{1}{2}$.
- (ii) Find the co-ordinates of M , the midpoint of AB .
- (iii) Hence show that the equation of the perpendicular bisector of AB (that is, line MN) is $y = 2x - 6$.
- (iv) Find the co-ordinates of point N , where MN intersects the y -axis.
- (v) Find the area of $\triangle AMN$.
- (vi) Hence, or otherwise, find the area of $\triangle MBN$, giving reasons for your answer.

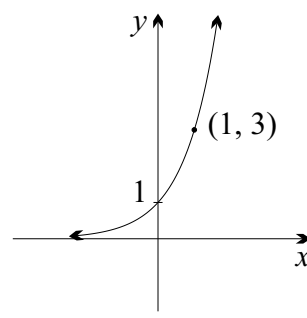
(b)



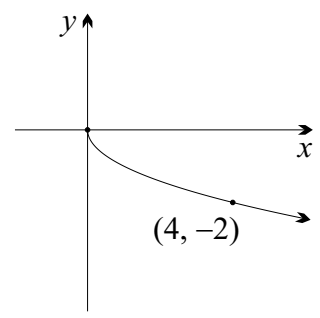
Graph A



Graph B



Graph C



Graph D

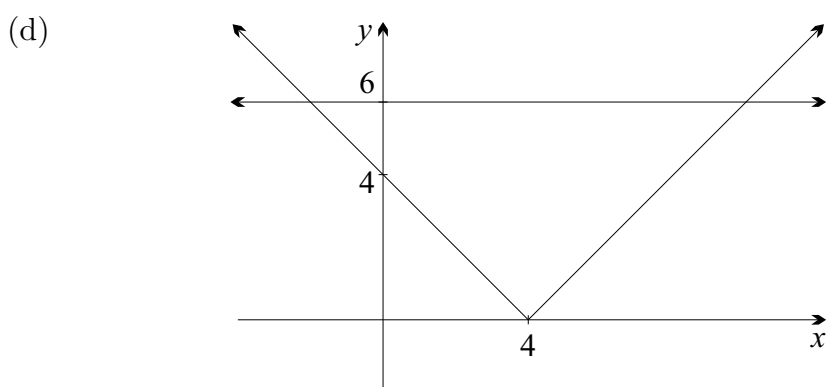
- (i) For each of the four graphs above, write down the equation of the function.
- (ii) State the domain of Graph B.
- (iii) State the range of Graph D.
- (iv) Which graph has odd symmetry?

QUESTION FOURTEEN (15 marks) Use a separate writing booklet.

- (a) The graph of $y = 2^x$ is reflected in the y -axis. Write down the equation of this reflected graph.
- (b) Find the perpendicular distance from the point $(-2, 1)$ to the line $4x + 3y + 1 = 0$.
- (c) (i) Sketch the graph of $2x + 3y = 6$ by finding the x and y intercepts.
(ii) On your graph, shade the region where $2x + 3y \leq 6$. Show clear working for your choice of region.
- (d) If $\sin \theta = \frac{5}{13}$ and θ is obtuse, find $\cos \theta$.
- (e) Solve the following equations, giving your answers as exact values or correct to the nearest degree:
- (i) $\cos x = -\frac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$.
- (ii) $\cot x = 2$, for $-180^\circ \leq x \leq 180^\circ$.
- (iii) $\sin(x + 60^\circ) = \frac{1}{\sqrt{2}}$, for $0^\circ \leq x \leq 360^\circ$.

QUESTION FIFTEEN (15 marks) Use a separate writing booklet.

- (a) Determine algebraically whether the function $f(x) = \frac{x}{(2-x)(2+x)}$ is odd, even or neither.
- (b) (i) Write down the exact values of $\sin 60^\circ$ and $\cos 60^\circ$.
- (ii) Hence find the exact value of $\frac{\cos 60^\circ + 1}{\sin 60^\circ - 1}$ in simplest form.
- (c) Simplify $\tan x \operatorname{cosec}^2 x - \tan x$.



The diagram above shows the graphs of $y = |x - 4|$ and $y = 6$.

- (i) Find the x co-ordinates of their two points of intersection.
- (ii) Hence, or otherwise, solve $|x - 4| > 6$.
- (e) (i) Prove the identity $\frac{\sec^2 x - 1}{(1 - \cos x)(1 + \cos x)} = \sec^2 x$.
- (ii) Hence solve the equation $\frac{\sec^2 x - 1}{(1 - \cos x)(1 + \cos x)} = 4$, for $0^\circ \leq x \leq 360^\circ$.

_____ End of Section II _____

END OF EXAMINATION

Tear-off pages follow ...

B L A N K P A G E



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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

NAME:

CLASS: MASTER:

Question One

A B C D

Question Two

A B C D

Question Three

A B C D

Question Four

A B C D

Question Five

A B C D

Question Six

A B C D

Question Seven

A B C D

Question Eight

A B C D

Question Nine

A B C D

Question Ten

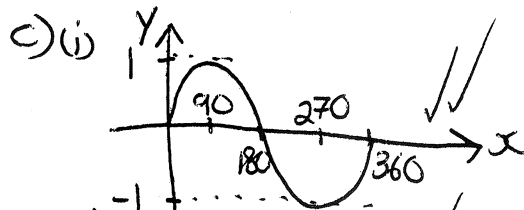
A B C D

FORM V HALF YEARLY 2012

- ① $(1, -1)$ B ✓ ② $\sin 45^\circ = \frac{1}{\sqrt{2}}$ C ✓
 ③ $(x+3)^2 + (y-3)^2 = 4$ C ✓ ④ $m_1 = 2$ $m_2 = \frac{1}{2}$ D ✓
 ⑤ $25 - 5x - 2x^2$ C ✓ ⑥ $\frac{5^2 + 6^2 - 7^2}{2 \times 5 \times 6}$ A ✓
 ⑦ $y = 2x + 4$
 $2x - y + 4 = 0$ B ✓
 $y - 2x - 4 = 0$
 ⑧ $(2x - y)(4x^2 + 2xy + y^2)$ D ✓
 ⑨ $\cos^2 \theta = 1 - \sin^2 \theta$ B ✓ ⑩ D ✓

Section II

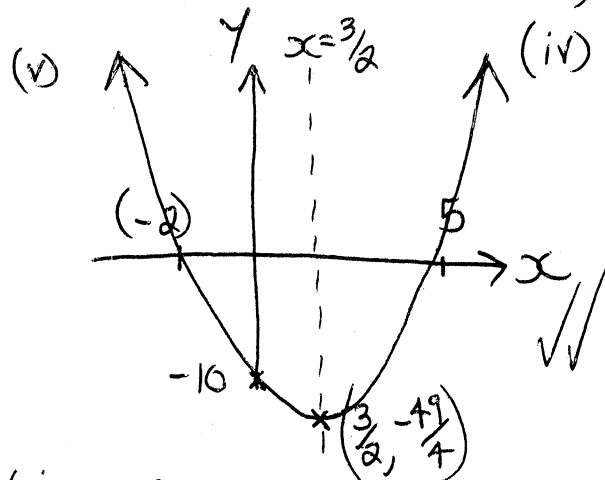
- ⑪ a) (i) $x^2 + 3x$ ✓
 (ii) $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$ ✓
 (iii) $a^2 - 6a + 9$ ✓
 b) (i) $x^2 - 4 = (x+2)(x-2)$ ✓
 (ii) $x^2 - 4x = x(x-4)$ ✓
 (iii) $x^2 - 4x - 5 = (x-5)(x+1)$ ✓



- (ii) $-1 \leq y \leq 1$ ✓
 (iii) $0^\circ, 180^\circ, 360^\circ$ ✓
 d) $\frac{2 + \sqrt{3}}{\sqrt{2}} \left(\frac{x\sqrt{2}}{x\sqrt{2}} \right) = \frac{2\sqrt{2} + \sqrt{6}}{2}$ ✓
 (or $\sqrt{2} + \frac{1}{2}\sqrt{6}$) ✓
 e) A(6,3) B(9,4)
 (i) $AB = \sqrt{3^2 + 1^2} = \sqrt{10}$ ✓
 (ii) $m = \frac{4-3}{9-6} = \frac{1}{3}$ ✓
 (iii) $y - 3 = \frac{1}{3}(x - 6)$ ✓
 $y = \frac{1}{3}x + 1$ ✓

- ⑫ a) (i) $x = 3$ OR $x = (-2)$ ✓
 (ii) $x - 1 = 5$ OR $x - 1 = -5$
 $x = 6$ ✓ $x = (-4)$ ✓
 b) $A = \frac{1}{2} \times 5 \times (x \sin 30^\circ)$
 $= 15 \times \frac{1}{2}$
 $= 7\frac{1}{2} \text{ cm}^2$ ✓

- c) $f(-3) = \frac{1 - (-3)}{(-3) + 1}$ ✓
 $= \frac{4}{-2}$ ✓
 $= (-2)$ ✓
 d) $y = x^2 - 3x - 10$
 (i) $(0, -10)$ ✓
 (ii) $y = (x-5)(x+2)$ ✓
 $(5, 0) \neq (-2, 0)$ ✓
 (iii) $x = \frac{3}{2}$ ✓



- (vi) $x^2 - 3x - 10 \leq 0$
 $(-2) \leq x \leq 5$ ✓ [OR correct interpretation from their part (v)]

NO MARK FOR $\frac{3}{2}$ $\frac{7}{2}$

⑬ a) i) A(0,4) B(8,0)

$$m = \frac{0-4}{8-0} = -\frac{4}{8} = -\frac{1}{2}$$

(ii) M(4,2)

(iii) $m_{\perp} = 2$ ✓ $y-2 = 2(x-4)$ ✓ show

(iv) Sub $x=0$ $y=2x-6$
 $y = -6$
 N(0,-6) ✓

(v) Area = $\frac{1}{2} \times AN \times 4$
 $= \frac{1}{2} \times 10 \times 4$
 $= 20 u^2$ ✓

(vi) $|\Delta MBN| = |\Delta ANB| - |\Delta AMN|$
 $= \frac{1}{2} \times 10 \times 8 - 20$
 $= 20 u^2$ ✓

(or $AB = 4\sqrt{5}$
 $MN = 4\sqrt{5}$
 $\frac{1}{2} \times AB \times MN = 20 u^2$)

(or else NM is perp bisector of AB hence ΔANB is isosceles
 $\therefore |\Delta NMB| = |\Delta ANM|$)

b) Graph A $y = -\frac{1}{2}x$ ✓

B $y = \sqrt{4-x^2}$ ✓

C $y = 3^x$ ✓

D $y = -\sqrt{x}$ ✓

(ii) $-2 \leq x \leq 2$ ✓

(iii) $y \leq 0$ ✓

(iv) Graph A ✓

⑭ a) Replace x with $(-x)$ i.e. $y = 2^{-x}$ ✓

b) (-2,1) $4x+3y+1=0$

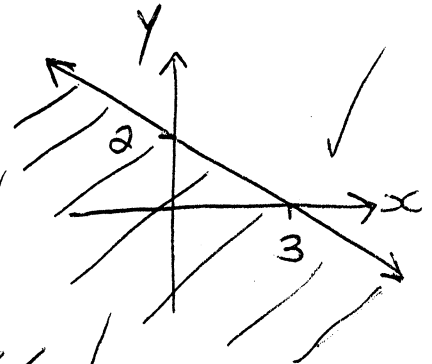
$$p = \frac{|4x(-2)+3y(1)+1|}{\sqrt{4^2+3^2}}$$

$$= \frac{|-8+3+1|}{\sqrt{25}}$$

$$= \frac{4}{5} u$$

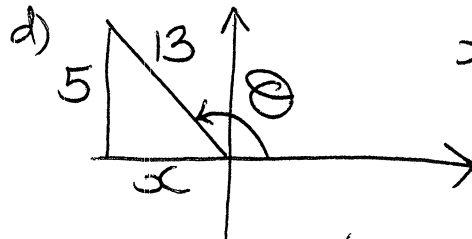
c) (i) $2x+3y=6$

$x=0$ $y=2$ (0,2) ✓
 $y=0$ $x=3$ (3,0) ✓



(ii) Test (0,0)

LHS $2 \times 0 + 3 \times 0 = 0 < 6$ ✓

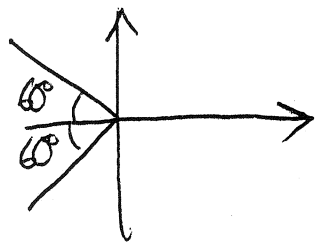


$x=12$ (5,12,13) ✓

$\therefore \cos \theta = \left(-\frac{12}{13}\right)$ ✓

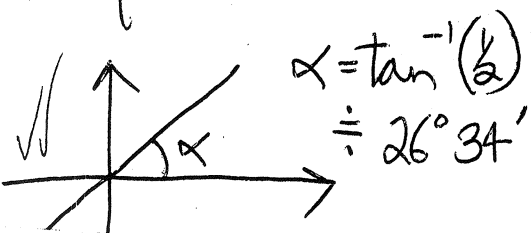
e) i) $\cos x = (-\frac{1}{2})$

$x = 120^\circ, 240^\circ$



ii) $\cot x = 2$
 $\tan x = \frac{1}{2}$

$x = 27^\circ, -153^\circ$



iii) $\sin(x+60) = \frac{1}{\sqrt{2}}$

Let $x+60 = u$

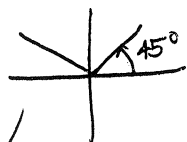
$\sin u = \frac{1}{\sqrt{2}}$

$u = 135^\circ, 405^\circ$

$x+60 = 135, 405$

$x = 75^\circ, 345^\circ$

$0^\circ \leq x \leq 360^\circ$
 $60^\circ \leq (x+60) \leq 420^\circ$



15) a) $f(x) = \frac{x}{(2-x)(2+x)}$

$f(-x) = \frac{-x}{(2+x)(2-x)}$

$= -x$
 $(2-x)(2+x)$

$= -f(x)$ ∴ Odd Function

b) i) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$

ii) $\frac{\frac{1}{2} + 1}{\frac{\sqrt{3}}{2} - 1} \times 2 = \frac{1+2}{\sqrt{3}-2} \times (\sqrt{3}+2)$

$= \frac{3\sqrt{3}+6}{3-4}$

$= -3\sqrt{3}-6$

c) $\tan x \operatorname{cosec}^2 x - \tan x$
 $= \tan x (\operatorname{cosec}^2 x - 1)$
 $= \tan x (1 + \cot^2 x - 1)$
 $= \tan x \times \cot^2 x$
 $= \cot x$

d) i) $x-4=6$ or $-(x-4)=6$
 $x=10$ or $x=-2$

ii) $(-2) > x$ or $x > 10$

e) i) LHS $\frac{\sec^2 x - 1}{(1-\cos x)(1+\cos x)} = \frac{\tan^2 x}{1-\cos^2 x}$
 $= \frac{\sin^2 x / \cos^2 x}{\sin^2 x}$
 $= \frac{1}{\cos^2 x}$ / SHOW
 $= \sec^2 x = \text{RHS}$

ii) $\sec^2 x = 4$

$\sec x = 2$ or $\sec x = -2$

$\cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{2}$

$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$