



2013 Half-Yearly Examination

FORM V

MATHEMATICS 2 UNIT

Wednesday 15th May 2013

General Instructions

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 85 Marks

- All questions may be attempted.

Section I — 10 Marks

- Questions 1–10 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II — 75 Marks

- Questions 11–15 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eleven.
- Write your name and master on this question paper and submit it with your answers.

5P: MLS

5Q: GMC

5R: BR

Checklist

- SGS booklets — 5 per boy
- Multiple choice answer sheet
- Candidature — 32 boys

Examiner

MLS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

What is 12.0957 correct to three significant figures?

- (A) 12.095
- (B) 12.096
- (C) 12.1
- (D) 12.0

QUESTION TWO

Which of the following is the solution of the quadratic equation $(7 - 2x)(3 + x) = 0$?

- (A) $x = 3$ or $x = -\frac{7}{2}$
- (B) $x = 3$ or $x = -\frac{2}{7}$
- (C) $x = -3$ or $x = \frac{2}{7}$
- (D) $x = -3$ or $x = \frac{7}{2}$

QUESTION THREE

Which of the following lines is perpendicular to the line $x - y + 3 = 0$?

- (A) $x + y + 3 = 0$
- (B) $-x + y + 3 = 0$
- (C) $-x + y - 3 = 0$
- (D) $x - y - 3 = 0$

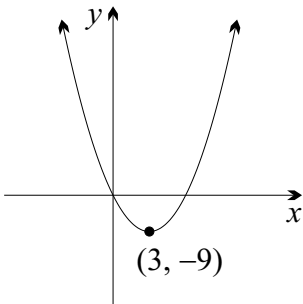
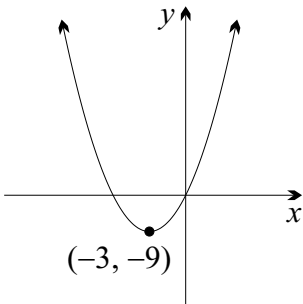
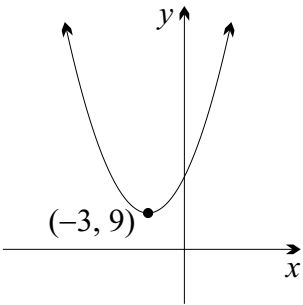
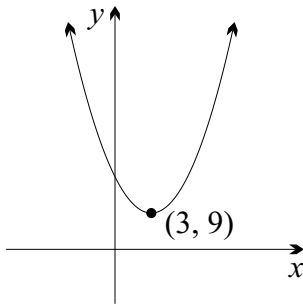
QUESTION FOUR

Which of the following is the equation of a circle graphed with centre $(-1, 1)$ and radius 3?

- (A) $x^2 - 2x + y^2 + y = 11$
- (B) $x^2 + 2x + y^2 - 2y = 11$
- (C) $x^2 - 2x + y^2 + 2y = 7$
- (D) $x^2 + 2x + y^2 - 2y = 7$

QUESTION FIVE

Which of the following could represent the graph of $y = (x - 3)^2 - 9$?

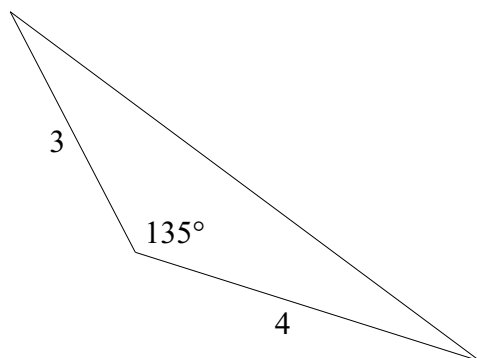
- (A) 
- (B) 
- (C) 
- (D) 

QUESTION SIX

Which of the following is the natural domain of $\frac{3}{\sqrt{2-x}}$?

- (A) $x \leq -2$
- (B) $x \leq 2$
- (C) $x > 2$
- (D) $x < 2$

QUESTION SEVEN



Which of the following is the exact area of the triangle above?

- (A) $6\sqrt{2}$
- (B) $\frac{3}{\sqrt{2}}$
- (C) $3\sqrt{2}$
- (D) $\frac{12}{\sqrt{2}}$

QUESTION EIGHT

Which of the following functions is an odd function?

- (A) $y = x(x - 1)(x + 1)$
- (B) $y = x^2 - 2x$
- (C) $y = \frac{1}{x + 1}$
- (D) $y = \sqrt{16 - x^2}$

QUESTION NINE

Which of the following is equivalent to $\sin^2 \theta$?

- (A) $1 + \cos^2 \theta$
- (B) $1 - \cos^2 \theta$
- (C) $1 - \tan^2 \theta$
- (D) $\cos^2 \theta - 1$

QUESTION TEN

Which of the following is equal to $\frac{2\sqrt{3} + 3\sqrt{2}}{3}$?

(A) $\frac{2}{2\sqrt{3} - 3\sqrt{2}}$

(B) $\frac{-2}{2\sqrt{3} + 3\sqrt{2}}$

(C) $\frac{-2}{2\sqrt{3} - 3\sqrt{2}}$

(D) $\frac{2}{2\sqrt{3} + 3\sqrt{2}}$

————— End of Section I —————

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION ELEVEN (15 marks) Use a separate writing booklet. **Marks**

(a) Factorise the following

(i) $x^2 - 25$

1

(ii) $x^3 - 8$

1

(iii) $x^2 + 2x - 35$

1

(b) Solve $|5x - 3| < 7$.

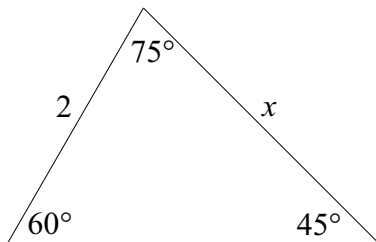
2

(c) Solve $(x - 1)^2 = 16$.

2

(d) (i) Write down the exact values of $\sin 60^\circ$ and $\sin 45^\circ$.

2



(ii) Use the sine rule to find the exact value of x in the above triangle.

2

(e) (i) Factorise $x^2 - 3x + 2$

1

(ii) Sketch the parabola $y = x^2 - 3x + 2$, showing clearly the x intercepts.

2

(iii) Solve $x^2 - 3x + 2 < 0$.

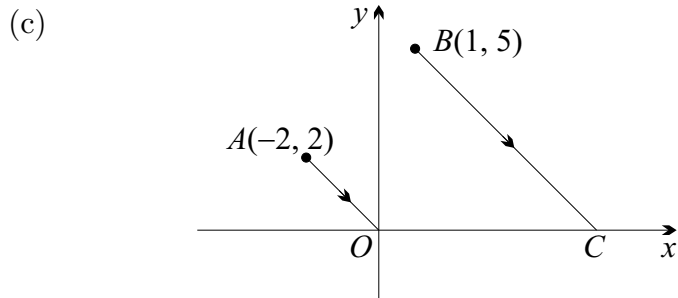
1

QUESTION TWELVE (15 marks) Use a separate writing booklet.

Marks

(a) Factorise $2x^2 - 13x + 15$. 2

(b) Solve $\cos x = \frac{\sqrt{3}}{2}$ for $0^\circ \leq x \leq 360^\circ$. 2



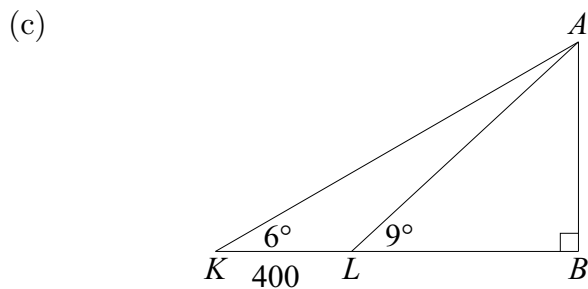
In the diagram above $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O , A and B are $(0, 0)$, $(-2, 2)$ and $(1, 5)$ respectively. The point C lies on the x -axis.

- (i) Find the length of OA . Give the length in exact simplified form. 2
 - (ii) Find the gradient of OA . 1
 - (iii) Find the size of $\angle AOC$. 1
 - (iv) Show that the equation of the line BC is $x + y - 6 = 0$. 2
 - (v) Find the coordinates of C . 1
 - (vi) Find the perpendicular distance between the lines OA and BC . Give your answer in exact form with a rational denominator. 2
 - (vii) Calculate the area of the trapezium $OABC$. 2
- Hint: The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$.

QUESTION THIRTEEN (15 marks) Use a separate writing booklet. **Marks**

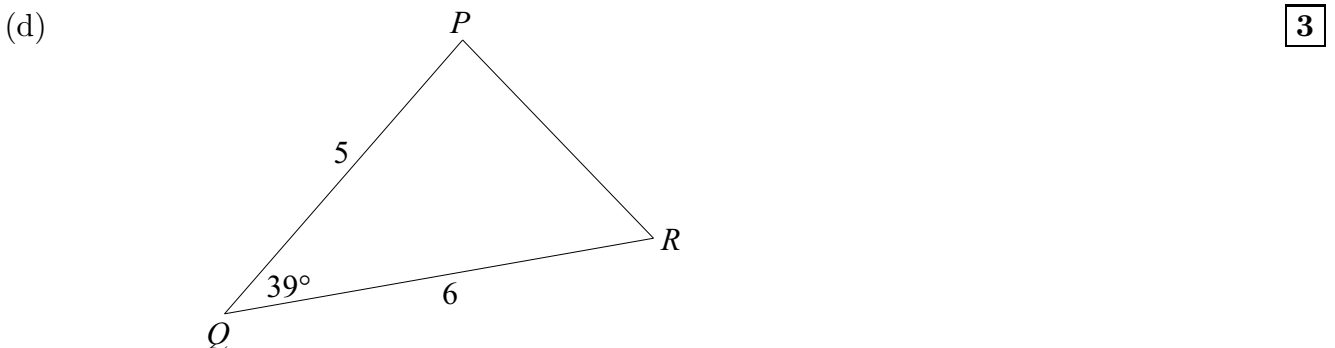
(a) The line $6x - ky = 4$ passes through the point $(3, 2)$. Find the value of k . **2**

- (b) Consider the circle with equation $(x - 4)^2 + y^2 = 25$.
- (i) Write down the length of the radius. **1**
 - (ii) Write down the coordinates of the centre. **1**
 - (iii) Find the y -intercepts. **1**
 - (iv) Find the x -intercepts. **1**
 - (v) Shade the region in the number plane where $(x - 4)^2 + y^2 \geq 25$. **1**



Kate observes a cliff AB from the royal yacht at position K . She then sails 400 metres closer to the cliff to position L . The angle of elevation of the cliff-top from K is 6° and from L is 9° .

- (i) Write down the size of $\angle KAL$. **1**
- (ii) Use the sine rule to calculate AL to the nearest metre. **2**
- (iii) Find AB to the nearest metre. **2**



Find the length of PR using the cosine rule and give your answer correct to two decimal places.

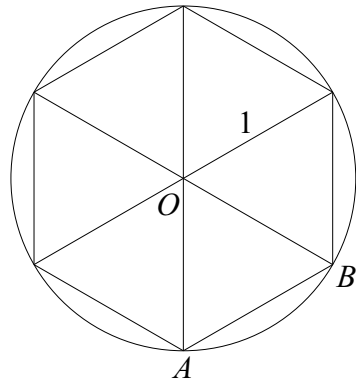
QUESTION FOURTEEN (15 marks) Use a separate writing booklet. **Marks**

- (a) (i) Sketch the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes. Mark clearly any x and y intercepts. **3**
- (ii) Find the coordinates of the point of intersection of these two graphs. **2**
- (b) Given $f(x) = \frac{1}{x^2 + 1}$, find $f(-x)$ and hence determine whether $f(x)$ is even, odd or neither. **2**
- (c) Show that $2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$. **2**
- (d) If $\sin \alpha = -\frac{2\sqrt{6}}{7}$ and $-90^\circ < \alpha < 90^\circ$, find the values of $\cos \alpha$ and $\tan \alpha$. **3**
- (e) The lines $2x - y - 4 = 0$ and $x + 2y + 1 = 0$ intersect at P .
- (i) Write down the general equation of a line through P . **1**
- (ii) Hence find the equation of the line through P and $(-1, -3)$. **2**

QUESTION FIFTEEN (15 marks) Use a separate writing booklet. **Marks**

- (a) Solve $\tan^2 \theta = \frac{1}{3}$ for $0 \leq \theta \leq 360^\circ$. **3**
- (b) (i) Sketch the graphs of $y = \sin x$ and $y = \cos x$ for $-180^\circ \leq x \leq 180^\circ$ on the same diagram. **3**
- (ii) Use your diagram to determine the number of solutions to the equation $\sin x = \cos x$ for $-180^\circ \leq x \leq 180^\circ$. **1**
- (iii) Write down these solutions. **1**

(c)



A regular hexagon is drawn inside a circle with centre O so that its vertices lie on the circumference, as shown in the diagram. The circle has radius 1 cm.

(i) Prove that $\triangle OAB$ is equilateral.

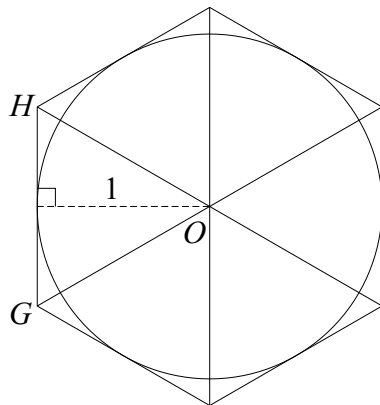
2

(ii) Find the area of $\triangle OAB$ and hence find the area of this hexagon. Leave your answer in surd form.

2

(iii)

2



Another regular hexagon is drawn outside the circle as shown. The altitude of $\triangle OGH$ is 1 cm.

Find the area of $\triangle OGH$ and hence find the area of this outer hexagon. Leave your answer in surd form.

(iv) By considering the results in (ii) and (iii), show that $\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$.

1

————— End of Section II —————

END OF EXAMINATION

Tear-off pages follow ...



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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

NAME:

CLASS: MASTER:

Question One

A B C D

Question Two

A B C D

Question Three

A B C D

Question Four

A B C D

Question Five

A B C D

Question Six

A B C D

Question Seven

A B C D

Question Eight

A B C D

Question Nine

A B C D

Question Ten

A B C D

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Q1. C

Q2. D

Q3. A

Q4. D

Q5. A

Q6. D

Q7. C

Q8. A

Q9. B

Q10. C

Q11.

(a) (i) $x^2 - 25 = (x+5)(x-5)$ ✓

(ii) $x^3 - 8 = (x-2)(x^2 + 2x + 4)$ ✓

(iii) $x^2 + 12x - 35 = (x+7)(x-5)$ ✓

(b) $|5x - 3| < 7$

$-7 < 5x - 3 < 7$ ✓

$-4 < 5x < 10$
 $-\frac{4}{5} < x < 2$ ✓

(c) (i) $(x-1)^2 = 16$
 $x-1 = 4$ or -4 ✓
 $x = 5$ or -3 ✓

(d) (i) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ ✓

$\sin 45^\circ = \frac{1}{\sqrt{2}}$ ✓

(ii) $\frac{x}{\sin 60^\circ} = \frac{2}{\sin 45^\circ}$ ✓

$x = \frac{2}{\frac{1}{\sqrt{2}}} \times \frac{\sqrt{3}}{2}$

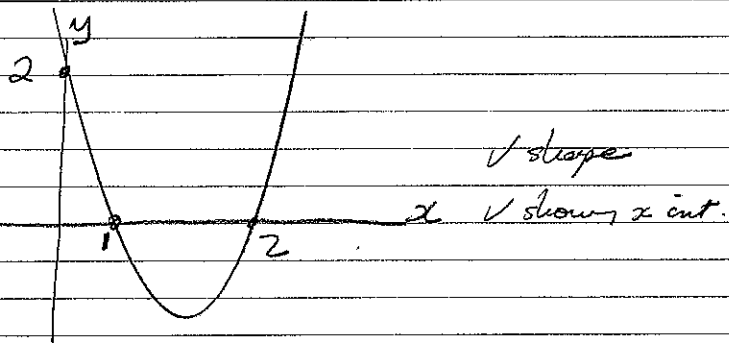
$= \frac{2 \times \sqrt{2} \times \sqrt{3}}{2}$

$= \sqrt{6}$ ✓

(e)

(i) $x^2 - 3x + 2 = (x-2)(x-1)$ ✓

(ii)



(iii)

$1 < x < 2$ ✓

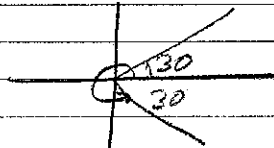
Q 12.

(a) $22x^2 - 13x + 15$
 $= 22x^2 - 3x - 10x + 15$
 $= x(22x-3) - 5(22x-3)$
 $= (x-5)(22x-3)$ ✓✓

$x \cdot 30 = -3x - 10$
 $+ -13 = -3 + -10$

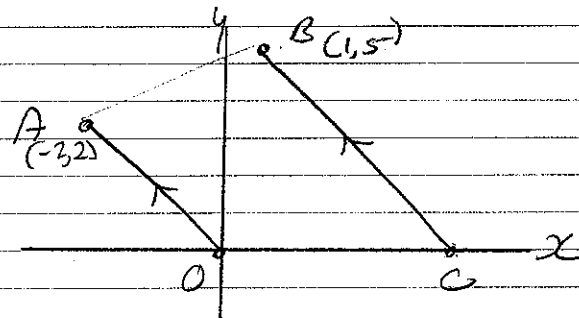
(b) $\cos x = \frac{\sqrt{3}}{2}$

related angle is 30° ✓



$x = 30^\circ$ or 330° ✓

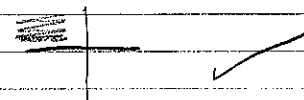
Q.



(i) $|OA| = \sqrt{4+4}$
 $= 2\sqrt{2}$ ✓

(ii) $m_{OA} = \frac{2}{-2}$
 $= -1$ ✓

(iii) $\tan \angle AOC = -1$
related angle is 45°
 $\angle AOC = 135^\circ$ ✓



(vi) $m_{BC} = -1$ ✓ $(1, 5)$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -1(x - 1)$$
 ✓

$$y - 5 = -x + 1$$

$$x + y - 6 = 0$$

(v) On the x axis, $y = 0$
so $x = 6$

and C is $(6, 0)$ ✓ accept 6.

(vi) $(-2, 2)$, $x + y - 6 = 0$

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|-2 + 2 - 6|}{\sqrt{1+1}}$$
 ✓

$$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\sqrt{2}}{2}$$

$$= 3\sqrt{2}$$
 ✓

(vii) $h = 3\sqrt{2}$, $a = 2\sqrt{2}$ need length BC.

length BC $(1, 5) - (6, 0)$

$$d = \sqrt{25 + 16}$$

$$= 5\sqrt{2}$$
 ✓

$$A = \frac{1}{2} \times 3\sqrt{2} (2\sqrt{2} + 5\sqrt{2})$$

$$= \frac{3}{2} \times \sqrt{2} \times 2\sqrt{2}$$

Q13.

(a) $6x - 6y = 4$ $(3, 2)$ ✓

$$18 - 2A = 4$$

$$-2A = -14$$

$$A = 7$$
 ✓

(b) $(x-4)^2 + y^2 = 25$

(i) $= 5$ ✓

(ii) $(4, 0)$ ✓

(iii) on y axis, $x = 0$, $16 + y^2 = 25$
 $y^2 = 9$

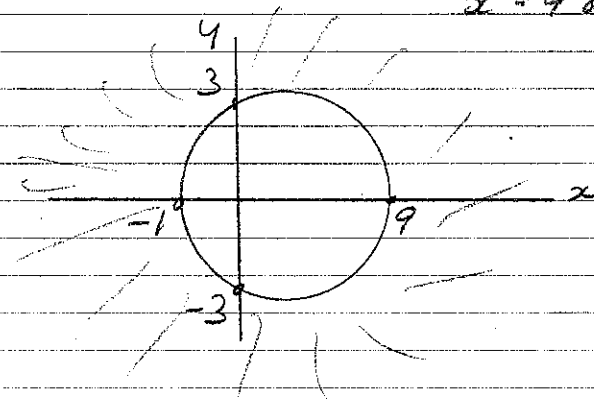
$$y = 3 \text{ and } -3$$
 ✓

(iv) on x axis, $y = 0$, $(x-4)^2 = 25$

$$x-4 = 5 \text{ or } -5$$

$$x = 9 \text{ or } -1$$
 ✓

(v)



(i) $\angle KAL = 3^\circ$ ✓

(ii) $\frac{AL}{\sin 6^\circ} = \frac{400}{\sin 3^\circ}$ ✓

$AL = \frac{400}{\sin 3^\circ} \times \sin 6^\circ$
 $\approx 799 \text{ m}$

(iii) $\sin 9^\circ = \frac{AB}{AL}$ ✓

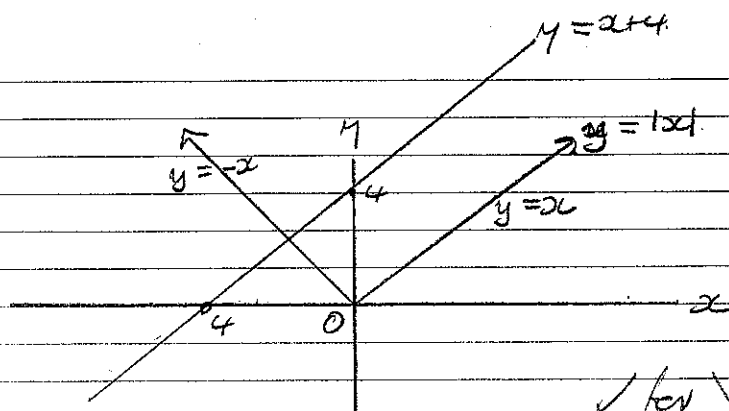
so $AB = AL \times \sin 9^\circ$
 $\approx 125 \text{ m}$ ✓

cd) $PR^2 = 5^2 + 6^2 - 2 \times 5 \times 6 \times \cos 39^\circ$ ✓
 $= 25 + 36 - 60 \cos 39^\circ$
 $= 14.37124231$ ✓
 $PR \approx 3.79$ ✓

Q14.

(a)

(i)



✓ for ✓ slope
 ✓ for line then $y=4$
 ✓ for line then $x=-4$

(ii) $y = -x$ and $y = x + 4$.

$x + 4 = -x$

$2x = -4$

$x = -2$ ✓

$y = 2$

So pt of intersection is $(-2, 2)$ ✓

(b) $f(x) = \frac{1}{x^2 + 1}$

$f(-x) = \frac{1}{(-x)^2 + 1}$ ✓

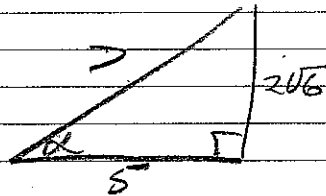
$= \frac{1}{x^2 + 1}$

$= f(x)$ so even ✓

✓ for many LHS, RHS.

$$\begin{aligned}
 \text{c) LHS} &= 2 \cos^2 \theta - 1 \\
 &= 2(1 - \sin^2 \theta) - 1 \\
 &= 2 - 2 \sin^2 \theta - 1 \\
 &= 1 - 2 \sin^2 \theta \\
 &= \text{RHS}
 \end{aligned}$$

$$\text{d) } \sin \alpha = \frac{-2\sqrt{6}}{5}$$



$$\cos \alpha = \frac{5}{7} \quad \checkmark$$

$$\tan \alpha = \frac{-2\sqrt{6}}{5} \quad \checkmark$$

✓ for correct signs

(probably 0 if just tried to find \alpha using calc).

$$\text{(e) (i) } 2x - y - 4 + k(x + 2y + 1) = 0 \quad \checkmark$$

$$\begin{aligned}
 \text{(ii) find } k: \\
 (-1, -3) \quad -2 + 3 - 4 + k(-1 - 6 + 1) &= 0 \\
 -3 - 6k &= 0 \\
 6k &= -3 \\
 k &= -\frac{1}{2} \quad \checkmark
 \end{aligned}$$

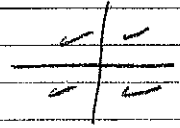
$$\begin{aligned}
 2x - y - 4 - \frac{1}{2}(x + 2y + 1) &= 0 \\
 4x - 2y - 8 - x - 2y - 1 &= 0 \\
 3x - 4y - 9 &= 0. \quad \checkmark
 \end{aligned}$$

Q15.

$$\text{(a) } \tan \theta = \frac{1}{3}$$

$$\tan \theta = \frac{1}{3} \text{ or } -\frac{1}{3}$$

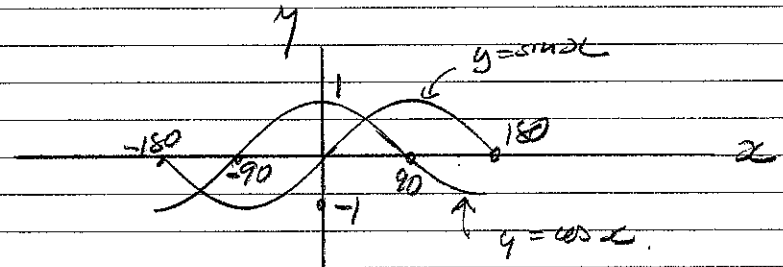
related angle is 30° ✓



$$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ \quad \checkmark \checkmark$$

(b)

(i)



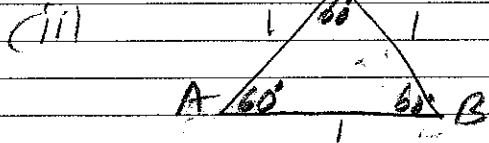
- ✓ 1 and -1
- ✓ $-150, 150^\circ$
- ✓ shapes

(ii) 2 solutions ✓

(iii) 45° and -135° ✓

(c) (i) $OA = OB$ radii of circle
 So $\triangle OAB$ is isosceles.
 $\angle AOB = 360^\circ = 60^\circ$

So $\triangle OAB$ is equilateral

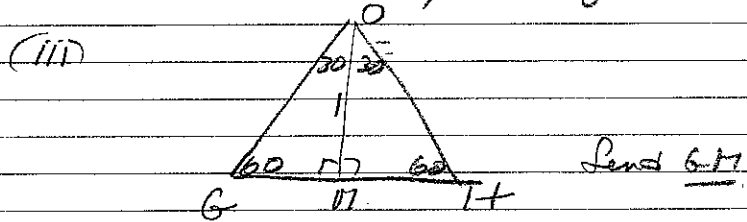


$$A = \frac{1}{2} ab \sin C$$

$$\triangle OAB = \frac{1}{2} \times 1 \times 1 \times \sin 60^\circ$$

$$= \frac{\sqrt{3}}{4}$$

So hexagon is $\frac{3\sqrt{3}}{2}$



$$\tan 60^\circ = \sqrt{3} = \frac{1}{OM} \text{ so } OM = \frac{1}{\sqrt{3}}$$

$$\text{So area } OGH = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 1$$

$$= \frac{1}{2} \text{ or } \frac{\sqrt{3}}{3}$$

∴ use pythagoras ✓

$$\text{Area of hexagon is } \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ sq units}$$

(iv) area inner < area circle < area outer.
 $\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$ ✓ for logic

$$\frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3}$$