SYDNEY GRAMMAR SCHOOL

2014 Half-Yearly Examination

FORM V
MATHEMATICS 2 UNIT

Wednesday 14th May 2014

General Instructions
- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 80 Marks
- All questions may be attempted.

Section I – 8 Marks
- Questions 1–8 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 72 Marks
- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection
- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Nine.
- Write your name and master on this question paper and submit it with your answers.

5A: BDD	5B: MLS	5C: LYL	5D: LRP
5E: PKH	5F: BR	5G: SG	5P: REJ
5Q: NL	5R: TCW		

Checklist
- SGS booklets — 6 per boy
- Multiple choice answer sheet

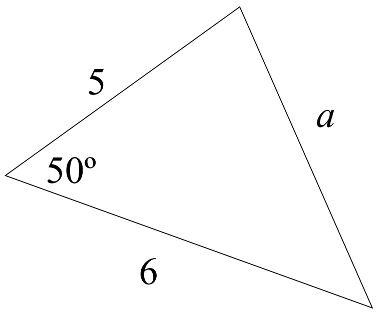
Examiner
PKH

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which statement is true for the diagram below?



(A) $a^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \cos 50°$

(B) $a^2 = 6^2 + 5^2 + 2 \times 6 \times 5 \times \cos 50°$

(C) $a^2 = 6^2 + 5^2 + 2 \times 6 \times 5 \times \sin 50°$

(D) $a^2 = 6^2 + 5^2 - 2 \times 6 \times 5 \times \sin 50°$

QUESTION TWO

What is the exact value of $\sqrt{50} - \sqrt{18}$?

(A) $\sqrt{32}$ (B) $2{\cdot}828$

(C) $2\sqrt{2}$ (D) $2\sqrt{5} - 2\sqrt{3}$

QUESTION THREE

Which of the following is the natural domain of $\sqrt{x-4}$?

(A) $x \geq 4$ (B) $x > 4$ (C) $x \leq 4$ (D) $x < 4$

QUESTION FOUR

The expression $\dfrac{K^6 + K^3}{K^3}$ can be simplified to :

(A) K^6 (B) $2K^3$ (C) $K^3 + 1$ (D) $K^2 + 1$

Exam continues next page ...

QUESTION FIVE

The solution to the equation $\tan\theta = -1$ for $0^\circ \leq \theta \leq 360^\circ$ is :

(A) $\theta = -45^\circ$ or $\theta = 45^\circ$

(B) $\theta = -45^\circ$

(C) $\theta = 135^\circ$ or $\theta = 225^\circ$

(D) $\theta = 135^\circ$ or $\theta = 315^\circ$

QUESTION SIX

What is the gradient of the line passing through $A(-1, 6)$ and $B(4, -4)$?

(A) -2

(B) $-\dfrac{1}{2}$

(C) $\dfrac{3}{2}$

(D) $-\dfrac{3}{2}$

QUESTION SEVEN

Which of the following expressions is equal to $a^3 - b^3$?

(A) $(a-b)(a^2+b^2)$

(B) $(a+b)(a^2-ab+b^2)$

(C) $(a-b)(a^2+2ab+b^2)$

(D) $(a-b)(a^2+ab+b^2)$

QUESTION EIGHT

Which of the following is equivalent to $\sin\theta$?

(A) $\cos(90^\circ + \theta)$

(B) $\cos(90^\circ - \theta)$

(C) $\dfrac{1}{\sec\theta}$

(D) $-\sin(180^\circ - \theta)$

End of Section I

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) Find $\sqrt{12\,645\,674}$ correct to 3 significant figures. **1**

(b) Find θ correct to the nearest degree if $\cos\theta = 0{\cdot}6$ and θ is acute. **1**

(c) Expand $(3x-2)^2$. **1**

(d) Find the mid-point of the interval joining $A(-1,5)$ and $B(3,-3)$. **1**

(e) Evaluate $\left|6\right| - \left|-12\right|$. **1**

(f) Factorise the following:

(i) $4x^2 - 9$ **1**

(ii) $2x^2 - 3x + 1$ **1**

(g) Write $\dfrac{2x+1}{x^2} + \dfrac{x-2}{x}$ as a single fraction in simplest form. **1**

(h) Solve the following equations:

(i) $\dfrac{4x}{3} = x + 1$ **1**

(ii) $-2(x+3) = 5(2x-3)$ **2**

(i) Solve the inequation $-3a - 2 \leq -8$. **1**

Exam continues next page ...

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) Find the distance between the points $C(2,-3)$ and $D(-5,1)$. Leave your answer as a surd. **2**

(b) Expand and simplify $(2-3\sqrt{3})^2$. **2**

(c) Rationalise the denominator of $\dfrac{4}{3-\sqrt{7}}$ and simplify. **2**

(d) Solve $x = \dfrac{2-x}{x}$. **2**

(e) Solve the quadratic inequation $x^2 - 2x - 8 \leq 0$. **2**

(f) Solve $|x+6| = 12$. **2**

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) If $g(x) = 5x - 3x^2$ find the value of $g(-2)$. **1**

(b) Sketch neat graphs of the following functions on separate axes showing any intercepts with the axes:

(i) $y = -\frac{1}{2}x + 3$ **2**

(ii) $y = 16 - x^2$ **2**

(iii) $y = \dfrac{1}{x+2}$ **2**

(iv) $y = -\sqrt{9-x^2}$ **2**

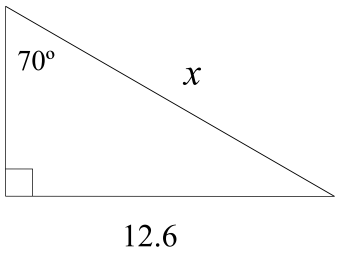
(c) Write down the natural domain and range of the function $y = 3^x$. **2**

(d) Sketch the graph of $y = |x-2|$. **1**

Exam continues overleaf ...

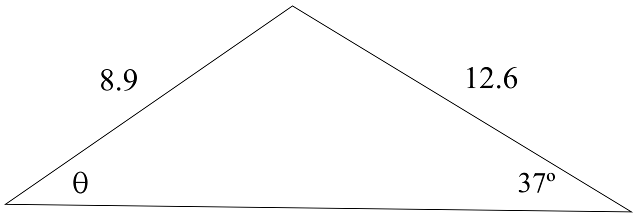
QUESTION TWELVE (12 marks) Use a separate writing booklet. **Marks**

(a) **2**



In the diagram above find x correct to two decimal places.

(b) **2**



In the diagram above find the acute angle θ correct to the nearest minute.

(c) Find the exact value of $\cos 225°$. **2**

(d) Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ for $0° \leq \theta \leq 360°$. **2**

(e) Draw a neat sketch of $y = \cos x$ for $-180° \leq x \leq 180°$. Label the intercepts with the axes. **2**

(f) Find the equation of the line through $P(-2, 4)$ which is perpendicular to $y = \frac{1}{2}x$. **2**

QUESTION THIRTEEN (12 marks) Use a separate writing booklet. **Marks**

(a) Which of the following numbers are rational? **1**

$\sqrt{7}\,, \pi\,, 0{\cdot}\dot{7},\ \sqrt{81}\,, 7^{\frac{1}{7}}$

(b) Given that $\cos\theta = -\frac{1}{4}$ and θ is obtuse, find the exact value of $\tan\theta$. **3**

(c) Prove that $\dfrac{\sec\theta\tan\theta}{1+\tan^2\theta} = \sin\theta$. **3**

(d) Find the perpendicular distance from the point $A(-1, 5)$ to the line $3x - 4y + 2 = 0$. **2**

(e) Sketch the region in the number plane which simultaneously satisfies $y \leq 3 - x$ and $y \geq x^2 - 1$. There is no need to find the points of intersection of the graphs. **3**

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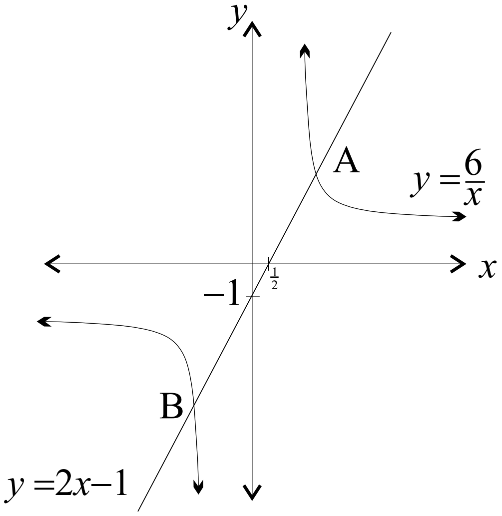
QUESTION FOURTEEN (12 marks) Use a separate writing booklet. **Marks**

(a)



In the diagram above the line $y = 2x - 1$ and the hyperbola $y = \dfrac{6}{x}$ are drawn. The points of intersection of the line and the hyperbola are labelled A and B.

(i) Use algebra to find the co-ordinates of A and B. **3**

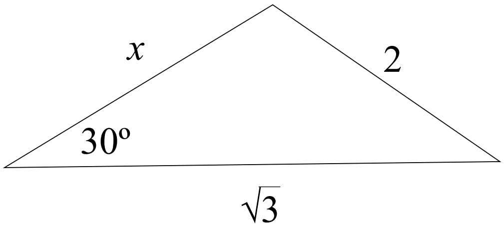
(ii) Hence solve $2x - 1 \leq \dfrac{6}{x}$. **2**

(b) Two ships leave port P. One travels 90 km on a bearing of 030° T and anchors at A. The other ship travels 40 km on a bearing of 120° T and anchors at B.

(i) Represent this situation on a neat diagram. **1**

(ii) Find the bearing of B from A to the nearest degree. **2**

(c)



In the diagram above:

(i) Show that $x^2 - 3x - 1 = 0$. **2**

(ii) Find the exact area of the triangle. **2**

End of Section II

END OF EXAMINATION

Tear-off pages follow ...

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2014
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FORM V
MATHEMATICS 2 UNIT
Wednesday 14th May 2014

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

NAME: ....

CLASS: .... MASTER: ....

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

Question Seven

A ◯ B ◯ C ◯ D ◯

Question Eight

A ◯ B ◯ C ◯ D ◯

PKH SOLUTION TO 2UNIT MATHS HALF YEARLY SGS 2014

/80

Multiple Choice

1. A 2 C 3 A 4 C 5 D /8

6 A 7 D 8 B

Q9

(a) $\sqrt{12645674} \doteqdot 3560$ ✓

(b) $\cos\theta = 0.6$

$\theta \doteqdot 53^\circ$ ✓

(c) $(3x-2)^2 = 9x^2 - 12x + 4$ ✓

(d) $M = \left(\frac{3-1}{2}, \frac{5-3}{2}\right) = (1,1)$ ✓

(e) $|6| - |-12| = 6 - 12 = -6$ ✓

(f) (i) $4x^2 - 9 = (2x-3)(2x+3)$ ✓

(ii) $2x^2 - 3x + 1 = (2x-1)(x-1)$ ✓

(g) $\frac{2x+1}{x^2} + \frac{x^2-2x}{x^2} = \frac{x^2+1}{x^2}$ ✓

(h) (i) $\frac{4x}{3} = x + 1$

$4x = 3x + 3$ ✓

$x = 3$

(ii) $-2(x+3) = 5(2x-3)$

$-2x - 6 = 10x - 15$ ✓

$x = -9$ ✓

$x = -\frac{3}{4}$

(i) $-3a - 2 \leq -8$

$-3a \leq -6$ ✓

$a \geq 2$

/12

Q10 (a) $d = \sqrt{7^2 + 4^2}$ ✓

$= \sqrt{65}$ ✓

(b) $(2 - 3\sqrt{3})^2$

$= 4 - 12\sqrt{3} + 27$ ✓

$= 31 - 12\sqrt{3}$ ✓

(c) $\dfrac{4}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \dfrac{4(3+\sqrt{7})}{9-7}$ ✓

$= 6 + 2\sqrt{7}$ ✓

(d) $x = \dfrac{2-x}{x}$

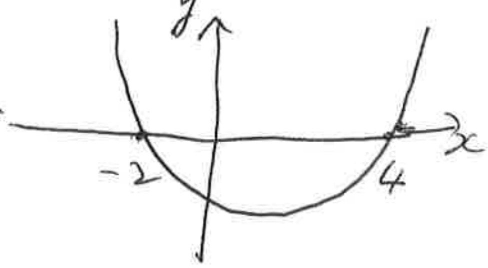
$x^2 + x - 2 = 0$ ✓

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$ ✓

(e) $x^2 - 2x - 8 \leq 0$

$(x+2)(x-4) \leq 0$ ✓



$-2 \leq x \leq 4$ ✓

(f) $|x+6| = 12$

$x + 6 = 12$ or $x + 6 = -12$

$x = 6$ ✓ $x = -18$ ✓

12

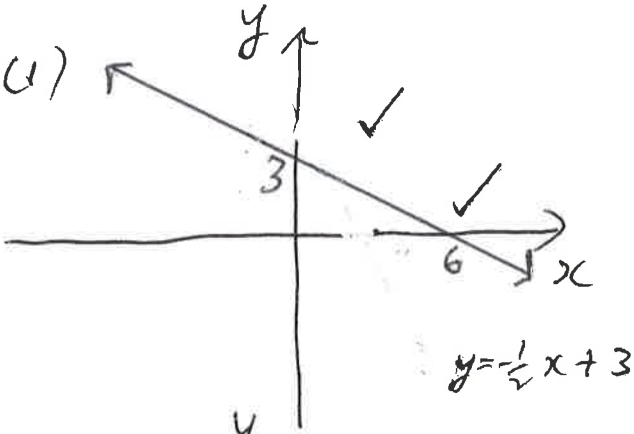
Q 11

(a) $g(-2) = 5(-2) - 3(-2)^2$

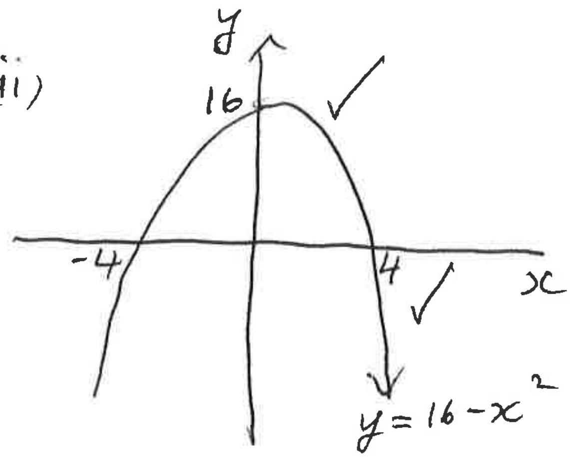
$= -10 - 12$

$= -22$

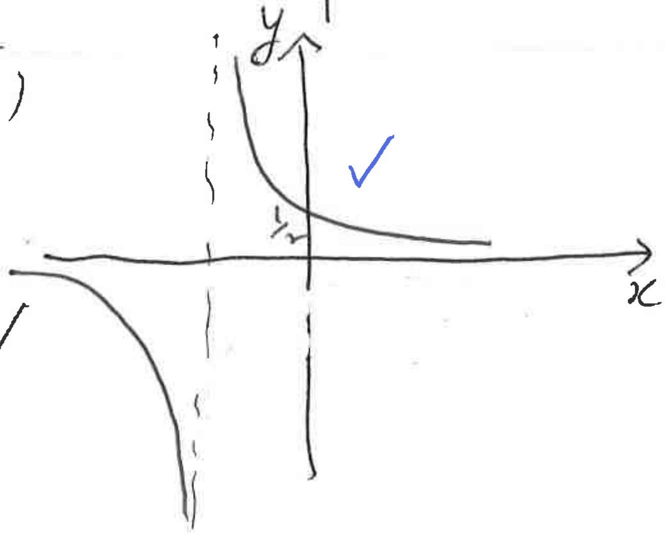
(b) (i)



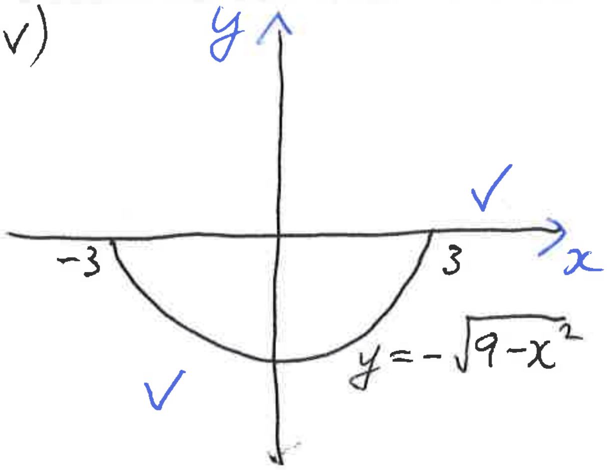
(ii)



(iii)



(iv)

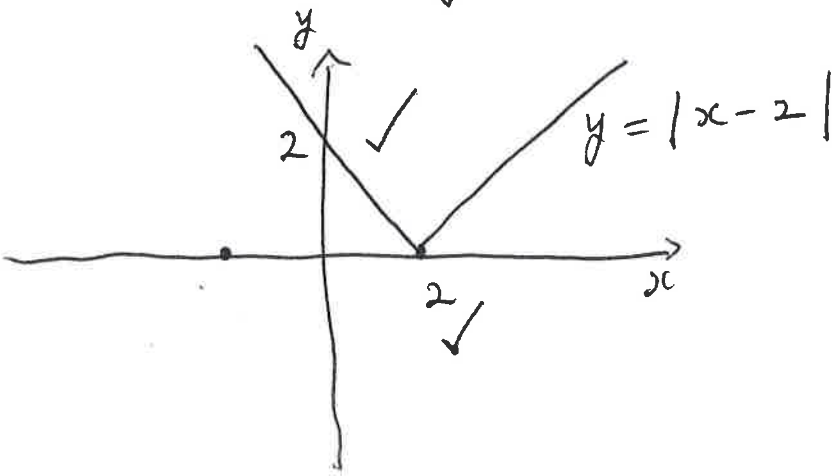


(c) $y = 3^x$

Domain : all real x

Range : $y > 0$

(d)



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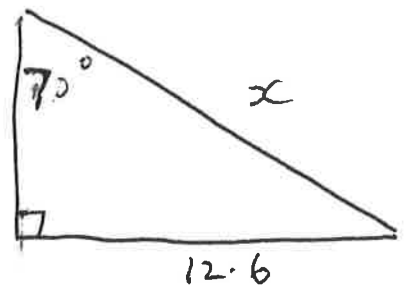
Q12 (a)



$\frac{12.6}{x} = \sin 70°$

$x = 12.6 \div \sin 70°$

$x \doteqdot 13.41$

(b) $\frac{\sin\theta}{12.6} = \frac{\sin 37°}{8.9}$

$\sin\theta = \frac{12.6 \sin 37°}{8.9}$

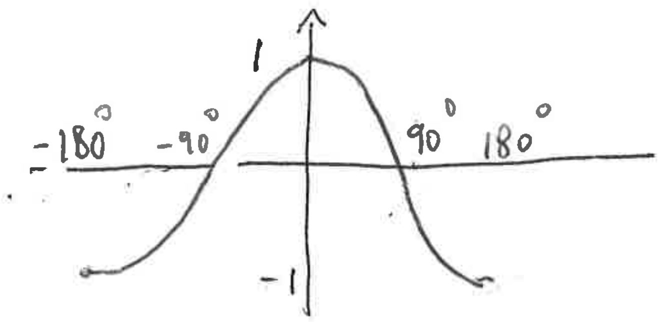
$\theta \doteqdot 58°26'$

(c) $\cos 225° = -\frac{1}{\sqrt{2}}$

(d) Solve $\sin\theta = -\frac{\sqrt{3}}{2}$ $0° \le \theta \le 360°$

$\theta = 240°$ or $300°$

(e)



(f) $y - 4 = -2(x+2)$

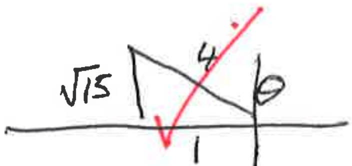
$y = -2x$

12

Q13

(a) $0.\dot{7}$, $\sqrt{81}$

(b) $\cos\theta = -\dfrac{1}{4}$



$\tan\theta = -\sqrt{15}$ (sign)

(c) $LHS = \dfrac{\sec\theta\tan\theta}{1+\tan^2\theta} = \dfrac{\sec\theta\tan\theta}{\sec^2\theta}$

$= \dfrac{\tan\theta}{\sec\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \cos\theta$

$= \sin\theta \quad = RHS$

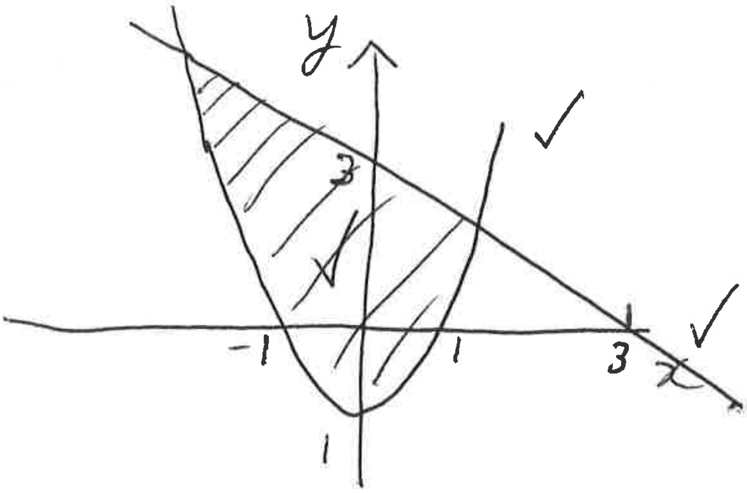
(d) $A(-1, 5) \qquad 3x - 4y + 2 = 0$

$(x_1, y_1) \qquad a \quad b \quad c$

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \dfrac{|-3 - 20 + 2|}{5} = \dfrac{21}{5}$ units

(e)



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Q14 (a) (i) Pts of ∩ where $2x-1 = \frac{6}{x}$ ✓

$2x^2 - x = 6$

$2x^2 - x - 6 = 0$

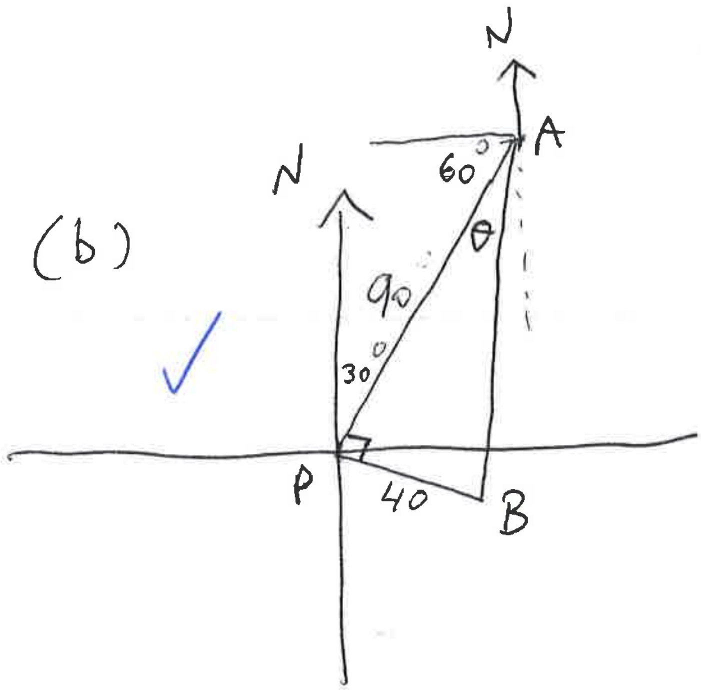
$(2x+3)(x-2) = 0$ ✓

$\left.\begin{matrix} x = -\frac{3}{2} \\ y = -4 \end{matrix}\right\}$ or $\left.\begin{matrix} x = 2 \\ y = 3 \end{matrix}\right\}$ ✓

(ii) Solution is ✓✓

$x \leq -\frac{3}{2}$ or $0 < x \leq 2$

(b)



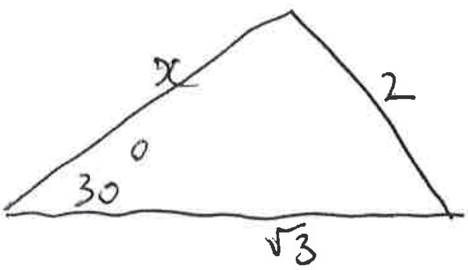
✓

Now $\tan\theta = \frac{4}{9}$

$\theta \doteqdot 24^\circ$ ✓

Bearing B from A is 186° T. ✓

(c) (i) By the cosine rule,



$4 = x^2 + 3 - 2\sqrt{3}\,x \cos 30^\circ$ ✓

$4 = x^2 + 3 - 2\sqrt{3}\,x \cdot \frac{\sqrt{3}}{2}$

$4 = x^2 + 3 - 3x$ ✓

$x^2 - 3x - 1 = 0$

(ii)

$x = \frac{3 \pm \sqrt{9+4}}{2}$ ✓

$x = \frac{3+\sqrt{13}}{2}$

Area $= \frac{1}{2} a b \sin 30^\circ$

$= \frac{1}{2}\sqrt{3}\left(\frac{3+\sqrt{13}}{2}\right) \cdot \frac{1}{2}$

$= \frac{3\sqrt{3}+\sqrt{39}}{8}$ ✓

12