NAME MASTER

SYDNEY GRAMMAR SCHOOL

2015 Half-Yearly Examination

FORM V

MATHEMATICS 2 UNIT

Monday 11th May 2015

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

Total — 80 Marks

- All questions may be attempted.

Section I – 8 Marks

- Questions 1–8 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 72 Marks

- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

5A: DS	5B: RCF	5C: SO	5D: DNW
5E: DWH	5F: REJ	5G: SJE	5H: KWM
5P: NL	5Q: TCW	5R: LRP	

Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 173 boys

Examiner

NL

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following is the exact value of $\cos 30^\circ$?

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{3}}$

QUESTION TWO

Which of the following CANNOT be expressed as $\sqrt{10}$?

(A) $\dfrac{\sqrt{40}}{2}$

(B) $\dfrac{10}{\sqrt{10}}$

(C) $5\sqrt{2}$

(D) $\sqrt{5} \times \sqrt{2}$

QUESTION THREE

Which of the following are the correct two factors of $5x^2 + 26x - 24$?

(A) $(5x - 1)$ and $(x + 24)$

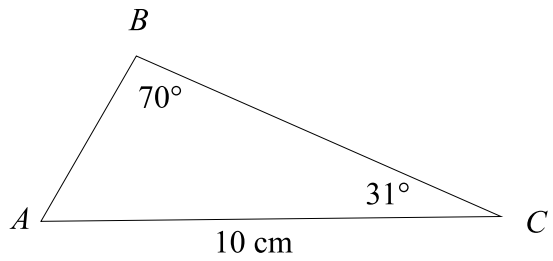
(B) $(5x + 2)$ and $(x - 12)$

(C) $(5x - 3)$ and $(x + 8)$

(D) $(5x - 4)$ and $(x + 6)$

Exam continues next page ...

QUESTION FOUR



In the diagram above, which is a correct expression for the length of side AB in centimetres?

(A) $\dfrac{10\sin 31^\circ}{\sin 70^\circ}$

(B) $10\tan 31^\circ$

(C) $\sqrt{10^2 - 20\cos 31^\circ}$

(D) $\dfrac{10\sin 70^\circ}{\sin 31^\circ}$

QUESTION FIVE

The expression $\dfrac{x-1}{x^2-1}$ simplifies to which of the following?

(A) $\dfrac{-1}{x-1}$

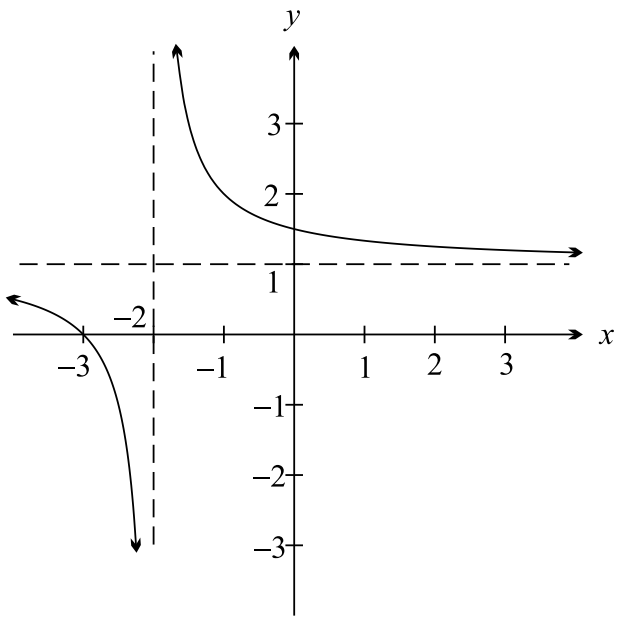
(B) $\dfrac{1}{x}$

(C) $\dfrac{1}{x+1}$

(D) $\dfrac{x}{x-1}$

Exam continues overleaf ...

QUESTION SIX



Which of the following could be the equation of the graph shown above?

(A) $y = \dfrac{1}{x-1} - 2$

(B) $y = \dfrac{1}{(x+2)^2} + 2$

(C) $y = \dfrac{1}{x+2} + 1$

(D) $y = \dfrac{1}{x-2} + 1$

QUESTION SEVEN

Which of the following is the equation of the line perpendicular to the line $y = -\frac{1}{3}x + 2$ and passing through the point $(1, 8)$?

(A) $y = -3x + 11$

(B) $y = 3x + 5$

(C) $y = 3x - 11$

(D) $y = \frac{1}{3}x + \frac{25}{3}$

Exam continues next page ...

QUESTION EIGHT

What is the natural domain of the function $y = \sqrt{9 - x^2} + 2$?

(A) $x > 9$

(B) $x \leq 9$

(C) $-3 \leq x \leq 3$

(D) $2 \leq y \leq 5$

End of Section I

Exam continues overleaf ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) Simplify:

(i) $4x^2 - x + 3x^2$ **1**

(ii) $3a^2 \times -5ab$ **1**

(b) Expand and simplify where possible:

(i) $(a+3)^2$ **1**

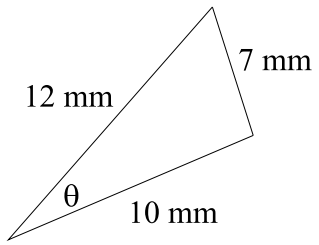
(ii) $(x-4)(x+4)$ **1**

(c) Factorise $x^2 - 11x + 24$. **1**

(d) Simplify $\dfrac{x^2+4x-5}{3x^2-3x}$. **2**

(e) Find the midpoint of the interval joining $A(-3,4)$ and $B(5,-2)$. **1**

(f)



Find the value of θ in the diagram above, correct to the nearest degree. **2**

(g) Express $\dfrac{4}{6+\sqrt{2}}$ as a simplified fraction with a rational denominator. **2**

Exam continues next page ...

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) Consider the parabola with equation $y = x^2 + 4x + 3$.

(i) Find the y-intercept. **1**

(ii) Find the x-intercepts. **2**

(iii) Find the equation of the axis of symmetry. **1**

(iv) Find the coordinates of the vertex. **1**

(v) Sketch the graph of $y = x^2 + 4x + 3$, clearly marking all the above features. **2**

(vi) Hence, or otherwise, solve the inequation $x^2 + 4x + 3 \geq 0$. **2**

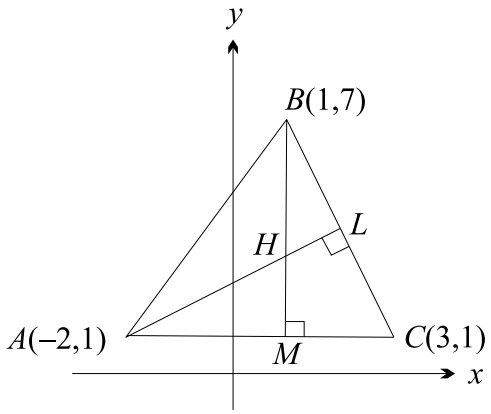
(b) Find the perpendicular distance from the point $R(-1, 5)$ to the line $y = -2x + 1$. **2**

(c) Factorise $x^3 + 27$. **1**

Exam continues overleaf ...

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a)



The diagram above shows a triangle with vertices $A(-2,1)$, $B(1,7)$ and $C(3,1)$. The point L is the foot of the perpendicular from A to BC, and M is the foot of the perpendicular from B to AC.

(i) Write down the equation of the vertical line BM. **1**

(ii) Show that the gradient of the line BC is -3. **1**

(iii) Show that the equation of the line AL is $x - 3y + 5 = 0$. **2**

Suppose the lines AL and BM meet at H.

(iv) Show that the coordinates of the point H are $(1,2)$. **2**

(v) Find the ratio of the length BH to the length HM. **2**

(b) (i) Sketch the graph of $y = \cos x$, for $0^\circ \leq x \leq 360^\circ$. **2**

(ii) Solve $\cos x = -\dfrac{1}{2}$, for $0^\circ \leq x \leq 360^\circ$. **2**

Exam continues next page ...

QUESTION TWELVE (12 marks) Use a separate writing booklet. **Marks**

(a) Consider the curve $y = x^3 + 3x^2 - 4x$.

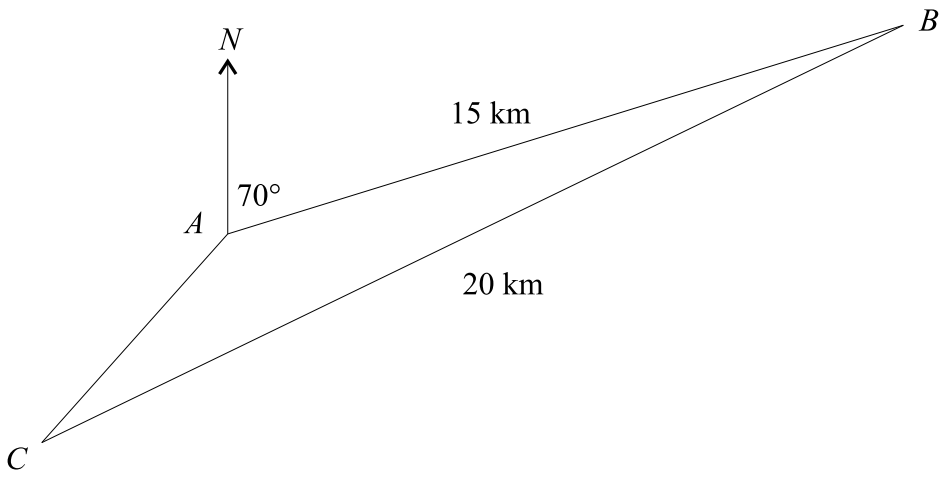
(i) Fully factorise $x^3 + 3x^2 - 4x$. **2**

(ii) Hence find the x-intercepts. **2**

(iii) Sketch the curve showing its x and y intercepts. **2**

(iv) Find a simplified equation for the curve obtained when the given curve is reflected in the y-axis. **2**

(b)



A drone, travelling at a constant height above the ocean, is programmed to search a triangular area ABC on the ocean surface. An aerial view of the flight path of the drone is represented in the diagram above.

Starting above point A, the drone flies on a bearing of 070° T for a distance of 15 km until above point B. It then changes direction and flies 20 km until above point C, as shown. Changing direction again, the drone then returns to be above point A.

(i) Given the search area is 42 km^2, show that $\angle ABC$ is 16° to the nearest degree. **1**

(ii) Calculate the bearing, to the nearest degree, on which the drone travels from point B to point C. **1**

(iii) Calculate the distance AC. **2**

Exam continues overleaf ...

QUESTION THIRTEEN (12 marks) Use a separate writing booklet. **Marks**

(a) (i) Find the radius and centre of the circle with equation $x^2 + y^2 = 6x + 8y$. **2**

(ii) Find the coordinates of the x-intercepts of this circle. **1**

(b) Solve $\sin\theta = 0{\cdot}39$, for $0^\circ \leq \theta \leq 450^\circ$. Give your answers correct to the nearest degree. **3**

(c) Find the exact value of $\sin\theta$, given $\cos\theta = \dfrac{2}{\sqrt{7}}$ and θ is a reflex angle. **3**

(d) Solve $3\sin^2\theta + 5\cos\theta - 1 = 0$, for $0^\circ \leq \theta \leq 180^\circ$. Give your answer correct to the nearest degree. **3**

Exam continues next page ...

QUESTION FOURTEEN (12 marks) Use a separate writing booklet. **Marks**

(a) A mountain hiker at point P can see two mountain peaks in the distance, peaks A and B. Peak A is on a bearing of 310° T and peak B is on a bearing of 015° T. After the hiker walks 500 m due north to point Q he finds the bearings of peaks A and B are now 300° T and 030° T respectively.

(i) Draw a diagram (not to scale) to show all the information given above. **1**

(ii) Calculate the distance between peaks A and B. **3**

(b) Determine whether the function $f(x) = \dfrac{x^3}{\sin x}$ is odd, even or neither. Justify your answer algebraically. **1**

(c) (i) Sketch the graphs $f(x) = |3x - 2|$ and $f(x) = -2x + 3$ on the same set of axes for $-2 \leq x \leq 2$. Show any x and y intercepts. **2**

(ii) Solve $|3x - 2| = -2x + 3$. **2**

(iii) Hence, or otherwise, solve $|3x - 2| \geq -2x + 3$. **1**

(d) Prove the identity $\dfrac{1 + \cos A}{1 - \cos A} = (\cot A + \operatorname{cosec} A)^2$. **2**

End of Section II

END OF EXAMINATION

B L A N K P A G E

NAME: .....

CLASS: .....MASTER: .....

SYDNEY GRAMMAR SCHOOL

2015
Half-Yearly Examination
FORM V
MATHEMATICS 2 UNIT
Monday 11th May 2015

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ○ B ○ C ○ D ○

Question Two

A ○ B ○ C ○ D ○

Question Three

A ○ B ○ C ○ D ○

Question Four

A ○ B ○ C ○ D ○

Question Five

A ○ B ○ C ○ D ○

Question Six

A ○ B ○ C ○ D ○

Question Seven

A ○ B ○ C ○ D ○

Question Eight

A ○ B ○ C ○ D ○

Form V 2 Unit Half-Yearly 2015 Solutions.

1.) $\cos 30^\circ = \frac{\sqrt{3}}{2}$ (A)

2.) $5\sqrt{2} = \sqrt{50}$ (C)
$\neq \sqrt{10}$

3.) $(5x-4)(x+6) = 5x^2+30x-4x-24$
$= 5x^2+26x-24$ (D)

4.) $\frac{AB}{\sin 31^\circ} = \frac{10}{\sin 70^\circ}$
$AB = \frac{10\sin 31^\circ}{\sin 70^\circ}$ (A)

5.) $\frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1}$ (C)

6.) $y = \frac{1}{x+2} + 1$ (C)

7.) $m_1 = -\frac{1}{3}$ $\therefore m_2 = 3$

$(1, 8)$

$y-8 = 3(x-1)$
$y = 3x-3+8$
$y = 3x+5$ (B)

8.) $9-x^2 \geq 0$
$x^2 \leq 9$
$-3 \leq x \leq 3$ (C)

9) a) i) $7x^2 - x$ (1) ii) $-15a^3b$ (1)

b) i) $a^2 + 6a + 9$ (1) ii) $x^2 - 16$ (1)

c) $(x-3)(x-8)$ (1)

d) $\frac{(x-1)(x+5)}{3x(x-1)}$ (1)

$= \frac{x+5}{3x}$ (1)

e) midpoint $= \left(\frac{5+-3}{2} , \frac{-2+4}{2} \right)$

$= (1 , 1)$ (1)

f) $\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\theta = \frac{12^2 + 10^2 - 7^2}{2 \times 12 \times 10}$ (1)

$= \frac{13}{16}$

$\theta = \cos^{-1}(13/16)$

$\approx 36^\circ$ (1)

g) $\frac{4(6-\sqrt{2})}{(6+\sqrt{2})(6-\sqrt{2})}$

$= \frac{4(6-\sqrt{2})}{34}$ (1)

$= \frac{2(6-\sqrt{2})}{17}$ (1)

10) a)

i) $x = 0 \quad y = 3 \quad (0, 3)$ (1)

ii) $0 = x^2 + 4x + 3$

$0 = (x+1)(x+3)$ (1)

$x = -1, -3$ (1)

(iii) $x = -\frac{b}{2a} = -\frac{+4}{2\times 1}$

$x = -2$ (1)

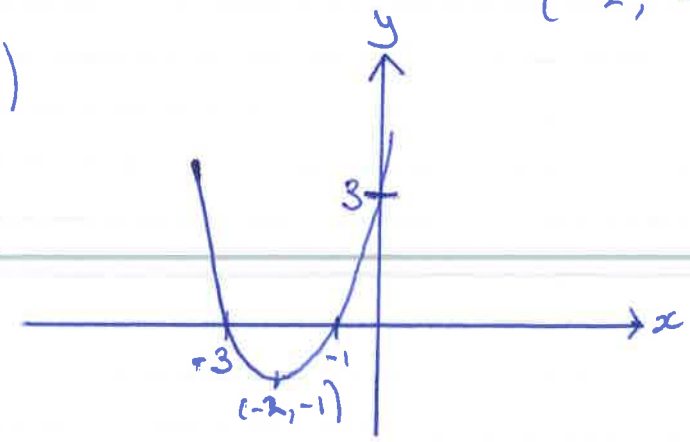
iv) $x = -2, \quad y = (-2)^2 + 4(-2) + 3$

$y = 4 - 8 + 3$

$y = -1$

$(-2, -1)$ (1)

v)



(-1 for missing feature)
(-2 for missing features)

vi) $x \leq -3, \quad x \geq -1$

$-3 \leq x \leq -1$ (1) mark.
$-3 < x < -1$ (0) marks

b) $d = \frac{|2\times(-1) + (5) - 1|}{\sqrt{2^2+1^2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

(1) for correct sub
(1) for answer

c) $x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$ (1)

Q11 a)

i) $x = 1$ (1).

ii) $m = \frac{7-1}{1-3} = \frac{6}{-2} = -3$

iii) $m_{AL} = \frac{1}{3}$ (1)

$y - 1 = \frac{1}{3}(x+2)$ (1)

$3y - 3 = x + 2$

$0 = x - 3y + 5$

$x - 3y + 5 = 0$

iv) $x = 1 \quad x - 3y + 5 = 0$ (1)

$1 - 3y + 5 = 0$

$6 - 3y = 0$ (1)

$3y = 6$

$y = 2$

$H.(1, 2)$

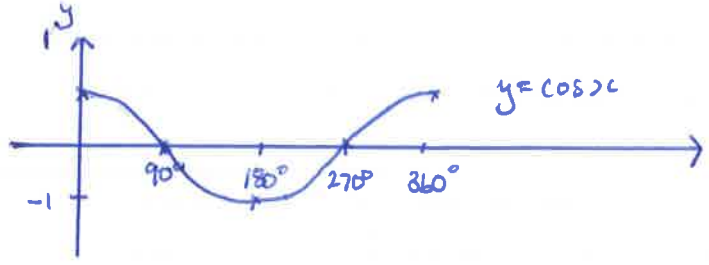
v) $B(1, 7) \quad H(1, 2) \quad M(1, 1)$

$BH = 7 - 2 = 5$

$HM = 2 - 1 = 1$ ((1) for either)

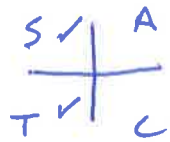
$\therefore \quad BH : HM = 5 : 1$ (1)

b)



(-1 for missing feature
-2 for missing features)

(ii) $\cos x = -\frac{1}{2}$



$x = 120°$ (1)

or

$x = 240°$ (1)

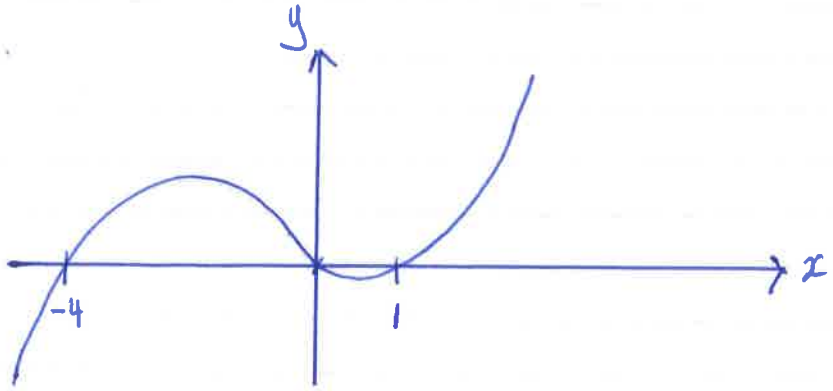
Q12. a)

i) $f(x) = x(x^2 + 3x - 4)$ (1)

$f(x) = x(x+4)(x-1)$ (1)

ii) $x = 0, 1, -4$ (-1 for each missing)

iii)



{ (1) Shape
{ -1 for missing feature).

iv) $f(x) = (-x)^3 + 3(-x)^2 - 4(-x)$ (1)

$= -x^3 + 3x^2 + 4x$ (1)

b) i) $A = \frac{1}{2}ab\sin C$

$42 = \frac{1}{2} \times AB \times BC \times \sin\theta$

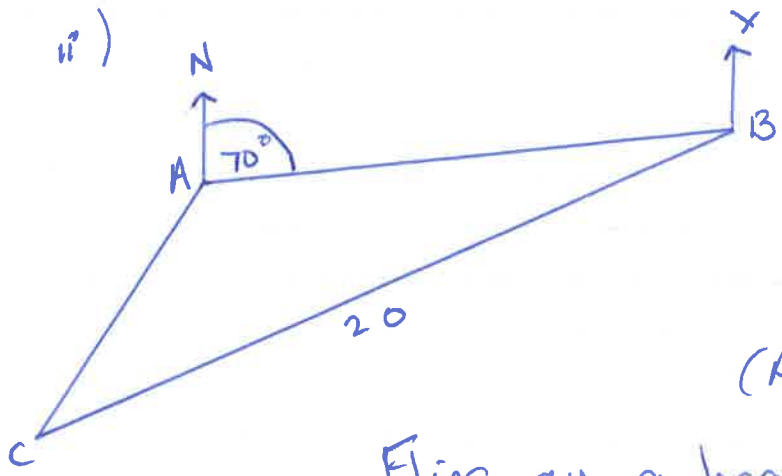
$42 = \frac{1}{2} \times 15 \times 20 \times \sin\theta$

$\sin\theta = \frac{7}{25}$

$\theta = 16.260...$

$\theta = 16^\circ$ (nearest degree). (1)

ii)



$\angle ABX = 110^\circ$
(Cointerior angles AN // BX)

$\angle XBC = 234^\circ$
(Angles in a revolution)

Flies on a bearing of $234^\circ T$ (1)

iii) $AC^2 = 15^2 + 20^2 - 2 \times 15 \times 20 \times \cos 16$ (1)

$AC^2 = 48.242\ldots$

$AC = 6.945\ldots$ km (1)

$AC = 6.95$ km (to 2 d.p)

Q13 a)

i) $x^2 - 6x + y^2 - 8y = 0$

$(x-3)^2 + (y-4)^2 = 25$ (1)

centre $(3, 4)$
radius 5 units. } (1)

(ii) when $y = 0$

$x^2 + 0 = 6x + 0$

$x(x-6) = 0$ ✓✓ (1)

$x = 0$ and $x = 6$ are the intercepts.

b) $\sin\theta = 0.39$

$\theta = \sin^{-1}(0.39)$

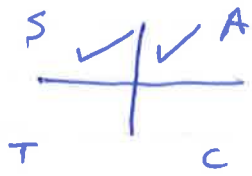
$\theta_1 = 22.9544994\ldots$ (1)

$\theta_2 = 180 - 22.95\ldots$

$= 157.0455$ (1)

$\theta_3 = 360 + 22.95$

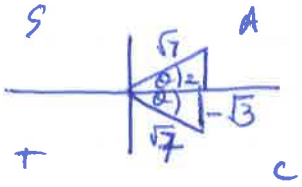
$= 382.95\ldots$ (1)



$\therefore \theta = 23^\circ, 157^\circ, 383^\circ$ (to nearest degree)

(-1 for rounding error)

c) $\cos\theta = \frac{2}{\sqrt{7}} = \frac{A}{H}$



$2^2 + x^2 = 7$

$x = \pm\sqrt{3}$ (1) reflex angle $\therefore 180^\circ < \theta < 360^\circ$

$\therefore \quad x = -\sqrt{3}$ (1)

and $\sin\theta = \frac{-\sqrt{3}}{+\sqrt{7}} = -\frac{\sqrt{21}}{7}$ (1)

d) $3\sin^2\theta + 5\cos\theta - 1 = 0 \qquad 0^\circ \leq \theta \leq 180^\circ$

$u = \cos\theta$
$u^2 = \cos^2\theta$

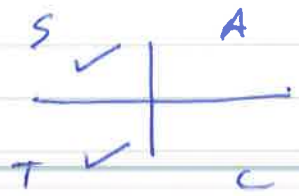
$\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta = 1 - \cos^2\theta$
$= 1 - u^2$

$\therefore \quad 3(1-u^2) + 5u - 1 = 0$ (1)
$3 - 3u^2 + 5u - 1 = 0$
$-3u^2 + 5u + 2 = 0$
$3u^2 - 5u - 2 = 0$
$(3u+1)(u-2) = 0$

$3u + 1 = 0$
$3u = -1$
$u = -1/3$

$u - 2 = 0$
$u = 2$
$\therefore \cos\theta = 2$ (1) invalid

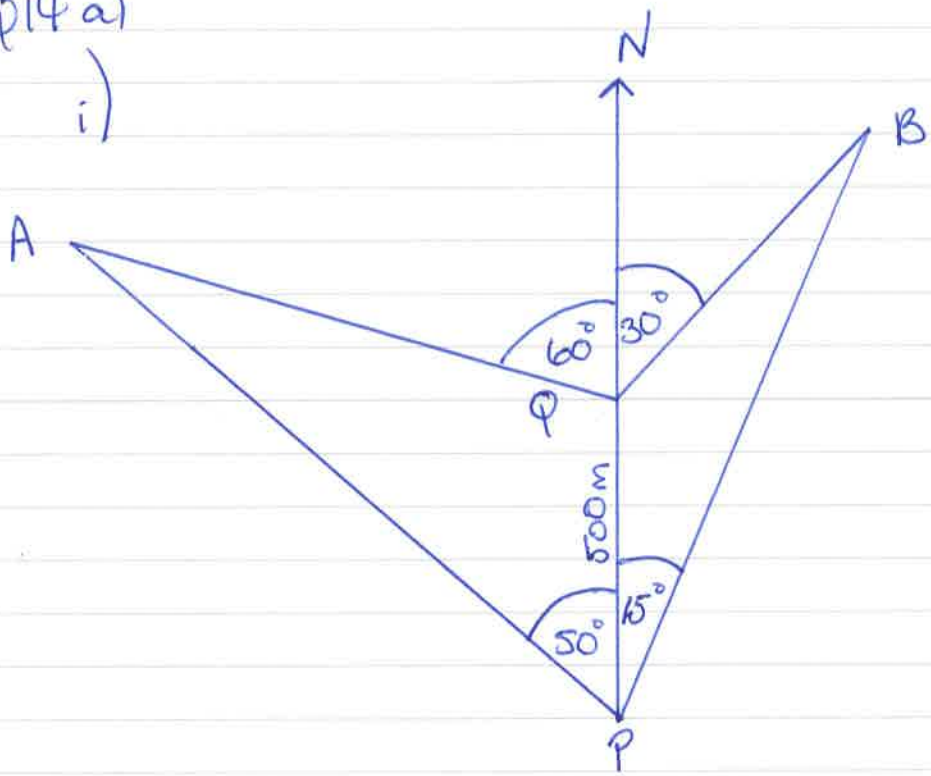
$\therefore \quad \cos\theta = -\frac{1}{3}$
$\theta = 180^\circ - 70.5^\circ$
$\theta = 109.471...$



$\therefore \quad \theta = 109^\circ$ (to nearest degree) (1)

Q14 a)

i)



(1)

(-1 for any missing feature.)

ii) ΔAPQ: $\dfrac{AQ}{\sin 50^\circ} = \dfrac{500}{\sin 10^\circ}$ $\left(\begin{array}{l}\angle AQP = 120^\circ \\ \angle PAQ = 10^\circ\end{array}\right)$

$AQ = \dfrac{500\sin 50^\circ}{\sin 10^\circ}$. (1)

ΔBPQ: $\angle PQB = 150^\circ$

$\angle QBP = 15^\circ$

$\therefore$ isosceles.

$\therefore$ $QB = 500$ m. (1)

ΔAQB is right-angled.

$AQ^2 + QB^2 = AB^2$.

$AB^2 = 500^2 + \left(\dfrac{500\sin 50^\circ}{\sin 10^\circ}\right)^2$

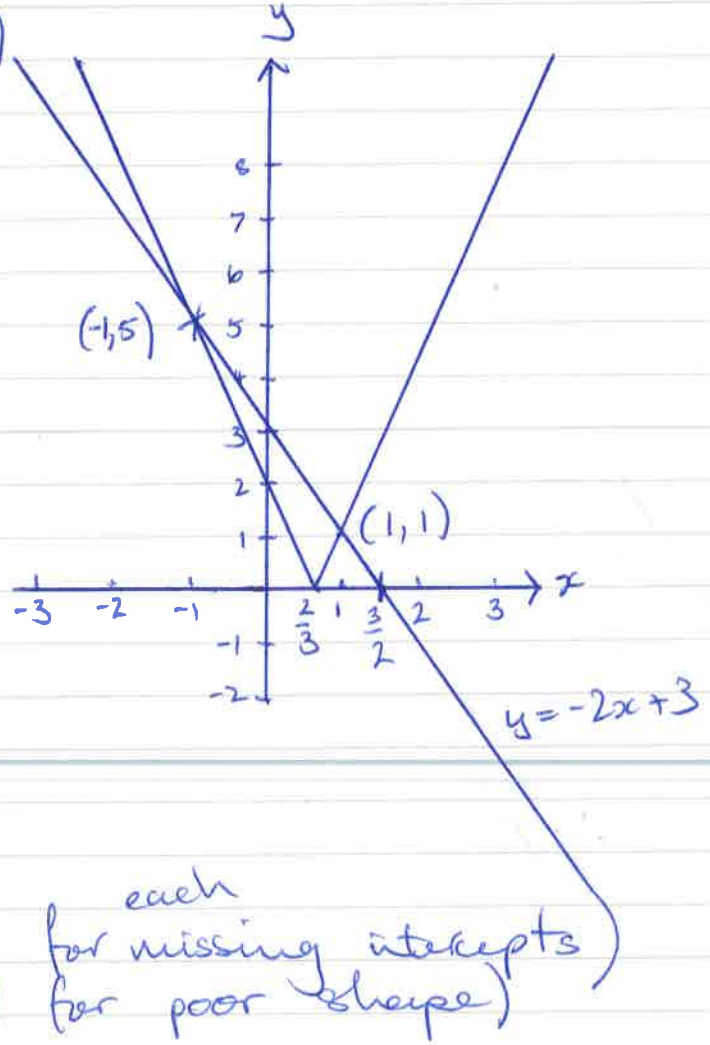
$AB = 2262$ m (to nearest metre) (1)

b) $f(x) = \frac{x^3}{\sin x}$ $\quad f(-x) = \frac{(-x)^3}{\sin(-x)}$

$= \frac{-x^3}{-\sin x}$ (1)

$= f(x)$ $\quad \therefore$ $f(x)$ is even. (1)

c) i)



(-1 each for missing intercepts)
(-1 for poor shape)

ii) $3x - 2 = -2x + 3$

$5x = 5$

$x = 1$ (1)

$3x - 2 = 2x - 3$

$x = -1$

$x = 1$ or $x = -1$ (1)

(iii) $x \leq -1$, $x \geq 1$ } (1)

(d) LHS $= \frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}$

$= \frac{1 + 2\cos A + \cos^2 A}{1 - \cos^2 A}$

$= \frac{1}{\sin^2 A} + \frac{2\cos A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A}$ (1)

$= \operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A + \cot^2 A$

$= (\operatorname{cosec} A + \cot A)^2$ (1)

$=$ RHS.

Alternatively:

$$RHS = (\cot A + \operatorname{cosec} A)^2$$

$$= \cot^2 A + 2\cot A \operatorname{cosec} A + \operatorname{cosec}^2 A$$

$$= \frac{\cos^2 A}{\sin^2 A} + \frac{2\cos A}{\sin^2 A} + \frac{1}{\sin^2 A} \qquad (1)$$

$$= \frac{\cos^2 A + 2\cos A + 1}{\sin^2 A}$$

$$= \frac{(1 + \cos A)^2}{1 - \cos^2 A}$$

$$= \frac{(1+\cos A)^2}{(1-\cos A)(1+\cos A)} \qquad (1)$$

$$= \frac{1 + \cos A}{1 - \cos A}$$

$$= LHS.$$