

SYDNEY GRAMMAR SCHOOL



2017 Half-Yearly Examination

# FORM V

## MATHEMATICS 2 UNIT

Tuesday 16th May 2017

### General Instructions

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Board-approved calculators and templates may be used.

### Total — 80 Marks

- All questions may be attempted.

### Section I – 8 Marks

- Questions 1–8 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

### Section II – 72 Marks

- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

### Collection

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

5A: RCF

5B: SO

5C: BR

5D: REJ

5E: LYL

5F: LJF

5G: SDP

5H: CMDB

5P: BDD

5Q: LL

5R: TCW

### Checklist

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 190 boys

Examiner

SDP

**SECTION I - Multiple Choice**

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

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**QUESTION ONE**

The expression  $\frac{x}{2} + \frac{x}{3}$  simplifies to which of the following?

(A)  $\frac{2x}{5}$

(B)  $\frac{x^2}{6}$

(C)  $\frac{5x}{6}$

(D)  $\frac{2x}{6}$

**QUESTION TWO**

Which of the following is NOT equal to  $\sqrt{24}$ ?

(A)  $2\sqrt{6}$

(B)  $\sqrt{12} \times \sqrt{2}$

(C)  $\frac{1}{2}\sqrt{48}$

(D)  $2\sqrt{3} \times \sqrt{2}$

**QUESTION THREE**

What is the natural domain of the function  $f(x) = \frac{1}{x+5}$ ?

(A) all real values of  $x$ , where  $x \neq 5$

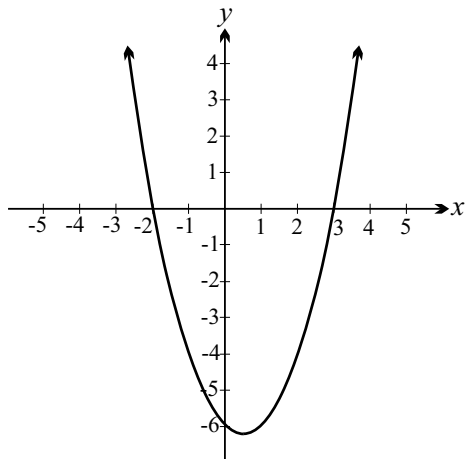
(B) all real values of  $x$ , where  $x \neq -5$

(C) all real values of  $x$ , where  $x < 5$

(D) all real values of  $x$ , where  $x > 5$

**Examination continues next page ...**

**QUESTION FOUR**



The above diagram shows the graph of  $y = (x - 3)(x + 2)$ .

Which of the following is the correct solution to the inequation  $(x - 3)(x + 2) < 0$ ?

- (A)  $x > 3$  or  $x < -2$
- (B)  $x > 2$  or  $x < -3$
- (C)  $-2 < x < 3$
- (D)  $-3 < x < 2$

**QUESTION FIVE**

Which of the following is the exact value of  $\operatorname{cosec}(-60^\circ)$ ?

- (A)  $-\frac{2\sqrt{3}}{3}$
- (B)  $\frac{2\sqrt{3}}{3}$
- (C) 2
- (D) -2

**QUESTION SIX**

What is the perpendicular distance between the point  $(3, 5)$  and the line  $3x - y + 2 = 0$ ?

- (A)  $\frac{8\sqrt{34}}{17}$
- (B)  $\frac{3\sqrt{34}}{17}$
- (C)  $\frac{8\sqrt{10}}{5}$
- (D)  $\frac{3\sqrt{10}}{5}$

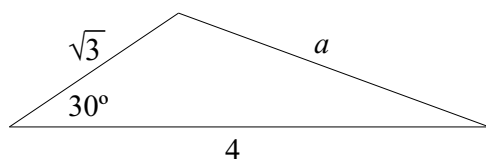
**QUESTION SEVEN**

Which of the following is the correct factorisation of  $54x^3 + 2y^3$ ?

- (A)  $2(3x - y)(9x^2 + 3xy + y^2)$
- (B)  $2(3x + y)(9x^2 - 3xy + y^2)$
- (C)  $2(3x - y)(3x + 3xy + y)$
- (D)  $2(3x + y)(3x - 3xy + y)$

**QUESTION EIGHT**

Which of the following is the correct value of  $a$ ?



- (A)  $\sqrt{31}$
- (B)  $\sqrt{13}$
- (C)  $\sqrt{7}$
- (D) 1

————— End of Section I —————

**SECTION II - Written Response**

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

**QUESTION NINE** (12 marks) Use a separate writing booklet. **Marks**

(a) Evaluate  $|2| - |-3 \times 4|$ . **1**

(b) Factorise fully:

(i)  $x^2 + 3x - 10$  **1**

(ii)  $9x^2 - 16$  **1**

(c) Express  $\frac{12}{3 - \sqrt{6}}$  as a fraction with a rational denominator. **2**

(d) Find the  $x$ -intercept of the line  $y = 3x - 6$ . **1**

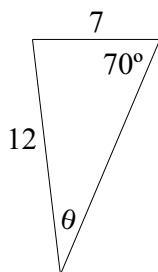
(e) Find the acute angle  $\theta$ , given that  $\cos \theta = \frac{\sqrt{3}}{2}$ . **1**

(f) Expand and simplify  $(x - 3)(x^2 + 3x + 9)$ . **1**

(g) (i) Find the midpoint  $M$  of the interval joining the point  $(4, 2)$  and the origin. **1**

(ii) Hence find the equation of the line through the point  $M$  with gradient 3. **1**

(h)



Find the value of  $\theta$  in the diagram above, correct to the nearest degree. **2**

**QUESTION TEN** (12 marks) Use a separate writing booklet. **Marks**

- (a) Find  $f(2)$  when  $f(x) = \frac{2}{\sqrt{x^2 + 12}}$ . **1**
- (b) (i) Write down the domain of  $y = \log_2 x$ . **1**  
 (ii) Write down the range of  $y = \log_2 x$ . **1**
- (c) Solve the following:
- (i)  $3 \leq 2x + 1 \leq 7$  **1**  
 (ii)  $|x - 2| = 10$  **2**
- (d) Find the equation of the line which is parallel to  $y = 2x - 5$  and passes through the point  $(1, 5)$ . Give your answer in general form. **2**
- (e) (i) Complete the square for  $f(x) = x^2 + 10x + 17$ . **1**  
 (ii) Hence solve  $f(x) = 0$ . Leave your answers in exact form. **1**
- (f) Solve  $\cos x = \frac{1}{2}$  for  $0^\circ \leq x \leq 360^\circ$ . **2**

**QUESTION ELEVEN** (12 marks) Use a separate writing booklet. **Marks**

- (a) Describe the transformation of the circle  $x^2 + y^2 = 16$  to the circle  $(x + 3)^2 + (y - 2)^2 = 16$ . **1**
- (b) Simplify  $\sqrt{8} + \sqrt{25} + \sqrt{50}$ . **1**
- (c) Simplify  $\frac{x^2 + 5x + 6}{2x + 4}$ . **2**
- (d) Find the coordinates of the vertex of  $y = x^2 - 4x - 32$ . **2**
- (e) Solve  $-x^2 - 2x + 15 \geq 0$ . **2**
- (f) Prove  $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \operatorname{cosec} x \sec x$ . **2**
- (g) Determine whether or not the following lines are concurrent: **2**

$$l_1 : y = 2x + 4 \quad l_2 : y = x + 14 \quad l_3 : y = 3x - 6$$

Clearly show your working.

**QUESTION TWELVE** (12 marks) Use a separate writing booklet. **Marks**

- (a) Solve  $x = \frac{1 + 4x}{x}$ . Leave your answer in exact form. **2**
- (b) Find the value of  $m$  such that  $\frac{2}{\sqrt{3} - \sqrt{2}} = m(\sqrt{2} + \sqrt{3})$ . **2**
- (c) (i) Solve  $|x - 2| < 22$ . **2**  
 (ii) Show your solution on a number line. **1**
- (d) (i) Fully factorise  $x^3 - 7x^2 + 10x$ . **2**  
 (ii) Hence solve  $x^3 - 7x^2 + 10x = 0$ . **1**  
 (iii) Sketch the curve  $y = x^3 - 7x^2 + 10x$ , showing any intercepts with the axes. **2**

**QUESTION THIRTEEN** (12 marks) Use a separate writing booklet. **Marks**

- (a) Simplify  $\frac{6x - 30}{3x^2 - 75} \div \frac{1}{4x + 20}$ . **2**
- (b) (i) Sketch  $y = \sqrt{9 - x^2}$ , showing any intercepts with the axes. **2**  
 (ii) Write down the domain of this function. **1**  
 (iii) Write down the range of this function. **1**
- (c) In  $\triangle ABC$ ,  $\angle BAC = 30^\circ$ ,  $\angle BCA = \theta$ , length  $AB = 4\sqrt{2}$  cm and length  $BC = 4$  cm. **3**  
 Find the two possible values of  $\theta$ .
- (d) Find the equation of the perpendicular bisector of the line joining the points  $(2, 10)$  and  $(8, 0)$ . Give your answer in gradient-intercept form. **3**

**QUESTION FOURTEEN** (12 marks) Use a separate writing booklet. **Marks**

- (a) Determine whether  $f(x) = x(x^2 - 4)$  is odd, even or neither. 1
  
- (b) (i) Sketch  $y = 2^x$ , showing any intercepts with the axes and any asymptotes. 1  
(ii) Hence sketch  $y = -2^x + 2$  and find the  $x$ -intercept. 2
  
- (c) (i) Accurately sketch  $y = 2|x + 2|$  and  $y = x + 5$  on the same set of axes. 1  
(ii) Write down the coordinates of any points of intersection. 1  
(iii) Hence or otherwise, solve  $2|x + 2| < x + 5$ . 1
  
- (d) Prove  $\operatorname{cosec} x - \sin x = \cot x \cos x$ . 2
  
- (e) If  $\sqrt{15} = ap^2 + b$  and  $p = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}}$ , find the values of  $a$  and  $b$ , where  $a$  and  $b$  are rational numbers. 3

————— End of Section II —————

**END OF EXAMINATION**



NAME: .....

CLASS: ..... MASTER: .....

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

**Question One**

A  B  C  D

**Question Two**

A  B  C  D

**Question Three**

A  B  C  D

**Question Four**

A  B  C  D

**Question Five**

A  B  C  D

**Question Six**

A  B  C  D

**Question Seven**

A  B  C  D

**Question Eight**

A  B  C  D

## Solutions Form V 2 Unit Huls-Yearly 2017

### Multiple Choice

$$1) \quad \frac{x}{2} + \frac{x}{3} = \frac{3x}{6} + \frac{2x}{6}$$

$$= \frac{5x}{6}$$

(C) ✓

$$2) \quad A \rightarrow 2\sqrt{6} = \sqrt{4} + \sqrt{6} \\ = \sqrt{24} \quad \checkmark$$

$$B \rightarrow \sqrt{12} + \sqrt{2} = \sqrt{24} \quad \checkmark$$

$$C \rightarrow \frac{1}{2}\sqrt{48} = \sqrt{\frac{1}{4}} + \sqrt{48} \\ = \sqrt{12} \quad \times$$

(C) ✓

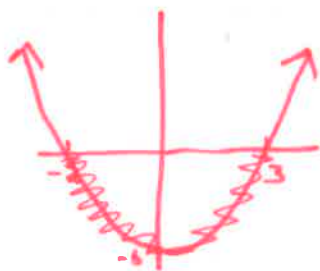
$$D \rightarrow 2\sqrt{3} + \sqrt{2} = \sqrt{4} + \sqrt{3} + \sqrt{2} \\ = \sqrt{24} \quad \checkmark$$

$$3) \quad \text{Denominator cannot equal } 0, \text{ so } x+5 \neq 0 \\ x \neq -5$$

(B) ✓

$$4) \quad (x-3)(x+2) < 0$$

as less than zero, solutions lie under x-axis

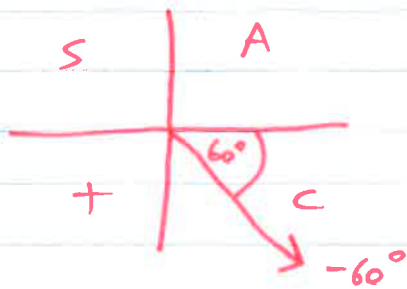


← WANT

$$\text{so } -2 < x < 3$$

(C) ✓

$$5) \quad \operatorname{cosec}(-60) = \frac{1}{\sin(-60)}$$



related angle =  $60^\circ$

negative sin quadrant

$$\text{so } \sin(-60) = -\sin(60) \\ = -\frac{\sqrt{3}}{2}$$

$$\text{so } \operatorname{cosec}(-60) = \frac{1}{-\frac{\sqrt{3}}{2}} \\ = -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ = -\frac{2\sqrt{3}}{3}$$

(A) ✓

$$6) \quad \text{perp. distance} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$x_1 = 3$$

$$a = 3$$

$$y_1 = 5$$

$$b = -1$$

$$c = 2$$

$$= \frac{|3 \times 3 + (-1) \times 5 + 2|}{\sqrt{3^2 + (-1)^2}}$$

$$= \frac{|9 - 5 + 2|}{\sqrt{9 + 1}}$$

$$= \frac{|6|}{\sqrt{10}}$$

$$= \frac{6}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \frac{6\sqrt{10}}{10} = \frac{3\sqrt{10}}{5}$$

(D) ✓

$$\begin{aligned}
 7) \quad 54x^3 + 2y^3 &= 2(27x^3 + y^3) \\
 &= 2((3x)^3 + y^3) \\
 &= 2(3x + y)((3x)^2 - (3x)(y) + y^2) \\
 &= 2(3x + y)(9x^2 - 3xy + y^2) \quad \text{(B) } \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8) \quad \text{cosine rule} \quad a^2 &= b^2 + c^2 - 2bc \cos A \\
 a^2 &= (\sqrt{3})^2 + 4^2 - 2 \times \sqrt{3} \times 4 \times \cos 30 \\
 &= 19 - 8\sqrt{3} \times \frac{\sqrt{3}}{2} \\
 &= 19 - 12 \\
 a^2 &= 7
 \end{aligned}$$

$$\text{so } a = \sqrt{7} \quad \text{(C) } \checkmark$$

### Question 9

$$\begin{aligned}
 a) \quad |21 - 1 - 3 \times 4| &= |21 - 1 - 12| \\
 &= |2 - 12| \\
 &= |-10| \quad \checkmark
 \end{aligned}$$

$$b) \quad x^2 + 3x - 10 = (x + 5)(x - 2) \quad \checkmark$$

$$c) \quad 9x^2 - 16 = (3x + 4)(3x - 4) \quad \checkmark$$

$$\begin{aligned}
 c) \quad \frac{12}{3 - \sqrt{61}} \times \frac{3 + \sqrt{61}}{3 + \sqrt{61}} &= \frac{36 + 12\sqrt{61}}{9 - 6} \\
 &= \frac{36 + 12\sqrt{61}}{3} \\
 &= 12 + 4\sqrt{61} \quad \checkmark
 \end{aligned}$$

g d) x-intercept when  $y=0$

$$0 = 3x - 6$$

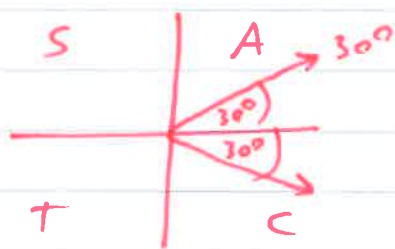
$$3x = 6$$

$$x = 2$$

✓ allow (2,0)

e)  $\cos \theta = \frac{\sqrt{3}}{2}$

related angle  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$



positive and acute angle so  $\theta = 30^\circ$  ✓

f)  $(x-3)(x^2+3x+9) = x^3-27$  ✓

g i) (4,2) and (0,0)  $M = \left(\frac{4+0}{2}, \frac{2+0}{2}\right)$

$$M = (2,1) \quad \checkmark$$

i)  $y - y_1 = m(x - x_1)$

$$y - 1 = 3(x - 2)$$

$$y - 1 = 3x - 6$$

$$y = 3x - 5$$

✓ allow  $3x - y - 5 = 0$

h) use sin rule  $\frac{\sin \theta}{7} = \frac{\sin 70}{12}$  ✓ or similar

$$\sin \theta = \frac{7 \sin 70}{12}$$

$$\theta = \sin^{-1}\left(\frac{7 \sin 70}{12}\right)$$

$$\theta = 33^\circ \quad (\text{nearest degree}) \quad \checkmark$$

## Question 10

$$\begin{aligned} \text{a)} \quad f(2) &= \frac{2}{\sqrt{2^2 + 12}} \\ &= \frac{2}{\sqrt{4+12}} \\ &= \frac{2}{\sqrt{16}} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \quad \checkmark \end{aligned}$$

bi) cannot put negatives into a log  
so all real values of  $x$ , where  $x > 0$  ✓

ii) can get any value out of logs  
so all real values of  $y$  ✓

$$\begin{aligned} \text{ci)} \quad 3 &\leq 2x + 1 \leq 7 \\ 2 &\leq 2x \leq 6 \\ 1 &\leq x \leq 3 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad |x-2| &= 10 \\ \swarrow \quad \searrow & \\ x-2=10 & \quad -(x-2)=10 \\ x=12 & \quad x-2=-10 \\ & \quad x=-8 \end{aligned}$$

$$x=12 \quad \checkmark \quad \text{or} \quad x=-8 \quad \checkmark$$

10a)

parallel so gradient = 2 ✓

$$y - 5 = 2(x - 1)$$

$$y - 5 = 2x - 2$$

$$y = 2x + 3$$

$$2x - y + 3 = 0 \quad \checkmark$$

ii)

$$f(x) = x^2 + 10x + 17$$

$$= (x + 5)^2 - 25 + 17$$

$$= (x + 5)^2 - 8 \quad \checkmark$$

iii)

$$0 = (x + 5)^2 - 8$$

$$8 = (x + 5)^2$$

$$x + 5 = \sqrt{8}$$

or  $x + 5 = -\sqrt{8}$

$$x + 5 = 2\sqrt{2}$$

$$x + 5 = -2\sqrt{2}$$

$$x = -5 + 2\sqrt{2}$$

$$\therefore x = -5 - 2\sqrt{2} \quad \checkmark \text{ both}$$

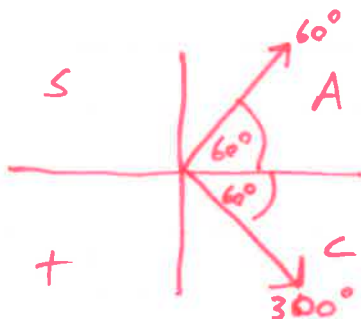
$$\text{allow } x = -5 \pm \sqrt{8}$$

f)

$$\cos x = \frac{1}{2}$$

related angle  $x = \cos^{-1}\left(\frac{1}{2}\right)$

$$= 60^\circ \quad \checkmark$$



$$\text{so } x = 60^\circ \text{ or } 300^\circ \quad \checkmark$$

## Question 11

a) Shift left 3  
Shift up 2 ✓ both

b)  $\sqrt{8} + \sqrt{25} + \sqrt{50} = 2\sqrt{2} + 5 + 5\sqrt{2}$   
 $= 5 + 7\sqrt{2}$  ✓

c)  $\frac{x^2 + 5x + 6}{2x + 4} = \frac{(x+2)(x+3)}{2(x+2)}$  ✓ factorise top  
 $= \frac{x+3}{2}$  ✓

d)  $y = x^2 - 4x - 32$

vertex  $x = -\frac{b}{2a}$

$$x = -\frac{(-4)}{2 \times 1}$$

$$x = \frac{4}{2}$$

$$x = 2$$
 ✓

when  $x=2$ ,  $y = 2^2 - 4 \times 2 - 32$

$$y = 4 - 8 - 32$$

$$y = -36$$
 ✓

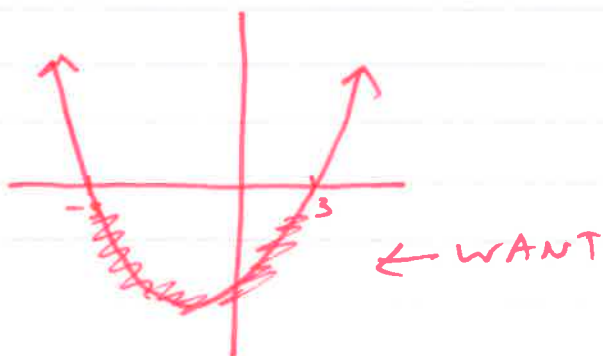
vertex  $(2, -36)$



$$11e) \quad -x^2 - 2x + 15 \geq 0$$

$$x^2 + 2x - 15 \leq 0$$

$$(x+5)(x-3) \leq 0$$



$$-5 \leq x \leq 3 \quad \checkmark \checkmark$$

$$f) \quad \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \operatorname{cosec} x \operatorname{sec} x$$

$$\begin{aligned} \text{LHS} &= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \\ &= \frac{\sin x \times \sin x}{\sin x \cos x} + \frac{\cos x \times \cos x}{\sin x \cos x} \\ &= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \quad \checkmark \\ &= \frac{1}{\sin x \cos x} \\ &= \frac{1}{\sin x} \times \frac{1}{\cos x} \\ &= \operatorname{cosec} x \operatorname{sec} x \\ &= \text{RHS} \quad \checkmark \end{aligned}$$

11g)  $l_1: y = 2x + 4$  concurrent  $\Rightarrow$  all intersect at  
 $l_2: -(y = x + 14)$  same point \*

$$0 = x - 10$$

$$x = 10$$

when  $x = 10$ ,  $l_1: y = 2 \times 10 + 4$

$$y = 24$$

✓ both  $x$  and  $y$

$x = 10, y = 24$  in  $l_3: y = 3x - 6$

$$\text{LHS} = 3x - 6$$

$$= 3 \times 10 - 6$$

$$= 30 - 6$$

$$= 24$$

$$\text{RHS} = y$$

$$= 24$$

LHS = RHS so  $l_1, l_2, l_3$  concurrent ✓

### Question 12

a)  $x = \frac{1 + 4x}{x}$

$$x^2 = 1 + 4x$$

$$x^2 - 4x - 1 = 0$$

$$(x-2)^2 - 4 - 1 = 0$$

$$(x-2)^2 - 5 = 0$$

$$x - 2 = \sqrt{5}$$

$$x = 2 + \sqrt{5}$$

or  $x - 2 = -\sqrt{5}$

$$x = 2 - \sqrt{5}$$

✓ both

$$12b) \quad \frac{2}{\sqrt{3}-\sqrt{2}} = m(\sqrt{2}+\sqrt{3})$$

$\frac{2}{(\sqrt{3}-\sqrt{2})(\sqrt{2}+\sqrt{3})} = m \quad \checkmark$ $m = \frac{2}{3-2}$ $m = \frac{2}{1}$ $m = 2 \quad \checkmark$	O R	$\frac{2}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \quad \checkmark$ $= \frac{2\sqrt{3}+\sqrt{2}}{3-2}$ $= 2(\sqrt{2}+\sqrt{3})$ <p>so <math>m=2</math> by inspection <math>\checkmark</math></p>
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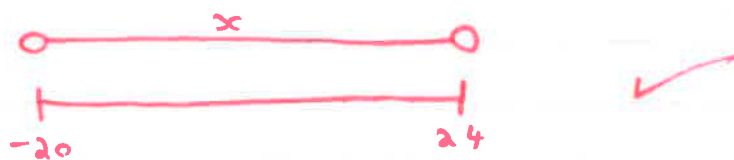
ci)

$$|x-2| < 22$$

$x-2 < 22$ $x < 24 \quad \checkmark$	/	$-(x-2) < 22$ $x-2 > -22$ $x > -20 \quad \checkmark$
--------------------------------------	---	--

so  $-20 < x < 24$

ii)



di)

$$x^3 - 7x^2 + 10x = x(x^2 - 7x + 10) \quad \checkmark$$

$$= x(x-2)(x-5) \quad \checkmark$$

ii)

$$0 = x(x-2)(x-5)$$

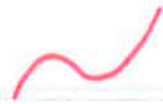
$$x=0 \quad \text{or} \quad x-2=0 \quad \text{or} \quad x-5=0$$

$$x=0 \quad \text{or} \quad x=2 \quad \text{or} \quad x=5 \quad \checkmark \text{ all 3}$$

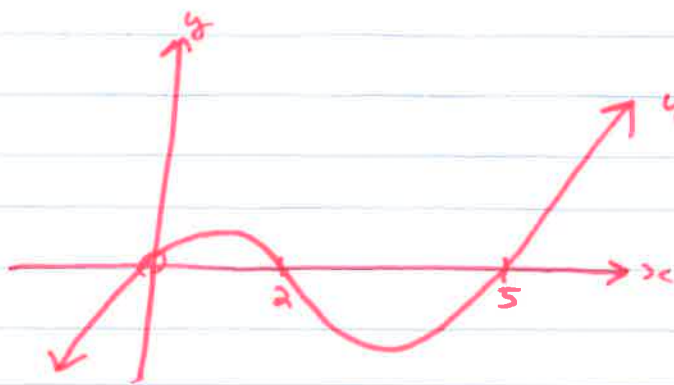
12 d iii)

when  $x=10$ ,  $y=400$  so

so



(not  $\cup$ )



$$y = x^3 - 7x^2 + 10x$$

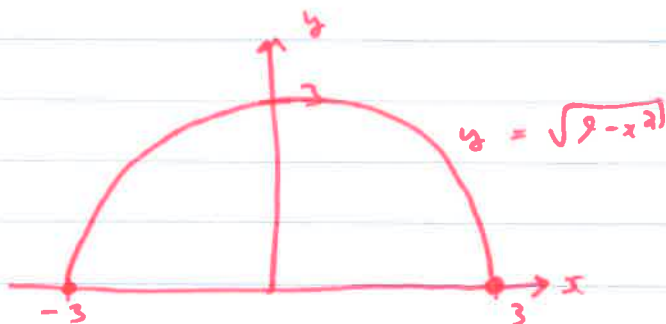
✓ shape

✓ intercepts

### Question 13

$$\begin{aligned} \text{a)} \quad \frac{6x-30}{3x^2-75} \div \frac{1}{4x+20} &= \frac{6(x-5)}{3(x^2-25)} \times \frac{4x+20}{1} \\ &= \frac{6(x-5)}{3(x+5)(x-5)} \times \frac{4(x+5)}{1} \quad \checkmark \\ &= \frac{6 \times 4}{3 \times 1} \\ &= \frac{24}{3} \\ &= 8 \quad \checkmark \end{aligned}$$

bi)



✓ shape  
✓ intercepts

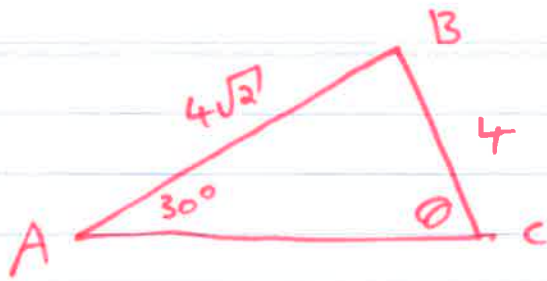
ii) values on x-axis, so

Domain: all real values of  $x$ , where  $-3 \leq x \leq 3$  ✓

iii) values on y-axis, so

Range: all real values of  $y$ , where  $0 \leq y \leq 3$  ✓

13c)



use sine rule  $\frac{\sin \theta}{4\sqrt{2}} = \frac{\sin 30}{4}$  ✓

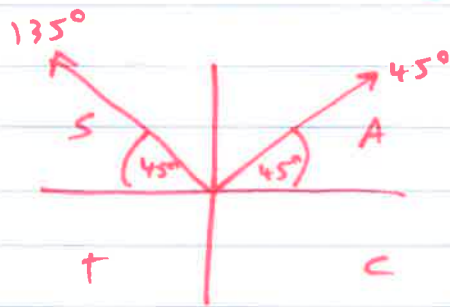
$$\sin \theta = \frac{4\sqrt{2} \sin 30}{4}$$

$$\sin \theta = \frac{4\sqrt{2} \times \frac{1}{2}}{4}$$

$$= \frac{2\sqrt{2}}{4}$$

$$\sin \theta = \frac{\sqrt{2}}{2}$$

related  $\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$   
 $= 45^\circ$



so  $\theta = 45^\circ$  ✓ or  $135^\circ$  ✓

d)  $M = \left(\frac{2+8}{2}, \frac{10+0}{2}\right)$   
 $= (5, 5)$  ✓

gradient  $m = \frac{10-0}{2-8}$   
 $= -\frac{10}{6}$

$$= -\frac{5}{3}$$

perpendicular gradient  $= \frac{3}{5}$  ✓

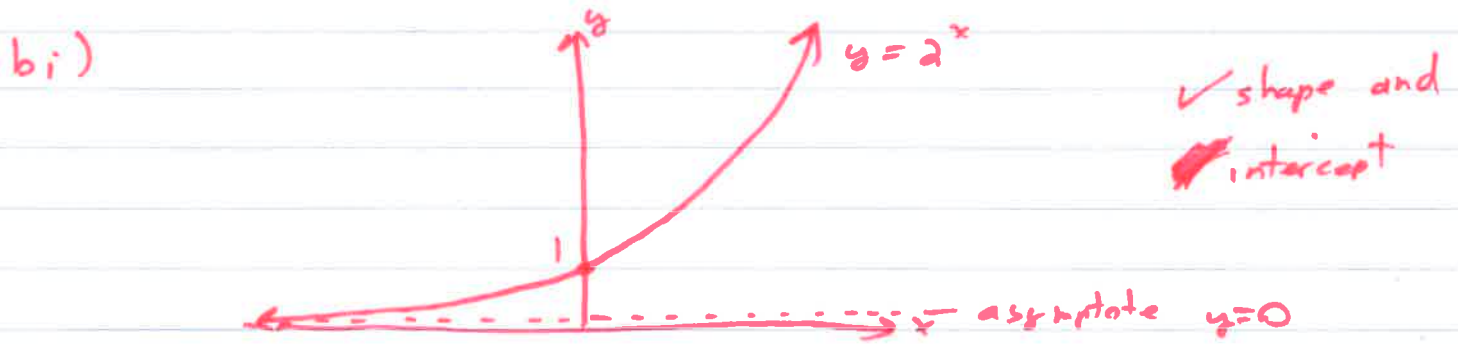
$$y - 5 = \frac{3}{5}(x - 5)$$

$$y - 5 = \frac{3}{5}x - 3$$

$$y = \frac{3}{5}x + 2$$
 ✓

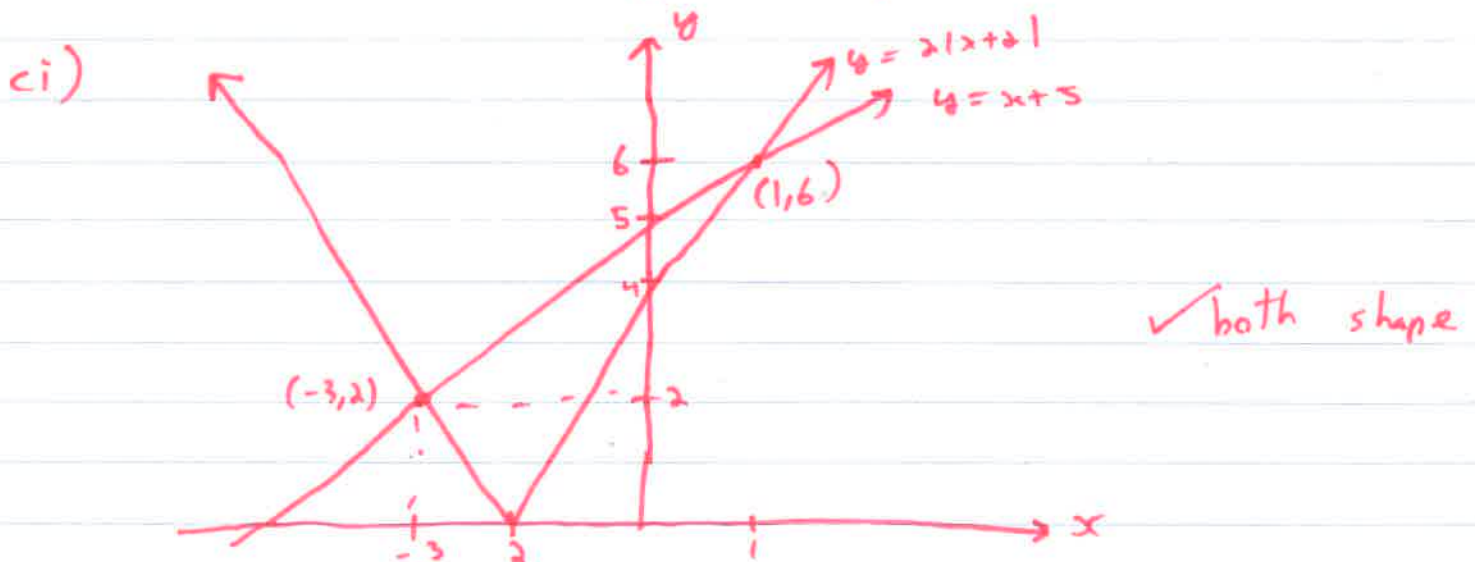
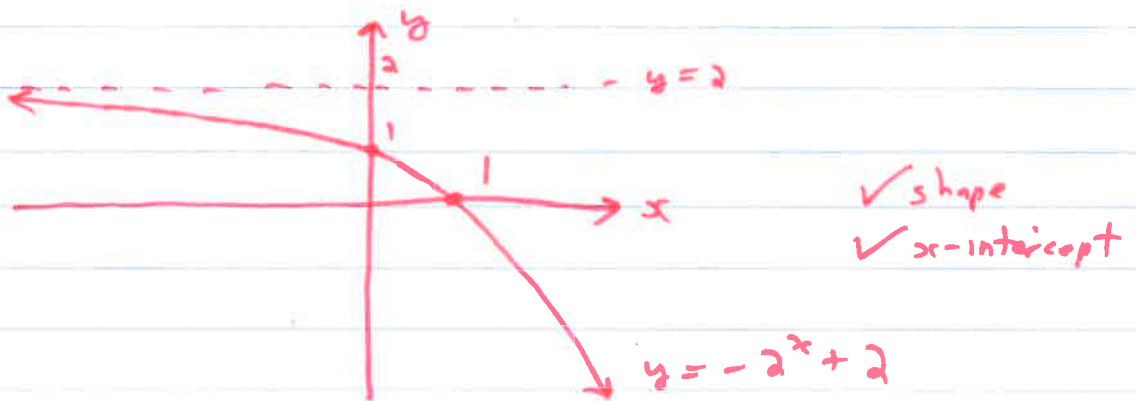
# Question 14

a)  $f(x) = x(x^2 - 4)$   
 $f(-x) = -x((-x)^2 - 4)$   
 $= -x(x^2 - 4)$   
 $= -f(x)$       so ODD ✓



ii)  $y = -2^x + 2$   
first reflect in x-axis

then shift up by 2:



14cii)

$$2|x+2| = x+5$$

$$2(x+2) = x+5$$

$$2x+4 = x+5$$

$$x = 1$$

$$y = 6$$

$$-2(x+2) = x+5$$

$$-2x-4 = x+5$$

$$-9 = 3x$$

$$x = -3$$

$$y = 2$$

(1, 6) and (-3, 2) ✓

iii) where  $y = 2|x+2|$  is below  $y = x+5$

$$-3 < x < 1$$
 ✓

d)

$$\operatorname{cosec} x - \sin x = \cot x \cos x$$

$$\text{L.H.S.} = \operatorname{cosec} x - \sin x$$

$$= \frac{1}{\sin x} - \frac{\sin x}{1}$$

$$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$
 ✓

$$= \frac{\cos^2 x}{\sin x}$$

$$= \frac{\cos x}{\sin x} \times \cos x$$

$$= \cot x \cos x$$
 ✓

e)

$$p^2 = \left( \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right)^2$$

$$= \frac{1}{3} + \frac{1}{5} + \frac{2}{\sqrt{15}}$$
 ✓

$$= \frac{8}{15} + \frac{2}{\sqrt{15}}$$

$$= \frac{8}{15} + \frac{2\sqrt{15}}{15}$$

$$= \frac{8 + 2\sqrt{15}}{15}$$

$$\sqrt{15} = ap^2 + b$$

PTO  
→

14 e continued...)

$$\text{so } \sqrt{15} = ap^2 + b = \frac{8a}{15} + \frac{2\sqrt{15}a}{15} + b$$

compare irrational numbers :  $\sqrt{15} = \frac{2\sqrt{15}}{15} \times a$

$$a = \frac{15}{2} \quad \checkmark$$

compare rational numbers :  $b + \frac{8a}{15} = 0$

$$b + 4 = 0$$

$$b = -4 \quad \checkmark$$

$$\text{so } a = \frac{15}{2} \quad \text{and } b = -4.$$