SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Year 11

2 Unit

May 2006

Common Test

Time allowed: 70 mins

Name: _____

Teacher: _____

Instructions:

- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/8	/7	/7	/7	/8	/7	/8	/6	/58

Question 1

a) Evaluate $\dfrac{1.3\times10^{-3}}{8.6-4.2}$. Give your answer in scientific notation using 2 significant figures. 2

b) Express $0.\dot{4}\dot{2}$ as a simplified fraction. 2

c) Simplify $\sqrt{27}-\sqrt{12}$ 2

d) Evaluate x^2-3x+1 when $x=3\sqrt{5}$. Leave your answer in simplest form 2

Question 2

a) Simplify: (i) $3x-4x(3-x)$ 1

(ii) $(4a^3b)^2\times\dfrac{a^2}{6b^3}$ 2

b) Express the answer to $\sqrt{x}\div\dfrac{1}{x}$ in simplified index form 2

c) Express $8^a\times4^{a+1}$ as a power of 2 2

Question 3

a) Factorise (i) x^2-5x-6 1

(ii) x^3-8 1

(iii) x^3+x^2+x+1 1

b) Simplify (i) $\dfrac{x^2-x}{x^2-1}$ 2

(ii) $\dfrac{4}{x+2}-\dfrac{1}{x}$ 2

Question 4

a) Evaluate $|-3|-|-4|^2$ 1

b) Solve (i) $\dfrac{2x-4}{3}=x+2$ 2

(ii) $2x^2+9x-5=0$ 2

(iii) $|1-2x|=6$ 2

Question 5

a) Solve simultaneously $3x - y = 0$ and $4x - 2y = 3$ 3

b) Solve $\frac{4}{x+2} + x = 3$ 2

c) Given $f(x) = 3x^2 - 2x$, find and simplify:

(i) $f(-3)$ 1

(ii) $f(x+h) + f(x)$ 2

Question 6

a) Sketch each function. Use a ruler for all straight lines and show any x, y intercepts:

(i) $y = \frac{1}{x+1}$ 1

(ii) $y = -\sqrt{4 - x^2}$ 1

(iii) $y = |3x - 6|$ 1

b) (i) Sketch $y = x^2 + 2x - 3$. Show all intercepts 2

(ii) Solve $x^2 + 2x - 3 \leq 0$. You may use the sketch above to help 1

(iii) What is the range of the function in (i) 1

Question 7

a) State the domain and range of the function $y = \sqrt{x-4}$ 2

b) Find the centre of the circle $x^2 - 4x + 4 + y^2 + 6y = 0$ 2

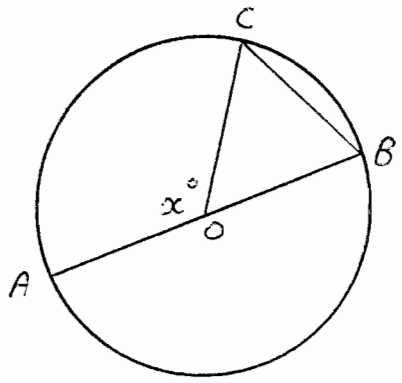
c) (i) Sketch $y = 2^x$ and $y = |x - 1|$ on the same axes 2

(ii) Hence or otherwise, solve $|x-1| > 2^x$ 1

(iii) On your graph in (i), neatly shade the intersected region of $x \geq 0$ *and* $y \geq 2^x$ 1

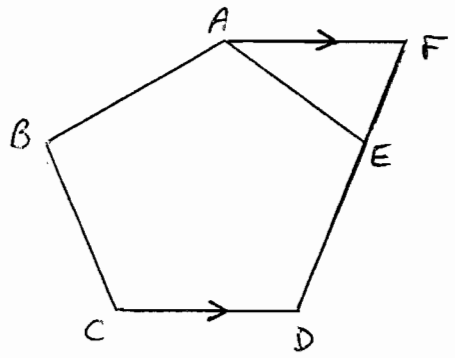
Question 8

a)



O is the centre of the circle. If $\angle AOC = x^o$, prove that $\angle B = \frac{x^o}{2}$ 3

b)



ABCDE is a regular pentagon. AF//CD and DE is produced to F.

Set out a geometrical proof to find the size of $\angle$FAE. 3

SOLUTIONS

1 a) 3.0×10^{-4} ← ① (① for 3.0)

b) $x = 0.4242\ldots$

$100x = 42.4242\ldots$

$99x = 42$ ① for correct method

$x = \frac{42}{99}$

$= \frac{14}{33}$ ← ①

c) $3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$ ← ①

d) $(3\sqrt{5})^2 - 3(3\sqrt{5}) + 1$

$= 9 \times 5 - 9\sqrt{5} + 1$ ① for substitution

$= 46 - 9\sqrt{5}$ ← ①

2 a) (i) $3x - 12x + 4x^2$

$= 4x^2 - 9x$ ①

(ii) $16a^6b^2 \times \frac{a^2}{6b^3}$ ①

$= \frac{8a^8}{3b}$ ← ①

b) $x^{\frac{1}{2}} \div x^{-1} = x^{\frac{3}{2}}$ ① ①

c) $(2^3)^n \times (2^2)^{n+1}$ ← ①

$= 2^{3n} \times 2^{2n+2}$

3 a) (i) $(x-6)(x+1)$ ①

(ii) $(x-2)(x^2+2x+4)$ ①

(iii) $x^2(x+1) + 1(x+1)$

$= (x+1)(x^2+1)$ ①

b) (i) $\frac{x(x-1)}{(x+1)(x-1)} = \frac{x}{x+1}$ ① ①

(iii) $\frac{4x}{x(x+2)} - \frac{1(x+2)}{x(x+2)}$ ①

$= \frac{4x - x - 2}{x(x+2)}$

$= \frac{3x-2}{x(x+2)}$ ①

4 a) $3 - 16 = -13$ ①

b) (i) $2x - 4 = 3x + 6$ ①

$-x = 10$

$x = -10$ ①

(ii) $(2x-1)(x+5) = 0$ ①

$\therefore x = \frac{1}{2}, -5$ ①

(iii) $1 - 2x = 6$ or $-(1-2x) = 6$ ①

$-2x = 5$ or $-1 + 2x = 6$

$x = -2\frac{1}{2}$ $2x = 7$

(5) a) $3x - y = 0$ —— ①

$4x - 2y = 3$ —— ②

① × −2 : $-6x + 2y = 0$ 1 mark for clear method

② + ③ : $-2x = 3$

$\boxed{x = -1\frac{1}{2}}$ 1 mark

Sub in ① : $-4\frac{1}{2} - y = 0$

$\boxed{y = -4\frac{1}{2}}$ 1 mark

b) $4 + x(x+2) = 3(x+2)$

$4 + x^2 + 2x = 3x + 6$

$x^2 - x - 2 = 0$ ← ①

$(x-2)(x+1) = 0$

$x = 2$ or -1 ← ①

c) (i) $f(-3) = 27 + 6$

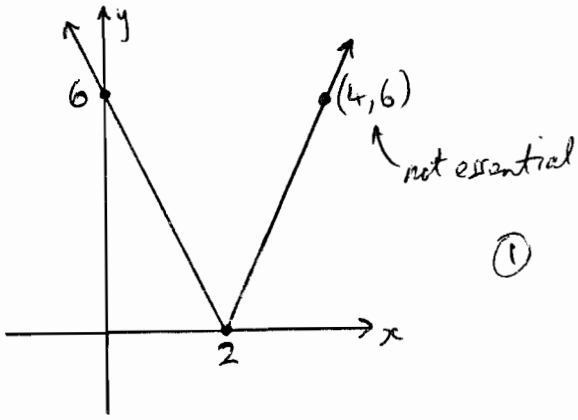
$= 33$ ← ①

(ii) $3(x+h)^2 - 2(x+h) + 3x^2 - 2x$ ← ①

$= 3x^2 + 6xh + 3h^2 - 2x - 2h + 3x^2 - 2x$

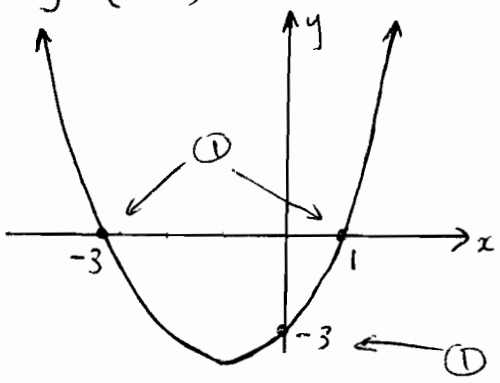
$= 6x^2 + 6xh + 3h^2 - 4x - 2h$ ← ①

(iii)



①

b) (i) $y = (x-1)(x+3)$



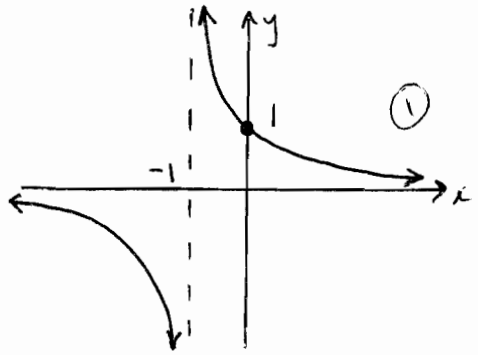
(ii) $-3 \leq x \leq 1$ ①

(iii) min. value when $x = -1$

$\therefore y = -4$

$\therefore$ range is $y \geqslant -4$

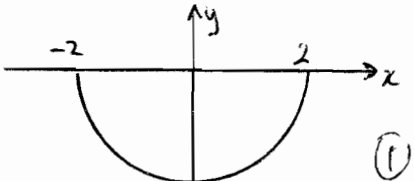
(6) a) (i)



must show marked info

①

(ii)



①

(7) a) $x - 4 \geq 0$

$\therefore D: x \geq 4$ ①

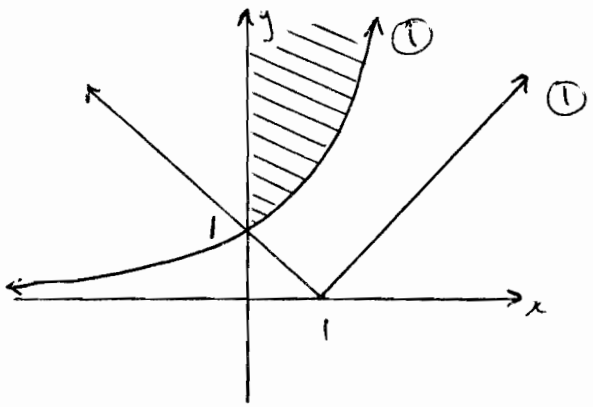
$R: y \geq 0$ ①

b) $(x-2)^2 + y^2 + 6y + 9 = 9$ ← ①

$(x-2)^2 + (y+3)^2 = 9$

$\therefore$ centre at $(2, -3)$ ← ①

c) (i)



① ①

(ii) $x < 0$ ①

(iii) above ①

(8) a) $x = \angle B + \angle C$ (exterior angle of $\triangle OBC$)

$\triangle OBC$ is isosceles ($OC = OB$, equal radii)

$\therefore \angle B = \angle C$ (base angles)

$\therefore \angle B = \angle C = \frac{x}{2}^\circ$

← 1 mark deducted for each error of reasoning

b) $\angle D = 108^\circ$ (angle sum for reg. pentagon is 540°)

$\therefore \angle F = 72^\circ$ (cointerior angles, $AF \parallel CD$)

Similarly, $\angle DEA = 108^\circ$

Now, $\angle DEA = \angle F + \angle FAE$ (exterior angle rule for $\triangle FAE$)

$\therefore \angle FAE = 108^\circ - 72^\circ$

$= 36^\circ$