

SYDNEY TECHNICAL HIGH SCHOOL



MATHEMATICS

Year 11

2 Unit

April 2007

Common Test

Time Allowed: 70 mins

Name :
Teacher :

Instructions:

- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read
- Write all answers in simplest form

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/8	/8	/8	/9	/8	/8	/8	/8	/65

Question 1

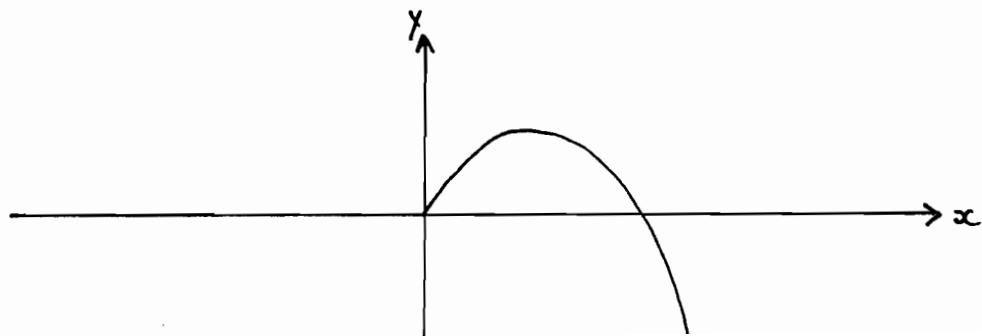
- a) Evaluate $\frac{2}{(1.11)^5 - 1}$ correct to 3 decimal places 2
- b) Express 68272 correct to 2 significant figures 1
- c) Write $\frac{x}{y} - 2$ as a single fraction 1
- d) Given that the surface area of an open cylinder is given by
$$A = 2\pi rh + \pi r^2$$
 find the value of h when $A = 250$ and $r = 6$ correct to 1 decimal place 2
- e) Subtract $3x^2 - 1$ from $x^3 - 2x^2 + 3$ 2

Question 2

- a) Find the exact value of $x^2 - 4x$ if $x = 4\sqrt{3}$ 2
- b) Rationalise the denominator of $\frac{\sqrt{3}}{6 - \sqrt{3}}$ 2
- c) Simplify $\sqrt{\frac{a^3 b^7}{ab^3}}$ 2
- d) State the range of the function $y = x^2 - 1$ 1
- e) The diagram shows part of a function $y = f(x)$ 1

Copy this diagram onto your answer sheet.

Complete the graph of $y = f(x)$ given that it is an odd function.



Question 3

- a) Factorise fully
- (i) $x^4 - x^2$ 2
- (ii) $3a^2 + 2a - 8$ 2
- (iii) $xy + 6x - y - 6$ 2
- b) Simplify $\frac{a^3 + 1}{a^2 - a + 1}$ 2

Question 4

- a) Solve the following
- (i) $\frac{x-5}{4} + 3 = \frac{5x}{3}$ 2
- (ii) $4x^2 - 4x + 1 = 0$ 2
- (iii) $|x+2| \leq 2$ 2
- (iv) $|x-1| = 3x-1$ 3

Question 5

- a) Sketch the following functions on separate number planes. Use a ruler to draw the axes. Label any important points.
- (i) $y = \frac{2}{x}$ 1
- (ii) $y = 3 - x$ 2
- (iii) $y = 9 - x^2$ 2
- b) Consider the function $y = \sqrt{16 - x^2}$
- (i) Explain why the domain of this function is restricted 1
- (ii) Find this domain 1
- (iii) Sketch the function on a number plane 1

Question 6

- a) (i) Solve simultaneously:

2

$$2x + 3y = 4$$

$$3x + 6y = 2$$

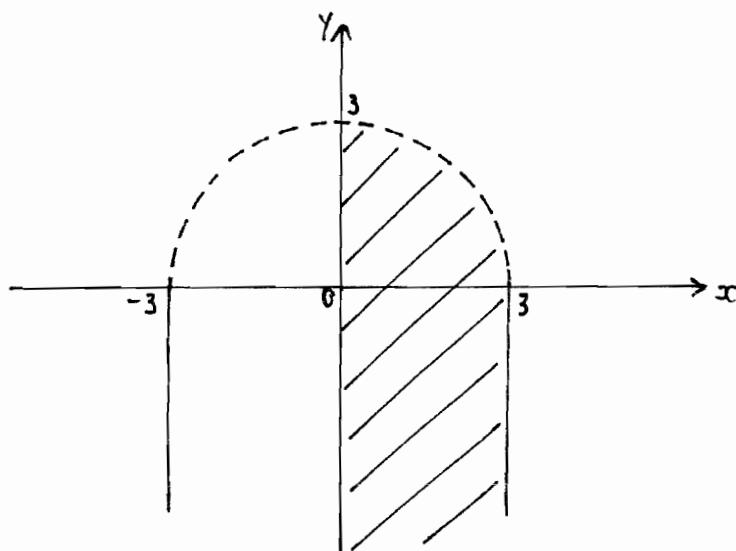
- (ii) What does this solution represent in relation to the graphs of

$$2x + 3y = 4 \text{ and } 3x + 6y = 2 ?$$

1

- b) State the two inequalities which represent the shaded region below:

2



- c) Express $0.\overline{36}$ as a simple fraction.

3

Question 7

- a) Find the values of a and b if $a + \sqrt{b} = 5 + 3\sqrt{6}$

2

- b) Simplify fully $\sqrt{98} \times \sqrt{48}$

2

- c) Solve $(x - 2)^2 = 3$ leaving your answer as a surd.

2

- d) The function $f(x)$ is defined as

2

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x > 3 \\ 3x & \text{if } -2 \leq x \leq 3 \\ 2 & \text{if } x < -2 \end{cases}$$

$$\text{Find } f(-3) + f(4) - f(3)$$

Question 8

a) Simplify $\frac{2}{x-y} + \frac{y}{x^2-y^2}$ 2

b) Simplify $f(x) + f(-x)$ if $f(x)$ is an odd function 1

c) Determine if the following function is odd, even or neither. 2

Show all working. $f(x) = \frac{x^2+1}{x^2-1}$

d) (i) Expand $(x - \frac{1}{x})^2$ 1

(ii) If $x - \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$ without solving for x 2

a) $\frac{2(x+y)}{x^2-y^2} - y = \frac{2x+3y}{x^2-y^2}$

b) $f(-x) = \frac{(-x)^2+1}{(-x)^2-1} = \frac{x^2+1}{x^2-1} = f(x)$ even

c)

Teacher's Name:

Student's Name/N^o:Year 11 2 Unit Ass. Task April 2007Question 1

a) 2.919 b) 6.8×10^4 or c) $\frac{x}{y} - 2$

$\frac{6.8 \times 10^4}{1}$

$= \frac{x - 2y}{y}$ ①

d) $A = 2\pi rh + \pi r^2$ e) $x^3 - 2x^2 + 3 - (3x^2 - 1)$ ①

$250 = 2\pi \times 6 \times h + \pi \times 6^2$ ①

$h = 3.6$ ①

$x^3 - 5x^2 + 4$ ①

Question 2

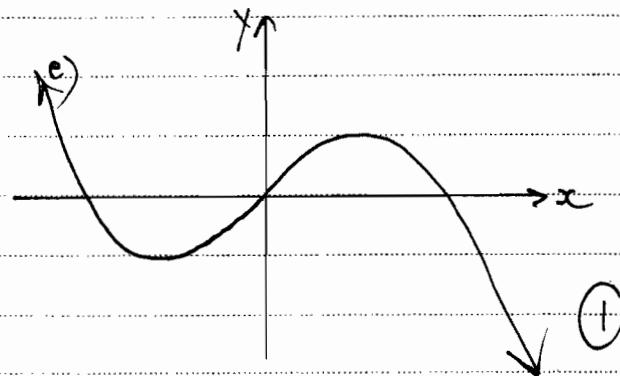
a) $x^2 - 4x$
 $(4\sqrt{3})^2 - 4(4\sqrt{3})$ ①
 $16 \times 3 - 16\sqrt{3}$
 $48 - 16\sqrt{3}$ ①

b) $\frac{\sqrt{3}}{6-\sqrt{3}} \times \frac{6+\sqrt{3}}{6+\sqrt{3}}$
 $\frac{6\sqrt{3}+3}{2\sqrt{3}+1}$ ①
 $\frac{33}{11}$ ①

c) $\frac{a^3 b^7}{a^2 b^4}$
 $= ab^3$ ①

d) $y = x^2 - 1$

Range: All real $y \geq -1$

Question 3

a) (i) $x^4 - x^2$
 $x^2(x^2 - 1)$ ①
 $x^2(x - 1)(x + 1)$ ①

(ii) $3a^2 + 2a - 8$
 $(3a - 4)(a + 2)$ ②

(iii) $xy + 6x - y - 6$
 $x(y + 6) - 1(y + 6)$ ①
 $(x - 1)(y + 6)$ ①

b) $a^3 + 1$ = $(a + 1)(a^2 - a + 1)$ ①
 $= (a + 1)$ ①

Teacher's Name:

Student's Name/N^o:Question 4

a) $\frac{x-5}{4} + 3 = \frac{5x}{3}$
 (i) $\frac{x-5}{4} + 3 = \frac{5x}{3}$

$$3(x-5) + 36 = 20x$$

$$3x - 15 + 36 = 20x \quad \textcircled{1}$$

$$21 = 17x$$

$$x = \frac{21}{17} \quad \textcircled{1}$$

(ii) $4x^2 - 4x + 1 = 0$

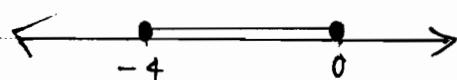
$$(2x-1)^2 = 0 \quad \textcircled{1}$$

$$x = \frac{1}{2} \quad \textcircled{1}$$

(iii) $|x+2| \leq 2$

$$x+2 \leq 2 \quad \text{or} \quad x+2 \geq -2 \quad \textcircled{1}$$

$$x \leq 0 \quad \text{or} \quad x \geq -4$$



$$-4 \leq x \leq 0 \quad \textcircled{1}$$

(iv) $|x-1| = 3x-1$

$$x-1 = 3x-1 \quad \text{or} \quad x-1 = -(3x-1)$$

$$0 = 2x$$

$$x-1 = -3x+1$$

$$x = 0 \quad \textcircled{1}$$

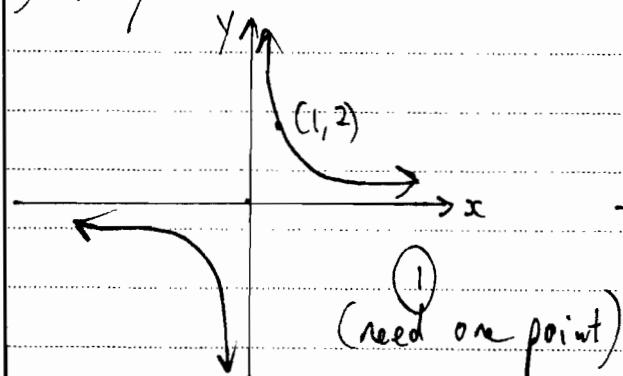
$$\text{or} \quad 4x = 2$$

$$x = \frac{1}{2} \quad \textcircled{1}$$

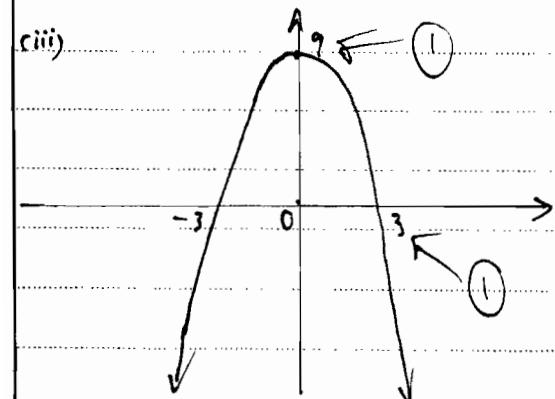
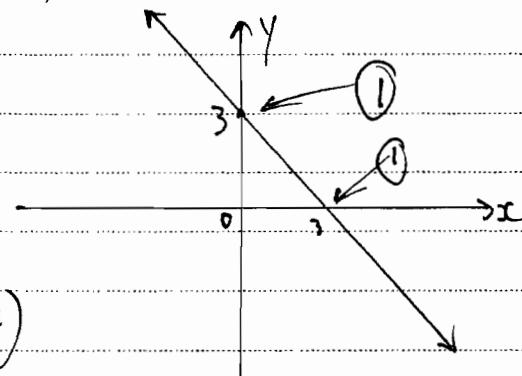
Check! $x = \frac{1}{2}$ only $\textcircled{1}$

Question 5

a) (i) $y = \frac{2}{x}$



(ii)



b) (i) You can't $\sqrt{-}$ a negative!

(iii) $16 - x^2 \geq 0$

$$(4-x)(4+x) \geq 0$$

$$-4 \leq x \leq 4 \quad \textcircled{1}$$



(iii)

① - need

Teacher's Name: _____ Student's Name/N^o: _____

Question 6

a) (i) $2x + 3y = 4 \quad \textcircled{1}$
 $3x + 6y = 2 \quad \textcircled{2}$

$$2x \textcircled{1} - \textcircled{2}$$

$$4x + 6y = 8 -$$

$$3x + 6y = 2$$

$$x = 6$$

$$\therefore 2x + 3y = 4$$

$$3y = -8$$

$$\underline{x = 6}, \underline{y = -\frac{8}{3}} \quad \textcircled{1}$$

(ii) Point of intersection $\textcircled{1}$

b) $x \geq 0 \quad \text{and} \quad \textcircled{1}$

$$y < \sqrt{9-x^2}$$

$\textcircled{1}$

c) Let $x = 0.3666\ldots$

$$\therefore 10x = 3.6666\ldots \textcircled{1}$$

$$\therefore 9x = 3.3$$

$$x = \frac{3.3}{9}$$

$$= \frac{33}{90}$$

$$\underline{x = \frac{11}{30}} \quad \textcircled{1}$$

Question 7

a) $a + \sqrt{b} = 5 + 3\sqrt{6}$

$$a + \sqrt{b} = 5 + \sqrt{54}$$

$$\therefore a = 5, b = 54$$

$\textcircled{1}, \textcircled{1}$

b) $\sqrt{28} \times \sqrt{48}$

$$= 7\sqrt{2} \times 4\sqrt{3} \quad \textcircled{1}$$

$$= 28\sqrt{6} \quad \textcircled{1}$$

c) $(x-2)^2 = 3$

$$x-2 = \pm\sqrt{3}$$

$$x = 2 \pm \sqrt{3} \quad \textcircled{1}$$

d) $f(-3) + f(4) - f(3)$

$$2 + 4^2 + 1 - 3 \times 3 \quad \textcircled{1}$$

$$2 + 17 - 9$$

$$= 10$$

$\textcircled{1}$

Teacher's Name:

Student's Name/Nº:

Question 8

$$\text{a) } \frac{2}{x-y} + \frac{y}{x^2-y^2}$$

$$\text{b) } f(x) + f(-x) = 0 \quad \textcircled{1}$$

$$\frac{2}{x-y} + \frac{y}{(x-y)(x+y)}$$

$$\frac{2(x+y)}{(x-y)(x+y)} + \frac{y}{(x-y)(x+y)} \quad \textcircled{1}$$

$$= \frac{2x+3y}{(x-y)(x+y)} \text{ or } \frac{2x+3y}{x^2-y^2} \quad \textcircled{1}$$

$$\text{c) } f(x) = \frac{x^2+1}{x^2-1}$$

odd if $f(-x) = -f(x)$

$$\frac{(-x)^2+1}{(-x)^2-1} = -\left(\frac{x^2+1}{x^2-1}\right)$$

even if $f(x) = f(-x)$

$$\frac{x^2+1}{x^2-1} = \frac{(-x)^2+1}{(-x)^2-1}$$

$$\frac{x^2+1}{x^2-1} \neq -\frac{x^2+1}{x^2-1}$$

$$\frac{x^2+1}{x^2-1} = \frac{x^2+1}{x^2-1} \quad \textcircled{1}$$

 \therefore Not odd \therefore Even

$$\text{d) i) } (x - \frac{1}{x})^2$$

$$= x^2 - 1 - 1 + \frac{1}{x^2}$$

$$\text{ii) } (x - \frac{1}{x})^2 = x^2 + \frac{1}{x^2} - 2$$

$$\therefore x^2 + \frac{1}{x^2} = (x - \frac{1}{x})^2 + 2 \quad \textcircled{1}$$

$$= x^2 + \frac{1}{x^2} - 2$$

$$x^2 + \frac{1}{x^2} = 11$$

 $\textcircled{1}$ $\textcircled{1}$