SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Year 11

2 Unit

Task 1
2013

Common Test

Time Allowed: 70 mins

Name : _____

Teachers Name : _____

Instructions:

- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read
- Write all answers in simplest form

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/8	/8	/8	/8	/8	/8	/8	/8	/64

Question 1 **8 marks**

a) Arrange in ascending order:

$$3^{\frac{1}{2}}, \quad 2^{\frac{5}{6}}, \quad \left(\frac{3}{11}\right)^{-\frac{1}{3}}$$ 1

b) Calculate $\dfrac{9.62 \times 10^{-4}}{3.67 \times 10^{-3} \times 2.67 \times 10^{2}}$ giving your answer correct to 2 significant figures 1

c) Write $\frac{x}{y} - 2$ as a single fraction 1

d) Given $s = \frac{n}{2}[2a + (n - 1)d]$ find s when n=16, a=5, d=4 1

e) Gold, a very soft metal, can be hammered into sheets of thickness 1.02×10^{-4} mm.
How many such sheets are needed to make a pile 1 cm thick? 1

f) Express $0.2\dot{9}\dot{8}$ as a fraction in lowest terms. 2

g) Rewrite $2(x^2 - 1)^{-\frac{1}{2}}$ as an expression with no negative or fractional indices 1

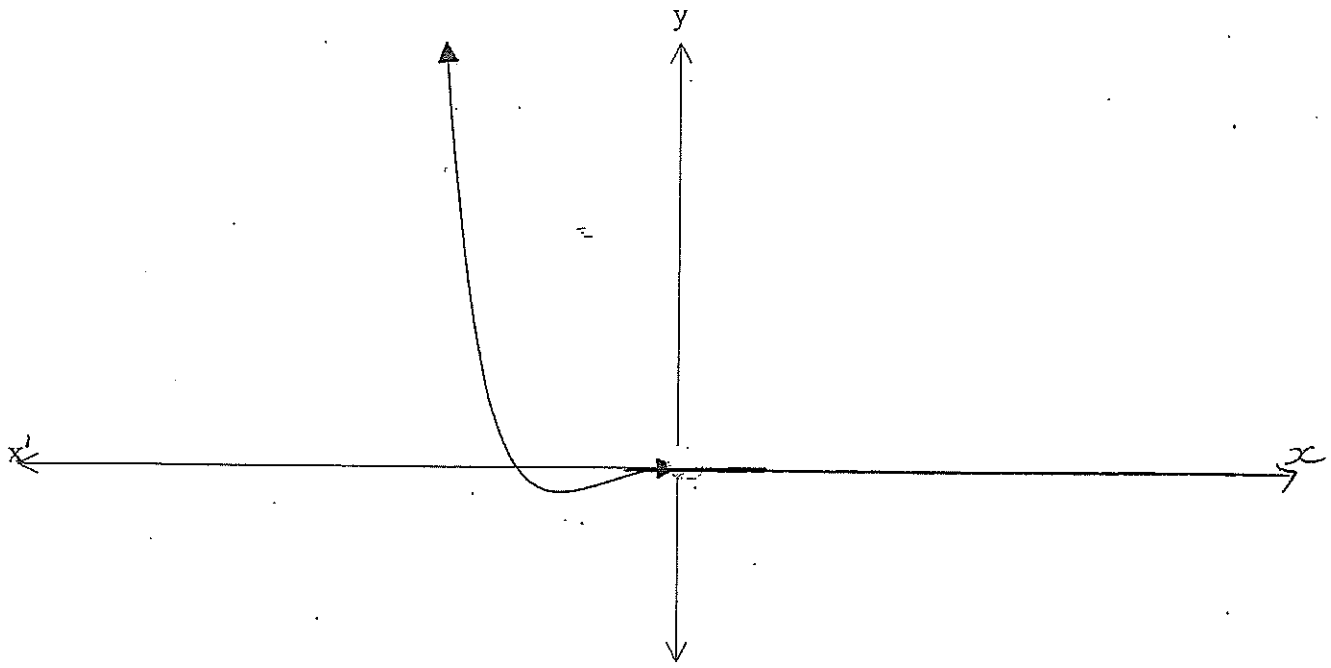
Question 2 **8 marks**

a) Simplify $\frac{1}{\sqrt{3}+\sqrt{x}} + \frac{1}{\sqrt{3}-\sqrt{x}}$ 2

b) Subtract $3x^2 - 1$ from $x^3 - 2x^2 + 3$ 1

c) The area of a circle is found using the formula $A = \pi r^2$. If the area of a particular circle is 45cm^2, find the radius correct to 1 decimal place 1

d) The diagram shows part of a function $y = f(x)$



Copy this diagram onto your answer sheet.

Complete the graph of $y = f(x)$ given that it is an even function. 1

e) Simplify $\sqrt{\frac{a^3b^7}{ab^3}}$ 2

f) Simplify $a^7 \times 5a^{-3} \div 15a^{-4}$ 1

Question 3 **8 marks**

Factorise fully

a) $x^4 - x^2$ 2

b) $3a^2 + 2a - 8$ 2

c) $x^2 - 12xy + 20y^2$ 2

d) $16x^4 - 2x$ 2

Question 4 **8 marks**

a) (i) Solve the following equations simultaneously:

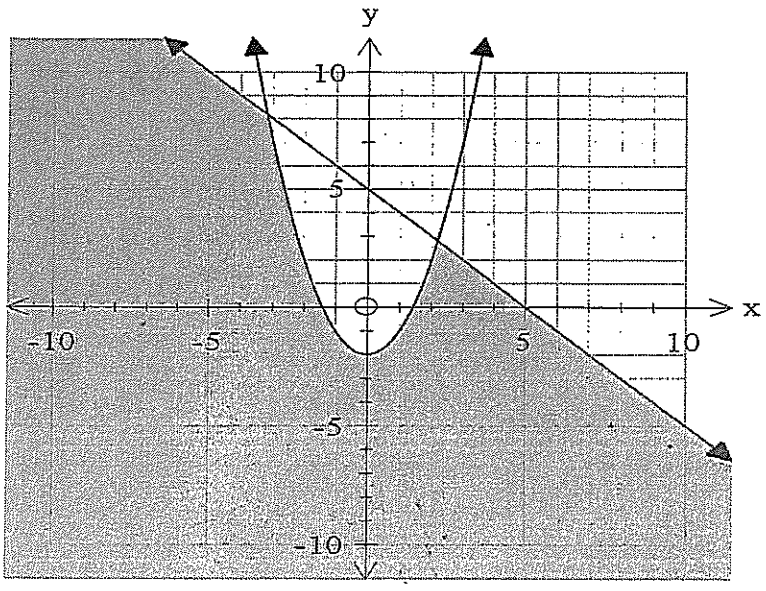
$$\left\{ \begin{array}{l} x^2 + y^2 = 21 \\ x + y = 3 \end{array} \right\}$$ 2

(ii) What does this solution represent in relation to the graphs of :

$x^2 + y^2 = 21$ and $x + y = 3$? 1

b) Using the process of completing the square, solve the following leaving your answer in surd form: $x^2 + 4x = 1$ 2

c) State the two inequalities which represent the shaded region below: 3



Question 5 **8 marks**

Solve the following

a) $\frac{x-5}{4}+3=\frac{5x}{3}$ 2

b) $4x^2-4x+1=0$ 2

c) $|x+2|\leq 2$ 2

d) $|2x+6|=3x-1$ 2

Question 6 **8 marks**

a) Sketch the following graphs on separate number planes. Use a ruler to draw all straight lines. Label any important points.

i. $y=|x-2|$ 2

ii. $x^2+y^2=4$ 2

iii. $y=(x+1)^2+2$ 2

b) From a visual perspective, which of the previous questions are functions? (Do not provide a formal proof). 2

Question 7 **8 marks**

a) Simplify fully $\sqrt{98} \times \sqrt{48}$ 2

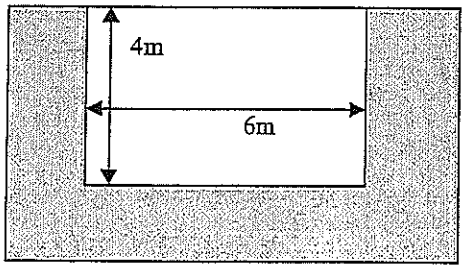
b) Simplify $\dfrac{a^3+1}{a^2-a+1}$ 1

c) The function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2+1 & if\ x>3 \\ 3x & if\ -2 \le x \le 3 \\ 2 & if x<-2 \end{cases}$$

Find $f(-3)+f(4)-f(3)$ 3

d) A rectangular garden bed; 6m long and 4m wide has a path of uniform width around three sides as shown



If the area of the path is 25.5m^2, use a quadratic equation to find the width of the path.

2

Question 8 **8 marks**

a) For the function $y = \dfrac{x^2 + 3x}{x + 3}$

i. State any discontinuities 1

ii. State the domain and range of the function 2

iii. Sketch the graph of the function, showing all important points. 3

b) Find $f(x)$ for all x given that $f(x-1) = x^2 - 1$ 2

End of Paper

21

) $\sqrt{3} = 1.732\ldots$

$2^{5/6} = 1.78179\ldots$

$3/11^{-1/3} = 1.54202\ldots$

$3/11^{-1/3}$, $\sqrt{3}$, $2^{5/6}$

) 9.8×10^{-4}

) $\frac{x-2y}{y}$

) $= {}^{16}/_{2}[10 + (15)4]$

$= 560$

) $1.02 \times 10^{-4}\ \overline{)10.00}$

$= 98039.21$

$= 98040$ sheets

) $1000n = 298.989898\ldots$

$10n = 2.9898\ldots$

$990n = 296$

$n = {}^{148}/_{495}$

) $= \frac{2}{\sqrt{(x^2-1)}}$

Q2

$$\frac{1}{\sqrt{3}+\sqrt{x}} + \frac{1}{\sqrt{3}-\sqrt{x}}$$

$$= \frac{\sqrt{3}-\sqrt{x} + \sqrt{3}+\sqrt{x}}{(\sqrt{3}+\sqrt{x})(\sqrt{3}-\sqrt{x})}$$

$$= \frac{2\sqrt{3}}{3-x}$$

(b) $x^3 - 2x^2 + 3 - (3x^2 - 1)$

$x^3 - 5x^2 + 4$

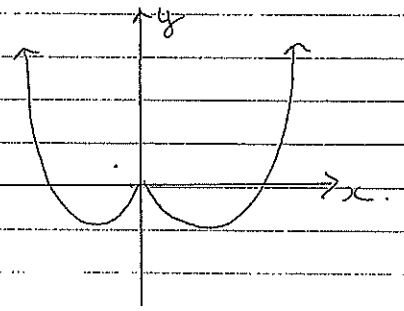
(c) $\pi r^2 = 45$

$r = \sqrt{45/\pi}$

$= 3.784\ldots$

$= 3.8$

(d)



(e) $= ab^2$

(f) $= \frac{a^8}{3}$

Q3

(a) $x^2(x-1)(x+1)$

(b) $(a+2)(3a-4)$

(c) $(x-2y)(x-10y)$

(d) $2x((2x)^3 - 1^3)$

$= (2x)(2x-1)(4x^2+2x+1)$

Q4

(a) (i) $x^2 + y^2 = 21$ — (1)

$x + y = 3$ — (2)

$y = 3 - x$ sub into (1)

$x^2 + (3-x)^2 = 21$

$x^2 + 9 - 6x + x^2 = 21$

$2x^2 - 6x + 12 = 0$

$2(x^2 - 3x - 6) = 0$

$x = \frac{3 \pm \sqrt{33}}{2}$ 4.37, −1.37

$x = \frac{3+\sqrt{33}}{2}$ $y = \frac{6}{2} - \left(\frac{3+\sqrt{33}}{2}\right)$

$= \frac{3-\sqrt{33}}{2}$

$x = \frac{3-\sqrt{33}}{2}$ $y = \frac{3+\sqrt{33}}{2}$

2

(ii) where graphs intersect.

1

(b) $x^2 + 4x = 1$

$(x+2)^2 = 1 + 4$

$x + 2 = \pm\sqrt{5}$

$x = \sqrt{5} - 2, -\sqrt{5} - 2$

2

(c) $y \leq -x + 5$

$y \leq x^2 - 2$

3

Q5

(a) $\frac{x-5}{4} + 3 = \frac{5x}{3}$

$\frac{x-5+12}{4} = \frac{5x}{3}$

$\frac{x+7}{4} = \frac{5x}{3}$

$3x + 21 = 20x$

$21 = 17x$

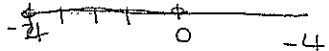
$x = {}^{21}/_{17}$

(c) $|x+2| \leq 2$

$x + 2 \leq 2$ $\quad -x - 2 \leq 2$

$x \leq 0$ $\quad -x \leq 4$

$x \geqslant -4$



$-4 \leq x \leq 0$

Q5

(d) $|2x+6| = 3x-1$

$2x+6 = 3x-1$

$7 = x$

check.

LHS: 20

RHS: 20

$-2x-6 = 3x-1$

$-5 = 5x$

$x = -1$

check

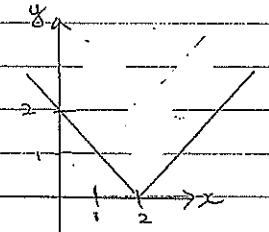
LHS: 4

RHS: -4

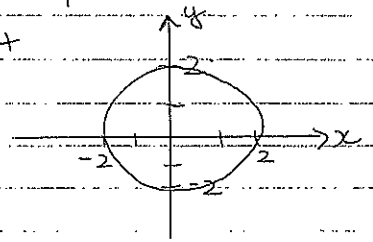
∴ only soln $x=7$

Q6

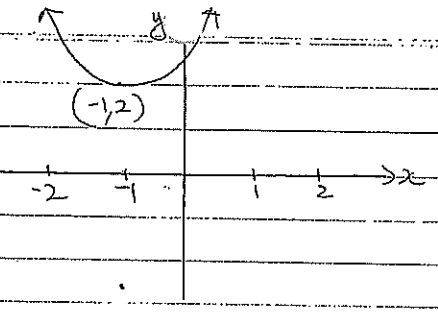
(i) $y = |x-2|$



(ii) $x^2+y^2=4$



(iii)



(b)

(i) & (iii)

Q7

$\sqrt{98} \times \sqrt{48} = \sqrt{4704}$

$= \sqrt{16 \times 294}$

$= 4\sqrt{49 \times 6}$

$= 28\sqrt{6}$

(b) $\frac{a^3+1}{a^2-a+1} = \frac{(a+1)(a^2-a+1)}{a^2-a+1}$

$= a+1$

(c) $f(-3) = 2$

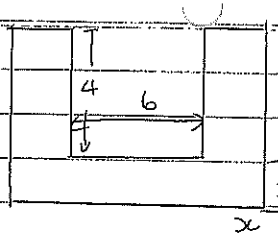
$f(4) = 17$

$f(9) = 9$

$2 + 17 - 9 = 10$

Q7

(d)



$A = 2(4+x)x + 6x$

$255 = 2x^2 + 14x$

$4x^2 + 28x - 51 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-28 \pm \sqrt{1600}}{8}$

$= \frac{-28 \pm 40}{8}$

$= 3/2, \quad x \neq -68/8$

∴ width = 3/2 m

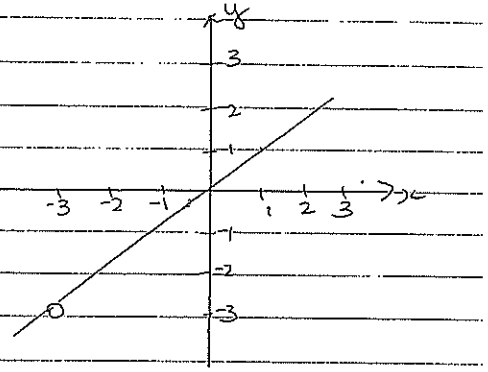
Q8

a) $y = \frac{x^2+3x}{x+3}$

(i) $x = -3$

(ii) domain $x: x \in R \; x \neq$

range $y: y \in R \; y \neq$



(b) $f(x-1) = x^2 - 1$

$f(x) = x^2 + 2x$