Name: .... MathsTeacher: ......File..........

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

Assessment 2

JULY, 2015

Time allowed: 90 minutes

- ***General Instructions:***
  - Marks for each question are indicated on the question.
  - Approved calculators may be used
  - All necessary working should be shown
  - Full marks may not be awarded for careless work or illegible writing
  - ***Begin each question on a new page***
  - Write using black or blue pen
  - All answers are to be in the writing booklet provided

- Section 1 Multiple Choice
  - Questions 1-5
  - 5 Marks
- Section II Questions 6-13
  - 63 Marks

Section I

Answers to be done on the multiple choice answer sheet in your answer booklet.

1. What are the solutions of $2x^2 - 5x - 1 = 0$?

   (A) $x = \frac{-5\pm\sqrt{17}}{4}$

   (B) $x = \frac{5\pm\sqrt{17}}{4}$

   (C) $x = \frac{-5\pm\sqrt{33}}{4}$

   (D) $x = \frac{5\pm\sqrt{33}}{4}$

2. Which inequality defines the domain of the function $f(x) = \frac{1}{\sqrt{x+3}}$?

   (A) $x > -3$

   (B) $x \geq -3$

   (C) $x < -3$

   (D) $x \leq -3$

3. Find the values of m for which $24 + 2m - m^2 \leq 0$

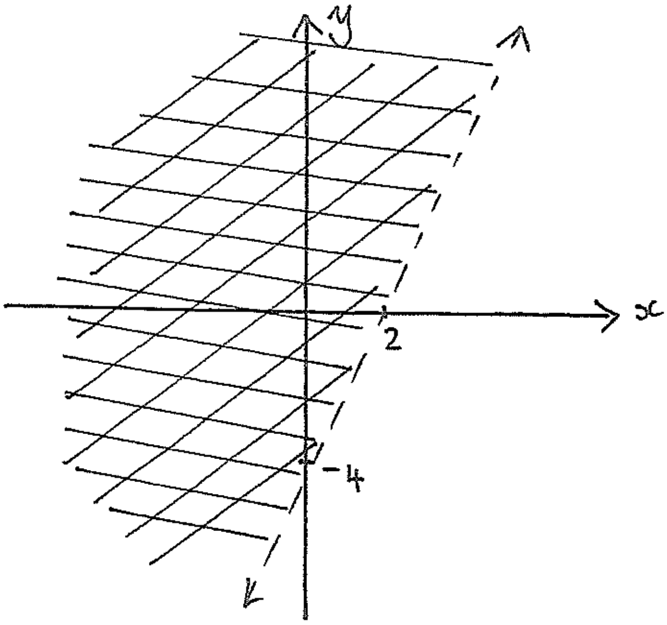
   (A) $m \leq -4 \ or\ m \geq 6$

   (B) $m \leq -6 \ or\ m \geq 4$

   (C) $-4 \leq m \leq 6$

   (D) $-6 \leq m \leq 4$

4.



The shaded region is best described by the inequality.

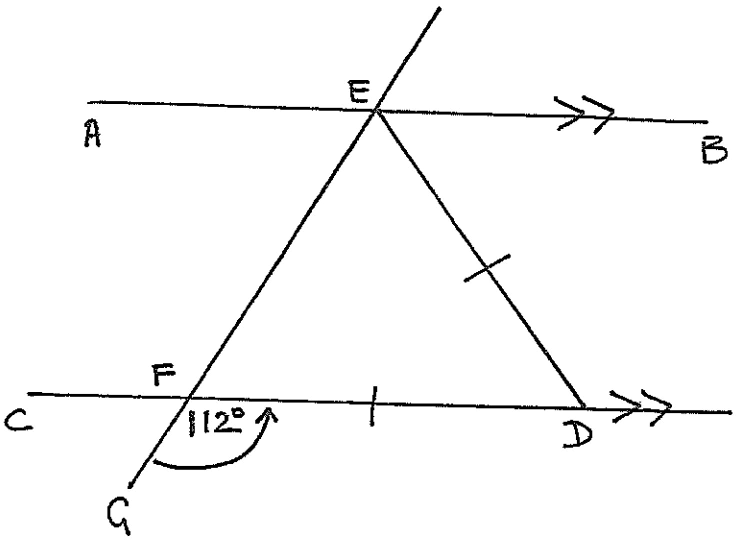
(A) $2x - y - 4 \geq 0$

(B) $2x - y - 4 \leq 0$

(C) $2x - y - 4 > 0$

(D) $2x - y - 4 < 0$

5.



If AB ∥ CD, ED = FD and ∠DFG = 112°
then ∠BED =

(A) 112° (B) 24° (C) 68° (D) 44°

Section II

Mark

Question 6 – (8 marks)

a) Evaluate $\sqrt[3]{\frac{651}{4\pi}}$ to four significant figures — 2

b) Solve $2 - 3x \leq 8$ and sketch your solution on a number line — 2

c) Solve $x^2 - 6x = 0$ — 2

d) Solve $4 < 4x - 3 < 9$ — 2

Question 7 – (8 marks) – Start a new page

a) Express $\frac{a^{-1}+b^{-1}}{a+b}$ in simplest fraction form without using negative indices. — 2

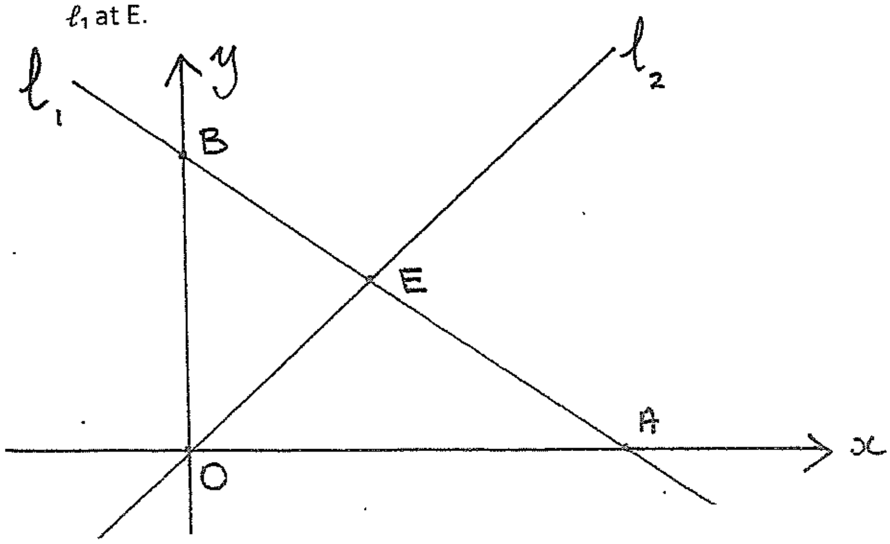
b) Solve $|5x - 2| = |3x + 4|$ — 2

c) Solve $\frac{5}{8}(x+4) = 4x - \frac{1}{2}$ — 2

d) Express $\frac{3\sqrt{2}}{3\sqrt{2}+2\sqrt{3}}$ in the form $a + b\sqrt{6}$ — 2

Question 8 – (8 marks) – Start a new page

a) The diagram shows a line ℓ_1 with equation $3x + 4y - 12 = 0$, which intersects the y axis at B.
A second line ℓ_2 with equation $4x - 3y = 0$, passes through the origin O and intersects ℓ_1 at E.



NOT TO SCALE

(i) Show that coordinates of B are (0, 3). 1

(ii) Show that ℓ_1 is perpendicular to . ℓ_2 2

(iii) Show that the perpendicular distance from O to ℓ_1 is $\frac{12}{5}$ units 1

(iv) Using Pythagoras' theorem, or otherwise, find the length of the interval BE. 1

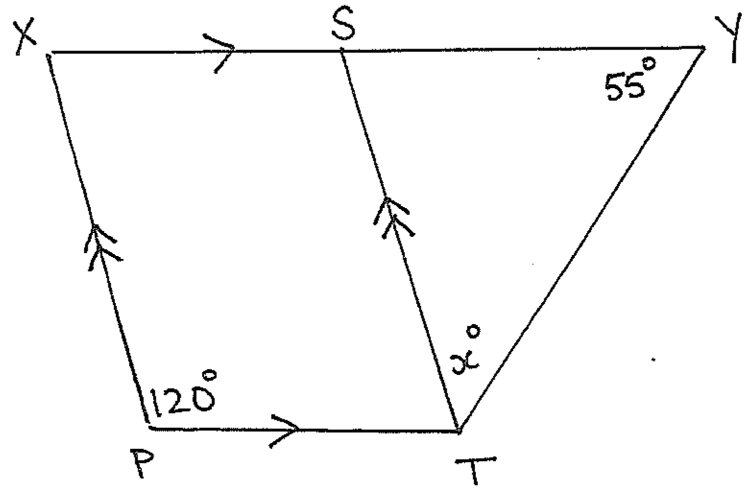
(v) Hence, or otherwise, find the area of ΔBOE. 1

b) Simplify $\frac{x^3-1}{x^2-1} \div \frac{3x^2+3x+3}{x^2-4x-5}$ 2

Question 9 – (7 marks) – Start a new page

Mark

a) 2



XY ll PT and XP ll ST
Redraw the diagram in your answer booklet.
Find x giving reasons for your answer.

b) A function is defined as follows

$$f(x)= \begin{cases} 0 & \text{if } x \leq -3 \\ -1 & \text{if } -3 < x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Find

i) $f(-3)+f(-2)+f(2)$ 1

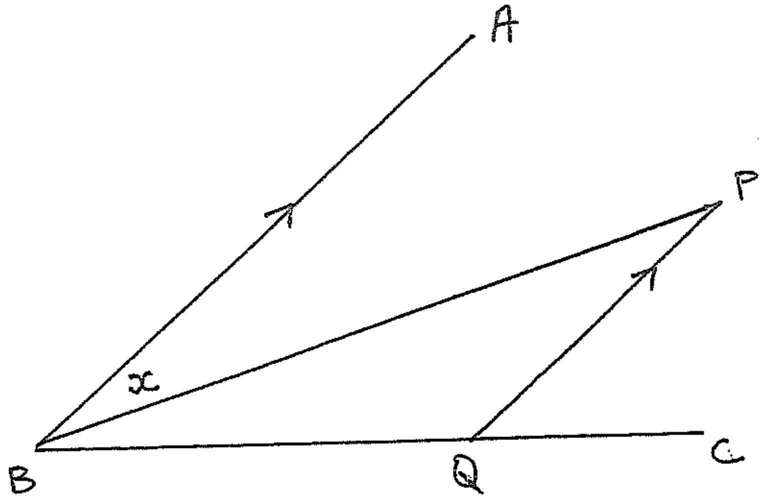
ii) $f(a^2)$ 1

c) i) Sketch $y = |x-1|$ and $y = x+1$ on the same axes. 2
Use a ruler and label each function carefully. Show any points of intersection with the x and y axes. Your sketch should be approximately half a page.

ii) Hence solve $|x-1| > x+1$ 1

Question 10 – (8 marks) – Start a new page

a)



2

Let $A\hat{B}P = x$
BP bisects $A\hat{B}C$ and $AB||PQ$
Redraw this diagram in your answer booklet. Use a ruler.
Your diagram should be approximately half a page in size.
Prove that BQ=PQ

b) Find the exact value of

i) $\sin 225^0$ 1

ii) $\tan(-30^0)$ 1

c) If θ is obtuse and $\tan\theta = \frac{-1}{5}$ find the exact value of $\cos\theta$ 1

d) Prove $\frac{1}{\sin\theta.\cos\theta}$ - $\tan\theta = \cot\theta$ 3

Question 11 – (8 marks) – Start a new page

a) Solve the following in the domain $0^{\circ} \le x \le 360^{\circ}$.
(write your answers correct to the nearest minute)

i) $\tan 2\theta = -1$ 2

ii) $3\sin^2\theta + 2\sin\theta = 0$ 2

iii) $3\sin\theta = 2\cos\theta$ 2

b) Find $\lim_{x \to 3} \frac{x-3}{x^2-9}$ 2

Question 12 – (8 marks) – Start a new page

a) Differentiate the following

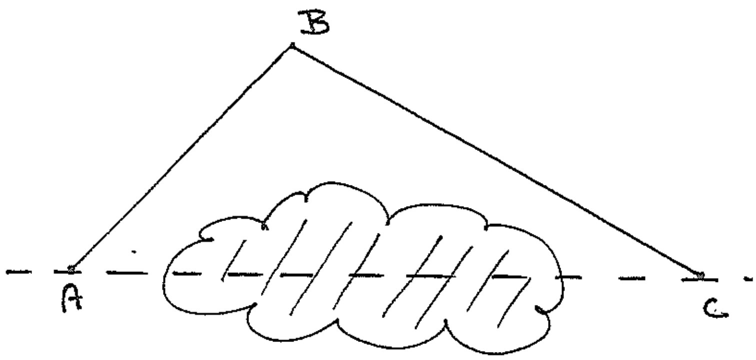
i) $y = 4x^3 - x + 5$ 1

ii) $y = (3x^2 - 4)^4$ 2

iii) $y = \frac{x+1}{x-1}$ 2

b)

A surveyor walking due east turns at A to avoid marshy country and walks 270 metres to B on a bearing of 048° and then turns and walks on a bearing of 112° to C. C is due east of A.

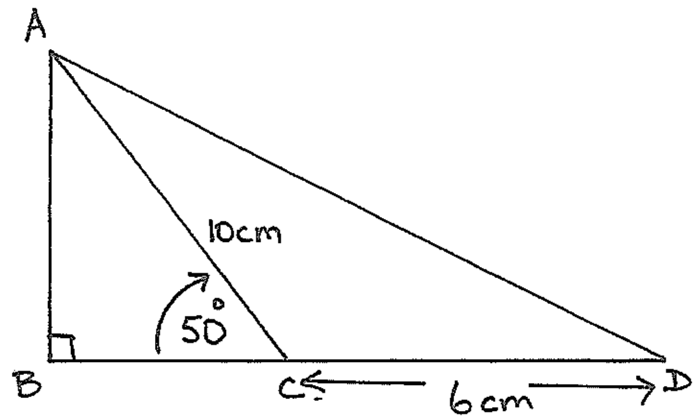


i) Redraw the diagram showing the size of angles BÂC, AB̂C and BĈA. 1

ii) Hence find the length of AC to the nearest metre. 2

Question 13 – (8 Marks) – Start a new page Mark

a) In the figure CD = 6cm, AC = 10cm, angle ACB = 50^0 and angle ABC = 90^0. Find:

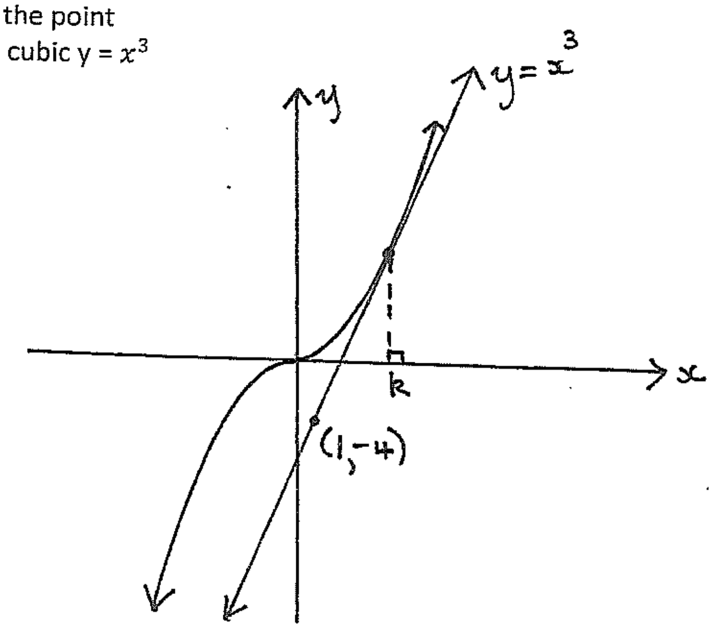


i) AD to the nearest cm 2

ii) Area of ΔACD to the nearest cm^2. 1

b) i) Show that $k = 2$ is a solution to the equation $2k^3 - 3k^2 - 4 = 0$ 1

ii) The diagram shows a tangent at the point where x = k (where k> 0) to the cubic y = x^3



∝. Find the gradient of the tangent at $x = k$ 1

β. Find the equation of the tangent at $x = k$ 2

γ. If the tangent is found to pass through (1,-4) find the value of k. 1

YEAR 11 TERM 3 TEST EXT 1 JULY 2015

Q1 D

2 A

3 A

4 D

5 D

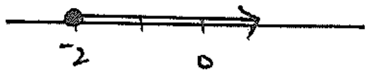
Question 6

a) 3.728 (4 sig. fig)

b) $2 - 3x \le 8$

$-3x \le 6$

$x \ge -2$



c) $x^2 - 6x = 0$

$x(x-6) = 0$

$\therefore x = 0,\ x = 6$

d) $4 < 4x - 3 < 9$

$7 < 4x < 12$

$\frac{7}{4} < x < 3$

Question 7

a) $\left(\frac{1}{a} + \frac{1}{b}\right) \div (a+b)$

$\frac{(b+a)}{ab} \times \frac{1}{(a+b)}$

$\frac{1}{ab}$

b) $5x - 2 = 3x + 4$

$2x = 6$

$\therefore x = 3$

$5x - 2 = -(3x+4)$

$5x - 2 = -3x - 4$

$8x = -2$

and $x = \frac{-1}{4}$

c) $\frac{5}{8}(x+4) = 4x - \frac{1}{2}$

$5(x+4) = 32x - 4$

$5x + 20 = 32x - 4$

$24 = 27x$

$x = \frac{24}{27}$

$\therefore x = \frac{8}{9}$

d) $\frac{3\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} \times \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

$\frac{3\sqrt{2}(3\sqrt{2}-2\sqrt{3})}{18-12}$

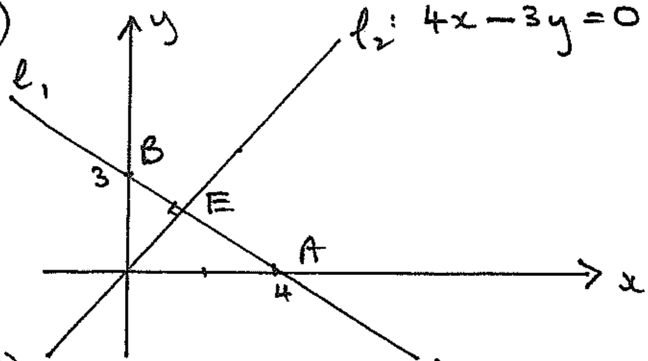
$\frac{18-6\sqrt{6}}{6}$

$\frac{6(3-\sqrt{6})}{6}$

$\therefore \frac{3\sqrt{2}}{3\sqrt{2}+2\sqrt{3}} = 3 - \sqrt{6}$

Question 8

a)



i) sub. $x = 0$ into l_1: $3x + 4y - 12 = 0$

$4y = 12$

$y = 3$

$\therefore B(0, 3)$

ii) $m_{l_1} = -\frac{3}{4}$ $m_{l_2} = \frac{4}{3}$

since $\frac{-3}{4} \cdot \frac{4}{3} = -1$

$\therefore l_1 \perp l_2$

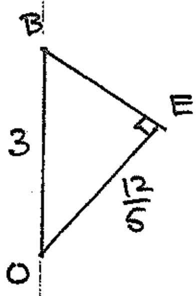
iii) $p = \left| \dfrac{3.0 + 4.0 - 12}{\sqrt{9+16}} \right|$

$l_1: 3x + 4y - 12 = 0$

$p = \left| \dfrac{-12}{5} \right|$

$\therefore \underline{\underline{p = \dfrac{12}{5} \text{ units}}}$

iv)



$3^2 - \left(\dfrac{12}{5}\right)^2 = (BE)^2$

$9 - \dfrac{144}{25} = (BE)^2$

$\dfrac{81}{25} = (BE)^2$

$\therefore \underline{\underline{BE = \dfrac{9}{5} \text{ units}}}$

v) Area $\Delta BOE = \dfrac{1}{2}\left(\dfrac{12}{5} \times \dfrac{9}{5}\right)$

$= \underline{\underline{\dfrac{54}{25} \text{ units}^2}}$

b) $\dfrac{x^3 - 1}{x^2 - 1} \times \dfrac{x^2 - 4x - 5}{3x^2 + 3x + 3}$

$\dfrac{(x-1)(x^2+x+1)}{(x-1)(x+1)} \times \dfrac{(x-5)(x+1)}{3(x^2+x+1)}$

$\underline{\underline{\dfrac{x-5}{3}}}$

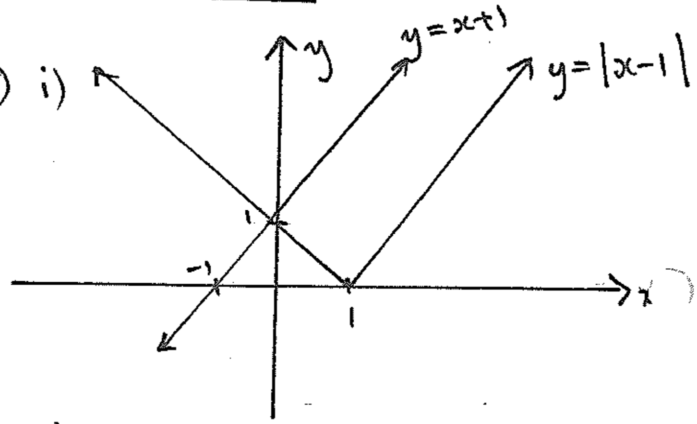
b) i) $f(-3) + f(-2) + f(2)$

$= 0 + -1 + 2$

$\underline{\underline{= 1}}$

ii) $\underline{\underline{f(a^2) = a^2}}$ since $a^2 \geq 0$

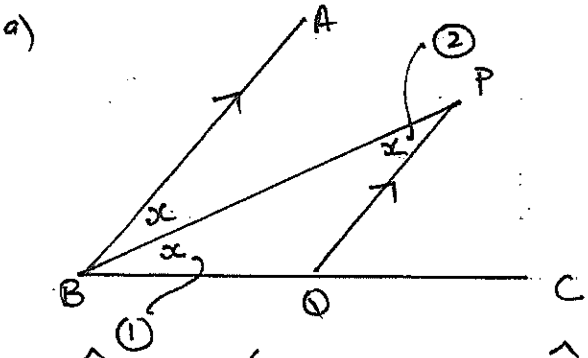
c) i)



ii) $\underline{\underline{x < 0}}$

Question 10

a)



$P\hat{B}Q = x$ (BP bisects $A\hat{B}C$)

$B\hat{P}Q = x$ (alternate angles $AB \parallel PQ$)

$\therefore PQ = BQ$ (sides opposite equal angles in isoceles triangl)

b) i) $\sin 225° = \sin(180 + 45)$

S	A
✓ T	C

$= -\sin 45°$

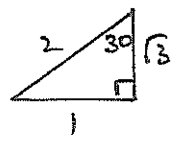
$= \underline{\underline{-1/\sqrt{2}}}$

ii) $\tan(-30°) = \tan(360 - 30)$

S	A
T	C ✓

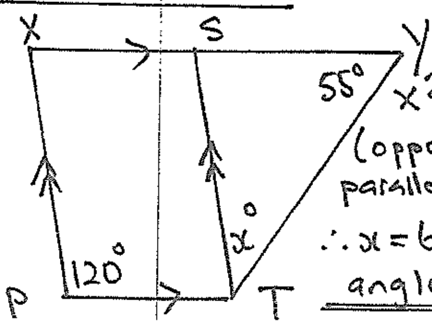
$= -\tan 30°$

$= \underline{\underline{-\dfrac{1}{\sqrt{3}}}}$



Question 9

a)

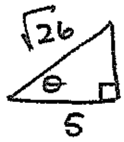


$X\hat{S}T = 120°$ (opposite angles of parallelogram equal)

$\therefore x = 65°$ (exterior angle of ΔSYT)

c) $\tan\theta = -\frac{1}{5}$

S ✓	A
T	C ✓



$\therefore \cos\theta = -\frac{5}{\sqrt{26}}$

d) $LHS = \frac{1}{\sin\theta.\cos\theta} - \tan\theta$

$= \frac{1}{\sin\theta.\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \frac{1-\sin^2\theta}{\sin\theta.\cos\theta}$

$= \frac{\cos^2\theta}{\sin\theta.\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= RHS$

QUESTION 11

a) i) $\tan 2\theta = -1$

S ✓	A
T	C ✓

"acute" $2\theta = 45°$

$\therefore 2\theta = 135°, 315°, 495°, 675°$

$\theta = 67\frac{1}{2}°, 157\frac{1}{2}°, 247\frac{1}{2}°, 337\frac{1}{2}°$

OR $67°30', 157°30', 247°30', 337°30'$

ii) $3\sin^2\theta + 2\sin\theta = 0$

$\sin\theta(3\sin\theta + 2) = 0$

$\sin\theta = 0 \qquad \sin\theta = -\frac{2}{3}$

$\theta = 0°, 180°, 360°$ and

S	A
T ✓	C ✓

$\theta = 221°49', 318°11'$

iii) $3\sin\theta = 2\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{2}{3}$

S	A ✓
T ✓	C

$\tan\theta = \frac{2}{3}$

$\therefore \theta = 33°41', 213°41'$

b) $\lim_{x\to 3} \frac{1(x-3)}{(x-3)(x+3)}$

$= \frac{1}{6}$

Question 12

a) i) $\frac{d}{dx}(4x^3 - x + 5) = 12x^2 - 1$

ii) $\frac{d}{dx}(3x^2-4)^4 = 4.6x(3x^2-4)^3$

$= 24x(3x^2-4)^3$

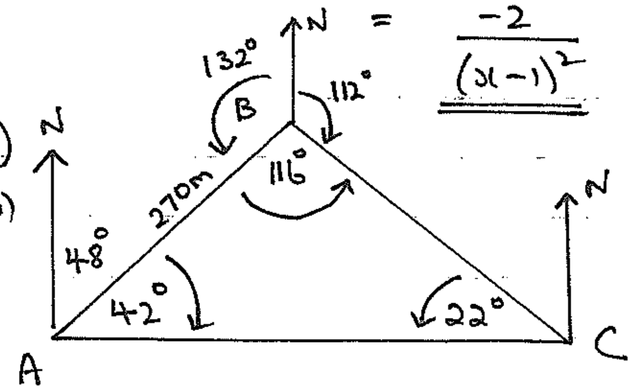
iii) Let $u = x+1 \qquad v = x-1$

$u' = 1 \qquad v' = 1$

$\therefore \frac{d}{dx}\left(\frac{x+1}{x-1}\right) = \frac{1(x-1) - 1(x+1)}{(x-1)^2}$

$= \frac{-2}{(x-1)^2}$

b) i)



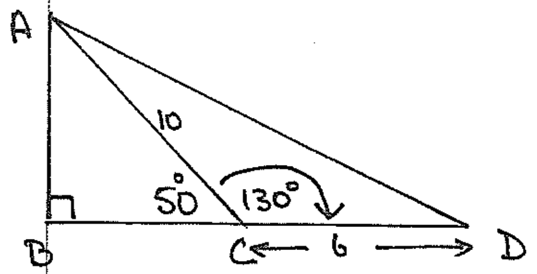
ii) $\frac{AC}{\sin 116°} = \frac{270}{\sin 22°}$

$\therefore AC = \frac{270\sin 116°}{\sin 22°}$

$AC = 648$ m (nearest m)

Question 13

a) i)



$AD^2 = 10^2 + 6^2 - 2.10.6\cos 130^\circ$

$= 100 + 36 - 120\cos 130^\circ$

$AD = 15\text{cm}$ (nearest cm)

ii) area $\triangle ACD = \frac{1}{2}.6.10.\sin 130^\circ$

$= 23\text{cm}^2$ (nearest cm^2)

b) i) sub $k = 2$ into

$2k^3 - 3k^2 - 4 = 0$

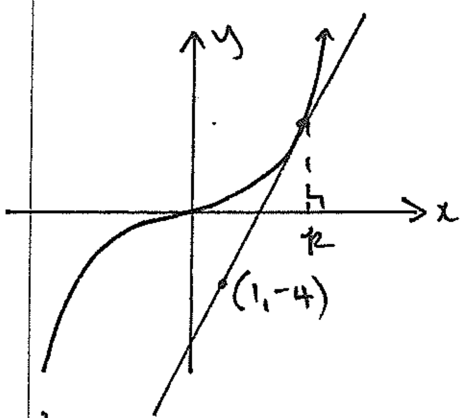
$LHS = 16 - 12 - 4$

$= 0$

$= RHS$

$\therefore k = 2$ is a solution

ii)



α. $y = x^3$

$\frac{dy}{dx} = 3x^2$

$\therefore m_T = 3k^2$ where $x = k$

β. pt (k, k^3)

tangent: $y - k^3 = 3k^2(x - k)$

$y - k^3 = 3xk^2 - 3k^3$

$y = 3xk^2 - 2k^3$

γ. sub $(1,-4)$ into tangent

$-4 = 3k^2 - 2k^3$

$\therefore 2k^3 - 3k^2 - 4 = 0$

$\therefore$ from part i)

$k = 2$