Gosford High School
Mathematics
Extension 1 Preliminary Course
Half yearly Exam 2004.

Time: 2 hours plus 5 minutes reading time.

Section A. To be attempted by all students.

Question 1.

a) Solve for x: $8^{3x-1} = 16^{x+1} \times 4^{x}$ (3)

b) Simplify $\dfrac{X^2Z}{Y^4}$ if $X = \left(\dfrac{2}{3}\right)^2, Y = \left(\dfrac{4}{3}\right)^4$ and $Z = \left(\dfrac{8}{3}\right)^7$ (3)

Give your answer as a fraction.

c) Simplify $\dfrac{2^n + 2^{n+3}}{2^n}$ (2)

d) Evaluate $\left(3^{\frac{1}{3}} - 2^{\frac{1}{3}}\right)\left(3^{\frac{2}{3}} + 3^{\frac{1}{3}}.2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$ (2)

e) Factorise (4)

i) $x^2 + 6x + 9 - y^2$

ii) $\sqrt{25x} + \sqrt{x^3}$

f) Given that $x = \sqrt{2} + 1$ show that $x + \dfrac{1}{x} = 2\sqrt{2}$ and hence find the value of

$x^2 + \dfrac{1}{x^2}$. (3)

Question 2.

a) Solve for x

i) $\dfrac{3}{x-2} - \dfrac{2}{x+3} = \dfrac{1}{x^2+x-6}$ (2)

ii) $(x-3)(x+2) < 0$ (1)

iii) $0 \le x^2 - 7x + 12$ (2)

iv) $x^2 \le 5x$ (2)

v) $\dfrac{2}{x+1} < 3$ (2)

vi) $0 < \dfrac{x^2-4}{x}$ (3)

vii) $(x-1)^2(x+1) \leq 0$ (2)

b) On a separate number line graph the solutions to part (a) (ii), (iii) and (vii) (3)

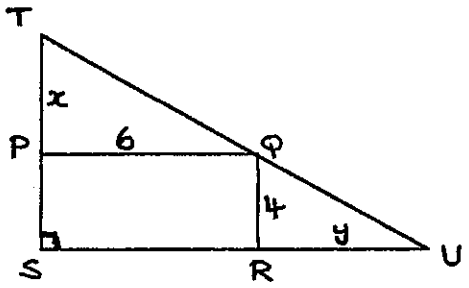
Question 3.

Solve simultaneously (4)

$2x - y = 4$

$8x^3 - y^3 = 28$

Question 4.



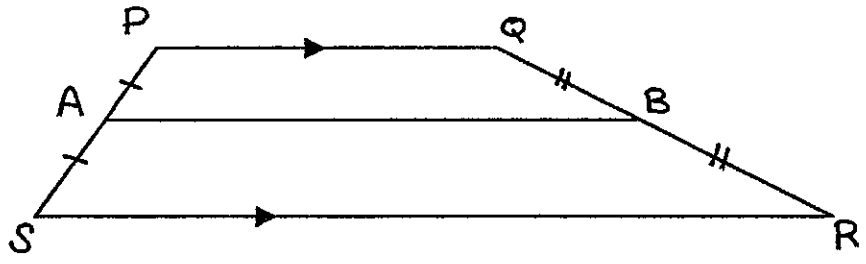
PQRS is a rectangle with PQ = 6*cm* and QR = 4*cm*. T and U lie on the lines SP and SR respectively, so that T, Q and U are collinear, as shown in the diagram. Let PT = *xcm* and RU = *ycm*.

a) Show that triangles TPQ and QRU are similar. (3)

b) Show that $xy = 24$ (1)

c) Show that the area, A, of triangle TSU is given by: $A = 24 + 3x + \dfrac{48}{x}$ (3)

Question 5.



PQRS is a trapezium with PQ parallel to SR. A and B are the mid-points of SP and RQ respectively.

a) State why AB is parallel to PQ and SR. (1)

b) Prove that $AB = \dfrac{1}{2}(PQ + SR)$ (3)

SECTION B. To be attempted by 11m1, 11m2, 11m3 and 11m4 only.

Question 1.
If $f(x) = x^3 + 3x^2 - x - 3$ find

a) f(-2) (1)
b) For what values of x does $f(x) = 0$? (2)

Question 2.
The function $f(x)$ is defined by the rule

$f(x) = 6$ for $x < 0$
$f(x) = 3x$ for $0 \le x \le 2$
$f(x) = x^2$ for $2 < x$

a) Find $f(-1) + f(2) - f(3)$ (2)

b) Sketch $f(x)$ in the domain $-1 \le x \le 3$ (3)

Question 3.
State the largest possible domain and range for the following

a) $y = \sqrt{9 - x}$ (2)

b) $y = \dfrac{3}{2x + 1}$ (2)

c) $y = x^2 - 6x + 8$ (2)

d) $y = \dfrac{5}{x^2 - 1}$ (2)

e) $y = \dfrac{5}{x^2 + 1}$ (2)

f) $y^2 = 3 - x$ (2)

Question 4.
Do a neat accurate sketch of the following.

a) $\frac{x}{2}+\frac{y}{3}=1$ (2)

b) $(x-1)^2+(y+2)^2=4$ (2)

c) $y=\frac{1}{x-1}$ (2)

d) $y=\frac{1}{|x|}$ (2)

e) $y<\sqrt{9-x^2}$ (2)

f) $y=|x+1|+|x-1|$ (3)

g) The region for which $x^2+y^2\leq 16$ and $x<y$ hold simultaneously. (3)

Question 5.
A circle has equation $x^2+y^2-4x+2y=0$

a) Find the centre and radius of the circle. (2)

b) The line $x+2y=0$ meets the circle in two points A and B. Find the coordinates of A and B. (3)

Question 6.
Let $f(x)=\frac{x}{x^2-1}$

a) For what values of x is f(x) undefined? (1)

b) Show that f(x) is an odd function. (2)

c) Sketch y = f(x). (2)

Question 7.

a) Show that $\frac{x^2-x+1}{x-1}=x+\frac{1}{x-1}$ (2)

b) hence or otherwise graph $y=\frac{x^2-x+1}{x-1}$ (3)

Section C. To be attempted by 11m5 only.

Question 1.
If $P(x) = 4x^3 - 5x^2 - x + 7$ and $Q(x) = x^4 - 2x^2 + 5$ find:

a) $P(x) - Q(x)$ (2)

b) $Q(-3)$ (2)

c) $(x+2).P(x)$ (2)

d) The degree of $Q(x)$ (1)

e) The remainder when $P(x)$ is divided by $(x+1)$ (2)

f) Divide $Q(x)$ by $(x+2)$ and write your answer in the form $Q(x) = (x+2)R(x) + \text{remainder}$ (3)

Question 2.
Do a neat sketch of the following clearly showing all zeros.

a) $y = x(x+2)(x-1)$ (2)

b) $y = (x-1)(x+3)^3$ (2)

Question 3.
If $P(x) = x^3 + 4x^2 + x - 6$

a) Show that $(x-1)$ is a factor of $P(x)$. (2)

b) Fully factorise $P(x)$. (3)

c) Do a neat sketch of $P(x)$. (2)

Question 4.
Fully factorise $2x^3 - 3x^2 + 1$ (4)

Question 5.

If α, β and γ are the roots of the polynomial equation $2x^3 - 14x - 1 = 0$ find:

a) $\alpha + \beta + \gamma$ (1)

b) $\alpha\beta + \alpha\gamma + \beta\gamma$ (1)

c) $\alpha\beta\gamma$ (1)

d) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (2)

e) $\alpha^2 + \beta^2 + \gamma^2$ (3)

Question 6.

Form a quadratic equation whose roots are $3 + \sqrt{2}$ and $3 - \sqrt{2}$ (2)

Question 7.

$P(x) = x^3 + ax^2 + bx - 18$ .Find the values of a and b if $(x+2)$ is a factor of $P(x)$ and -24 is the remainder when $P(x)$ is divided by $(x-1)$. (4)

Question 8.

Consider the equation $x^3 + 6x^2 - x - 30 = 0$ .One of the roots of this equation is equal to the sum of the other two roots. Find the value of the three roots. (4)