Gosford High School

Year 11

2008
Preliminary
Higher School Certificate

Mathematics
Extension 1
Assessment Task 2

Time Allowed - 60 minutes
+ 5 minutes reading time

The three questions are of equal value
Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

GOSFORD HIGH SCHOOL – HALF YEARLY EXAMINATION 2008

MATHEMATICS EXTENSION 1 – YEAR 11

Time Allowed: 1 Hour + 5 minutes reading time

Question 1 (18 Marks) **Marks**

a) Write $\left(1 + \sqrt{5}\right)^3$ in the form $a + b\sqrt{5}$ where a and b are integers. 3

b) By expressing $\dfrac{4}{2 + \sqrt{5}} - \dfrac{1}{9 - 4\sqrt{5}}$ 3

in its simplest form, show that it is a rational number.

c) Simplify $\dfrac{2^{n+3} - 2^{n-1}}{2^{2n+3} - 2^{2n-1}}$ showing all steps 3

d) Solve $9^x - 28(3^x) + 27 = 0$ 3

e) Solve and graph the solution on a number line

(i) $\dfrac{4}{5 - x} \geq 1$ 3

(i) $\dfrac{x + 1}{x^2 - 4} \geq 0$ 3

Question 2 (18 Marks) Begin a new sheet of paper

The points P and Q have coordinates (3, -2) and (1, 3) respectively.

a) The line k has equation $4x + 5y - 2 = 0$. Verify that P lies on k. 1

b)` The line l through Q has gradient $\dfrac{1}{3}$. Show that the equation of l is

$x - 3y + 8 = 0$. 2

c) Find the point of intersection R of k and l. 2

d) Find the perpendicular distance of P from l. 3

e) Hence find the area of the triangle PQR 3

f) Find the point T which divides PQ externally in the ratio 1:2 3

g) Find the equation of the line concurrent with

$4x + 5y - 2 = 0$ and $x - y + 1 = 0$ 4

which is perpendicular to $3x + 2y + 1 = 0$.

Question 3 (18 Marks) Begin a new sheet of paper

Marks

a) Solve for $0 \le \theta \le 360°$

(i) $3\sin\theta + 2\cos\theta = 0$ (Nearest minute) **2**

(ii) $\cos 2\theta = \cos\theta$ (Exact Answers) **3**

b) A yacht sails 16 Km from a port P on a bearing of 102° to a buoy Q. From Q, it then sails 23 Km on a bearing of 207° to another buoy R.

(i) Draw a large diagram showing this information. **1**

(ii) Prove that $\angle PQR$ is 75° **1**

(iii) Find how far the yacht is from P. (Answer to 2 decimal places) **2**

(iv) On what bearing must it sail to return to P? (Nearest minute) **3**

c) State the expansion of $\cos(\alpha+\beta)$ and hence find the exact value of $\cos 105°$ **3**

d) Prove showing all steps:

$$\frac{\cos(180°-A)}{\sin(180°+A)} - \frac{\sin(-A)}{\sin(90°-A)} = \sec A \operatorname{cosec} A$$ **3**

Year 11 Ext 1 H/Y Solutions

Question 1

a) $(1+\sqrt{5})^3 = (1+\sqrt{5})(1+2\sqrt{5}+5)$

$= (1+\sqrt{5})(6+2\sqrt{5})$

$= 6+2\sqrt{5}+6\sqrt{5}+10$

$= 16+8\sqrt{5}$

$\therefore a=16, \quad b=8$

b) $\frac{4}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} = \frac{8-4\sqrt{5}}{4-5}$

$\frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}} = \frac{9+4\sqrt{5}}{81-80}$

$\therefore -(8-4\sqrt{5}) - (9+4\sqrt{5})$

$= -8+4\sqrt{5}-9-4\sqrt{5}$

$= -17$ which is rational

c) $\frac{2^{n+3} - 2^{n-1}}{2^{2n+3} - 2^{2n-1}}$

$\frac{2^{n-1}\{2^4-1\}}{2^{2n-1}\{2^4-1\}} = 2^{n-1-2n+1}$

$= 2^{-n} = \frac{1}{2^n}$

d) $9^x - 28(3^x) + 27 = 0$

$3^{2x} - 28(3^x) + 27 = 0$

Put $u = 3^x$

$u^2 - 28u + 27 = 0$

$(u-27)(u-1) = 0$

$u = 27$ or 1

$3^x = 27$ or $3^x = 1$

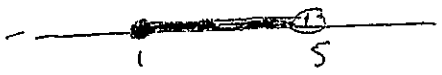
$x = 3$ or $x = 0$

e) i) $\frac{4}{5-x} \geq 1$

Critical Points: $x = 5$

And $4 = 5-x$

$x = 1$



Test $x=0$ $\frac{4}{5} \not\geq 1$

$1 \leq x < 5$

ii) $\frac{x+1}{x^2-4} \geq 0$

$x \neq \pm 2$

Solve $x+1 = 0$

$x = -1$

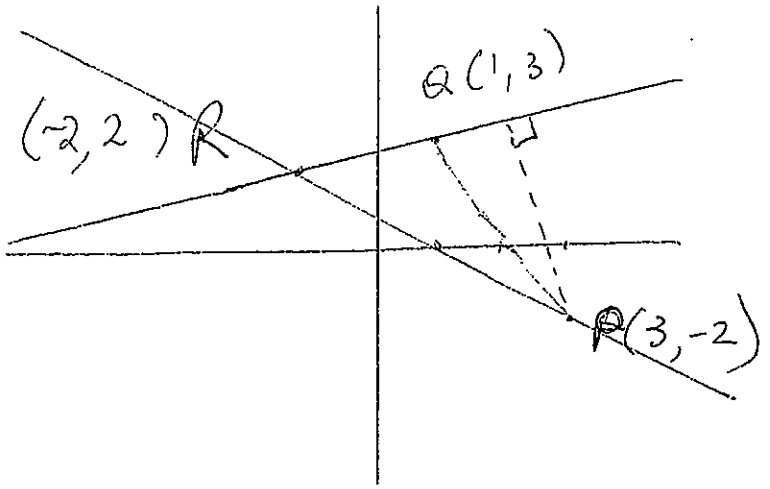
Test $x=0$ $\frac{1}{-4} \not> 0$

Test $x=3$ $\frac{4}{5} > 0$ ✓

Test $x=-\frac{1}{2}$ $\frac{-\frac{1}{2}}{-2\frac{3}{4}} > 0$ ✓

$-2 < x \leq -1$ or $x > 2$

Question 2



a) $P(3,-2)$ $4x+5y-2=0$

$LHS = 4\times3+5\times-2-2$

$= 12-10-2$

$= 0$

$\therefore$ P lies on k

b) $m=\frac{1}{3}$ $Q(1,3)$

$y-3=\frac{1}{3}(x-1)$

$3y-9=x-1$

$x-3y+8=0$ is l

c) $4x+5y=2$ —①

$x-3y=-8$ —②

② $\times -4$

$-4x+12y=32$ —③

① + ③ $17y=34$

$y=2$

Sub into ②

$x=6-8$

$x=-2$

R is $(-2,2)$

d) $d=\left|\frac{1\times3-3\times-2+8}{\sqrt{1^2+3^2}}\right|$

$d=\frac{17}{\sqrt{10}}$

e) QR:

$d^2=3^2+1^2$

$d=\sqrt{10}$

$\therefore$ Area $\triangle PQR=\frac{1}{2}bh$

$=\frac{1}{2}\times\sqrt{10}\times\frac{17}{\sqrt{10}}$

$A=\frac{17}{2}u^2$

f) $\frac{k}{l}=\frac{-1}{2}$

$x=\frac{-1\times1+2\times3}{-1+2}$ $y=\frac{-1\times3+2\times-2}{-1+2}$

$x=5$ $y=-7$

T is $(5,-7)$

g) $(4x+5y-2)+k(x-y+1)=0$

$m_1=-\frac{(4+k)}{5-k}$ $m_2=-\frac{3}{2}$

$\frac{-(4+k)}{5-k}\cdot\frac{-3}{2}=-1$

$3(4+k)=-2(5-k)$

$12+3k=-10+2k$

$k=-22$

$\therefore (4x+5y-2)-22(x-y+1)=0$

$4x+5y-2-22x+22y-22=0$

$-18x+27y-24=0$

$6x-9y+8=0$

Question 3

a) i) $3\sin\theta = -2\cos\theta$

$\tan\theta = -\frac{2}{3}$

2nd or 4th quads

$\theta = 180 + 33^\circ 41'$ or $360 - 33^\circ 41'$

$\theta = 146^\circ 19'$ or $326^\circ 19'$

ii) $\cos 2\theta = \cos\theta$

$2\cos^2\theta - 1 = \cos\theta$

$2\cos^2\theta - \cos\theta - 1 = 0$

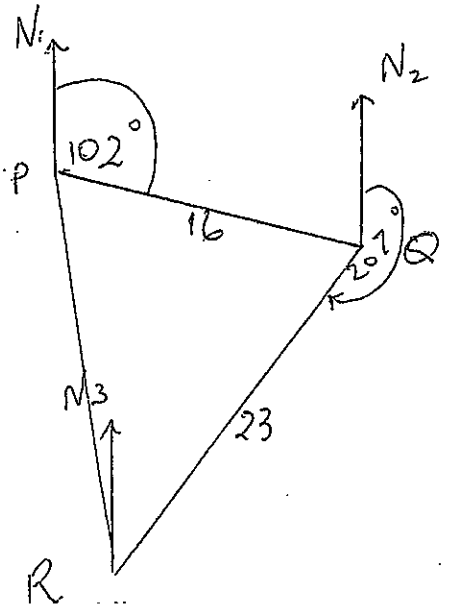
$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ or 1

$\theta = 180-60,\ 180+60,\ 0,\ 360$

$\theta = 120^\circ,\ 240^\circ,\ 0^\circ,\ 360^\circ$

b) i)



ii) $\angle PQN_2 = 78^\circ$ (corresponding $\angle$'s $PN_1 \parallel QN_2$)

$\therefore \angle PQR = 360 - 207 - 78$

$= 75^\circ$ (Angle sum of revolution)

iii) $q^2 = p^2 + r^2 - 2pr\cos Q$

$q^2 = 23^2 + 16^2 - 2\times 23\times 16\times \cos 75^\circ$

$= 574.509$

$q \doteq 24.38$ km

iv) First find either $\angle P$ or $\angle R$ in $\triangle PQR$:

$\frac{\sin P}{23} = \frac{\sin 75^\circ}{24.38}$

$\sin P = \frac{23\sin 75^\circ}{24.38}$

$= 0.9125$

$\angle P = 65^\circ 41'$

(Acute angle required as $23 < 24.38$)

$\therefore$ Bearing of yacht from P is $167^\circ 41'$

$\therefore$ Bearing of P from yacht is $180 + 167^\circ 41' = 347^\circ 41'$

c) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(60^\circ + 45^\circ) = \cos 60\cos 45 - \sin 60\sin$

$= \frac{1}{2}\times\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{1-\sqrt{3}}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$

d)

$\cos(180-A) = -\cos A$

$\sin(180+A) = -\sin A$

$\sin(-A) = -\sin A$

$\sin(90-A) = \cos A$

$\therefore LHS =$

$$\frac{-\cos A}{-\sin A} - \frac{-\sin A}{\cos A}$$

$$= \cot A + \tan A$$

$$= \frac{1}{\tan A} + \tan A$$

$$= \frac{1 + \tan^2 A}{\tan A}$$

$$= \frac{\sec^2 A}{\tan A}$$

$$= \frac{1}{\cos^2 A} \div \tan A$$

$$= \frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}$$

$$= \frac{1}{\cos A} \times \frac{1}{\sin A}$$

$$= \sec A \cdot \operatorname{cosec} A$$