Gosford High School

Year 11

2009 Preliminary Higher School Certificate

Mathematics Extension 1 Assessment Task 2

Time Allowed - 60 minutes
+ 5 minutes reading time

The four questions are of almost equal value
Remember to start each new question on a new page. Work down the page in one column only.
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

Linear Function	/12	
Inequalities	/12	
Polynomials	/7	
Algebra	/6	
Polynomials	/13	
Total	/50	

Mathematics Extension 1 Year 11 Half Yearly Examination – 2009

Question 1 (12 Marks) **Marks**

a) A (2, 4), B (5, –3) and C (–2, –6) are the vertices of a triangle on the number plane.

(i) Find the length of AB **1**

(ii) Find the gradient of AB **1**

(iii) Find the mid point of AC **1**

(iv) What type of triangle is ΔABC. Fully justify your answer. **3**

(v) Find the coordinates of D such that ABCD is a parallelogram. **2**

b) Find, in general form, the equation of the line perpendicular to $5x-3y-2=0$ concurrent with $y=3x+2$ *and* $2x-y=1$. **4**

Question 2 (12 Marks) Begin a New Sheet of Paper

a) Solve and graph your solution on a number line $\frac{1}{2x-1} \leq 2$ **3**

b) Solve $2x-1 \leq |x+4|$ **3**

c) Solve $\frac{3}{(x-2)(x+1)} \geq 0$ **3**

d) Solve $|x+1| + |x-2| = 3$ **3**

Question 3 (13 Marks) Begin a New Sheet of Paper **Marks**

a) Graph $f(x)=(x-3)^2(x+2)(2x-1)$ **2**

Hence or otherwise, solve $(x-3)^2(x+3)(2x-1) \leq 0$ **2**

b) Prove that, if $x^4 - x^3 + kx - 4$ has a factor of $(x+1)$, then it also has a factor of $(x-2)$. **3**

c) Expand and simplify $(2x-3y)(2x+3y) - (2x-3y)^2$ **1**

d) Factorise $a^2-b^2+2bc-c^2$ **2**

e) Simplify $\dfrac{x}{x^3-y^3} - \dfrac{1}{x^2-y^2}$ **3**

Question 4 (13 Marks) Begin a New Sheet of Paper

a) When the polynomial $P(x)$ is divided by x^2-1 the remainder is $3x-1$.
What is the remainder when $P(x)$ is divided by $x-1$? Justify your answer. **2**

b) (i) Factorise completely the polynomial $P(x)=x^3 - 3x + 2$ given that the equation $P(x) = 0$ has a repeated root. **3**

(ii) The polynomial $Q(x)$ has the form $Q(x)=P(x)(x+a)$ with $P(x)$ as in (i). **1**
The constant a is chosen so that $Q(x) \geq 0$ for all real values of x.
Find all possible values of a. (Hint: Draw a graph or two.)

c) If α, β *and* γ are the roots of $x^3 - 2x^2 + 3x + 7 = 0$, find the values of:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\dfrac{2}{\alpha} + \dfrac{2}{\beta} + \dfrac{2}{\gamma}$ **2**

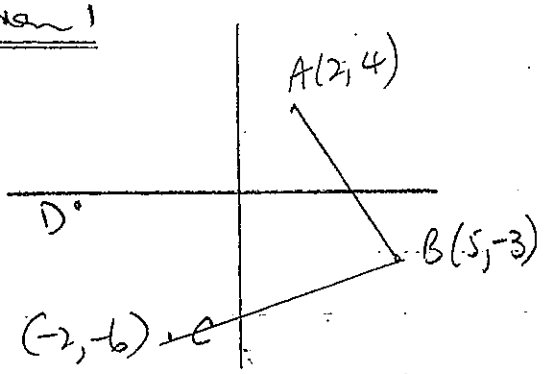
(iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(iv) **Hence** using (i) and (iii) find $\alpha^3 + \beta^3 + \gamma^3$ **2**

Solutions to 2009 Yr 11 Ext 1 Task 2

Question 1

a)



i) $d^2 = 3^2 + 7^2 = 9 + 49$

$d = \sqrt{58}$

ii) $m_1 = \frac{-3-4}{5-2} = -\frac{7}{3}$

iii) M $(0, -1)$

iv) For BC: $d^2 = 7^2 + 3^2$

$d = \sqrt{58}$

$m_2 = \frac{-3+6}{5+2} = \frac{3}{7}$

BC = AB and BC ⊥ AB

$\therefore m_1 m_2 = -1$

$\therefore$ $\triangle ABC$ is isosceles & right angled.

v) D $(-3, -1)$

b) $2x - (3x+2) = 1$

$2x - 3x - 2 = 1 \Rightarrow x = -3$

$y = -7$

$m_1 = \frac{5}{3} \quad \therefore m_2 = -\frac{3}{5}$

$\therefore y + 7 = -\frac{3}{5}(x+3)$

$5y + 35 = -3x - 9$

$3x + 5y + 44 = 0$

Question 2

a) $x > \frac{1}{2}$ | $x < \frac{1}{2}$

$1 \leq 4x - 2$ | $1 \geq 4x - 2$

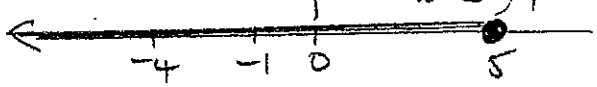
$\frac{3}{4} \leq x$ | $\frac{3}{4} \geq x$

$x < \frac{1}{2}$ or $x \geq \frac{3}{4}$

b) $2x - 1 \leq |x+4|$

For $x \geq -4$: $2x - 1 \leq x + 4$, $x \leq 5$

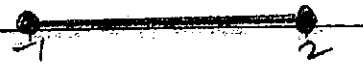
For $x < -4$: $2x - 1 \leq -x - 4$, $3x \leq -3$, $x \leq -1$



$x \leq 5$

d) $|x+1| + |x-2| = 3$

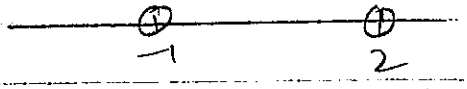
Distance of x from -1 plus distance of x from 2 equals 3.



$-1 \leq x \leq 2$

c) Critical points $x = 2, -1$

No solutions to $3 = 0$



Test $x = 0$ $\quad \frac{3}{-2} \neq 0$

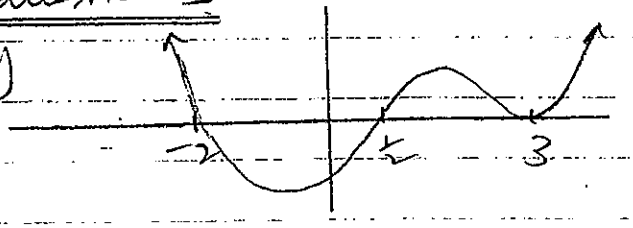
Test $x = 3$ $\quad \frac{3}{1 \times 4} > 0$ ✓

Test $x = -2$ $\quad \frac{3}{(-4)(-1)} > 0$ ✓

$\therefore x < -1$ or $x > 2$

Question 3

a)



$-2 \leq x \leq \frac{1}{2}$ or $x = 3$

b) $P(-1) = 1 + 1 - k - 4 = 0$

$k = -2$

$\therefore P(x) = x^4 - x^3 - x - 4$

$P(2) = 16 - 8 - 4 - 4 = 0$

$\therefore (x-2)$ is a factor

c) $4x^2 - 9y^2 - 4x^2 + 12xy - 9y^2$
$= 12xy - 18y^2$

d) $a^2 - (b^2 - 2bc + c^2)$
$a^2 - (b-c)^2$
$\therefore (a+b-c)(a-b+c)$

e) $\frac{x}{(x-y)(x^2+xy+y^2)} - \frac{1}{(x-y)(x+y)}$

$\frac{x(x+y) - (x^2+xy+y^2)}{(x-y)(x+y)(x^2+xy+y^2)}$

$\frac{x^2+xy - x^2 - xy - y^2}{(x-y)(x+y)(x^2+xy+y^2)}$

$= \frac{-y^2}{(x-y)(x+y)(x^2+xy+y^2)}$

Question 4

a) $P(x) = (x^2-1)Q(x) + 3x - 1$
$\therefore P(1) = 0 + 3\times 1 - 1$
$P(1) = 2$ Remainder is 2.

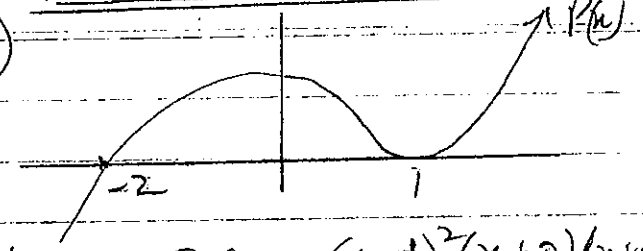
b) $P(1) = 1 - 3 + 2 = 0$

$x^2 + x - 2$
$x-1 \overline{)\ x^3 + 0x^2 - 3x + 2}$
$-x^3 + x^2$
$x^2 - 3x + 2$
$-x^2 + x$
$-2x + 2$
$-2x + 2$

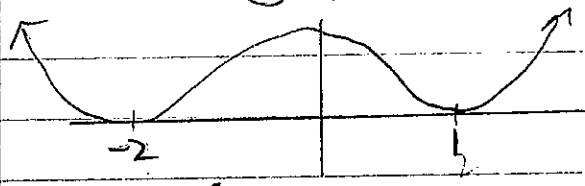
$P(x) = (x-1)(x^2+x-2)$
$= (x-1)(x-1)(x+2)$
$P(x) = (x-1)^2(x+2)$

$P(x) = (x-1)^2(x+2)$

ii)



Now $Q(x) = (x-1)^2(x+2)(x+a)$
For $Q(x) \geq 0$ for all x



is only choice.
$Q(x) = (x-1)^2(x+2)^2$
$a = 2$ only

c) i) $\alpha+\beta+\gamma = -\frac{b}{a} = 2$

ii) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} = \frac{2(\beta\gamma+\alpha\gamma+\alpha\beta)}{\alpha\beta\gamma}$
$= \frac{2\times 3}{-7} = -\frac{6}{7}$

iii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$
$= 2^2 - 2.3$
$= -2$

iv) Sub $x = \alpha$ into eqn
$\alpha^3 - 2\alpha^2 + 3\alpha + 7 = 0$
or $\alpha^3 = 2\alpha^2 - 3\alpha - 7$ ①
Sim $\beta^3 = 2\beta^2 - 3\beta - 7$ ②
Sim $\gamma^3 = 2\gamma^2 - 3\gamma - 7$ ③
① + ② + ③
$\alpha^3+\beta^3+\gamma^3 = 2(\alpha^2+\beta^2+\gamma^2)$
$-3(\alpha+\beta+\gamma) - 21$

$= 2(-2) - 3(2) - 21$
$= -4 - 6 - 21$
$= -31$