GOSFORD HIGH SCHOOL

2010 EXTENSION 1 MATHEMATICS

Preliminary Course Assessment Task 2

PART 1

Special Instructions

- Students are to hand in their papers in three bundles.
- Questions 1 to 3 are to be in one bundle, Questions 4 to 6 in another and Questions 7 to 9 in another.

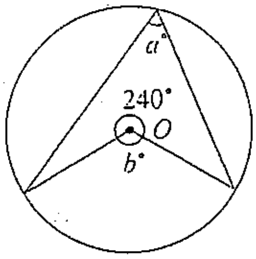
- **Students must start Question 4 on a new page and Question 7 on a new page.**

General Instructions

Time Allowed: 1 hour plus 5 minutes reading time
- Attempt all questions.
- Approved calculators may be used.
- Write using blue or black pen.
- Full marks may not be awarded where necessary working is not shown.

Question 1. Find the value of the pronumerals in each of the following.
(No reasons needed.)

a



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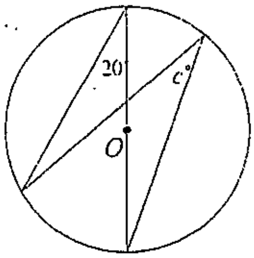

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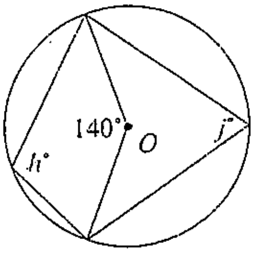

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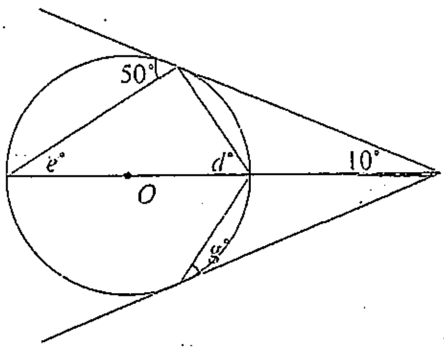

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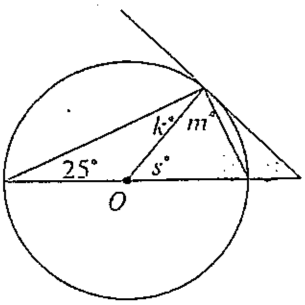

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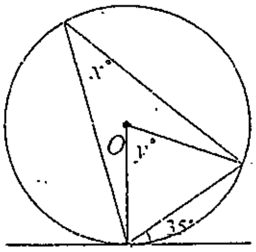
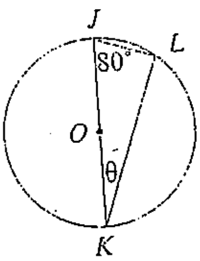


(13)

Question 2. Find the value of the pronumerals in each of the following giving reasons
for your answers.

a



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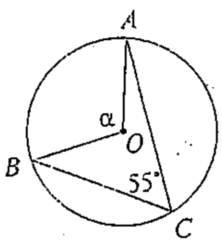

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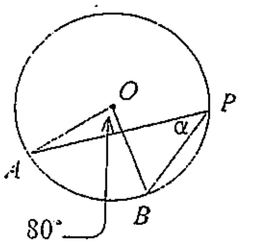

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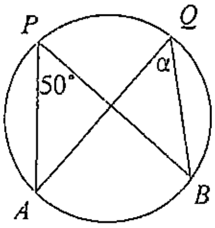

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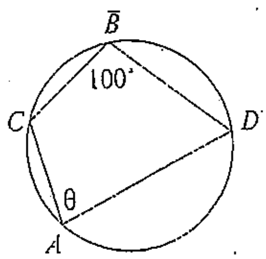

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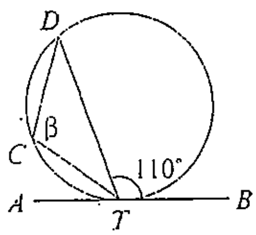
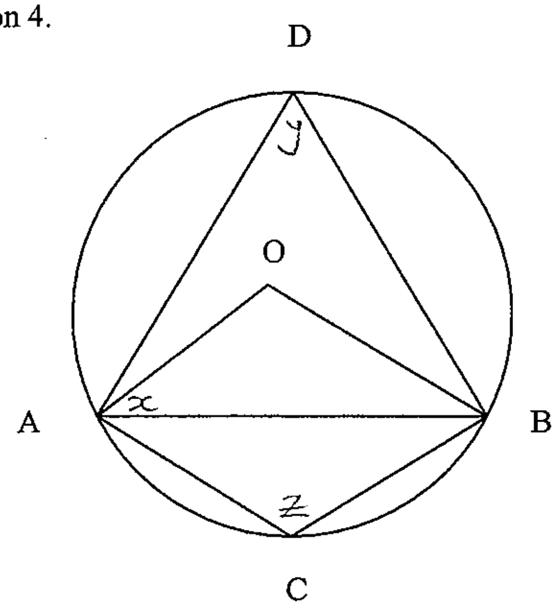


(12)

Question 3. Two parallel chords of length 50cm and 104cm are drawn in a circle; radius 65cm. How far apart are the chords? (5)

Question 4.



NOT TO SCALE

ABCD is a cyclic quadrilateral.

O is the centre of the circle. $\angle OAB = x^0, \angle ACB = z^0$ and $\angle ADB = y^0$.

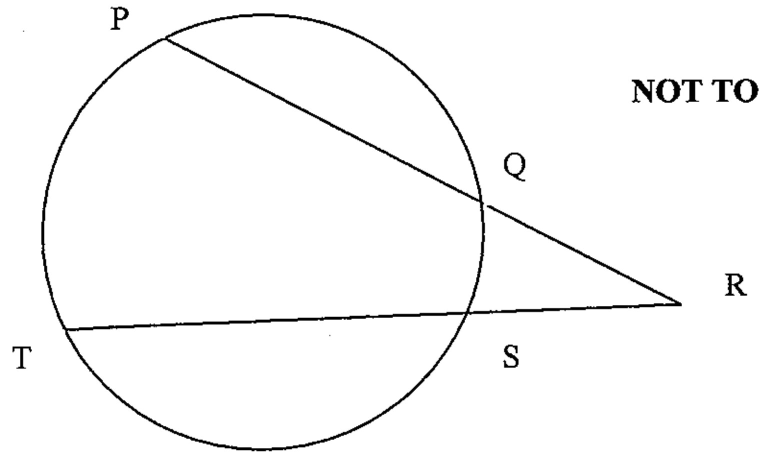
If $\angle ACB$ is obtuse, prove that $z - y = 2x$. (4)

Question 5. AB is a chord of a circle and XAY is the tangent at A. AK and AL are chords bisecting $\angle$ XAB and $\angle$ YAB respectively.

(a) Draw a diagram clearly showing all of this information. (2)

(b) Prove that: (i) AL = BL. (ii) KL is a diameter of the circle. (4)

Question 6. PQ and TS are two secants of a circle intersecting externally at R. Given that PQ = 7cm, QR = 5cm and SR = 4cm, find the length of TS.

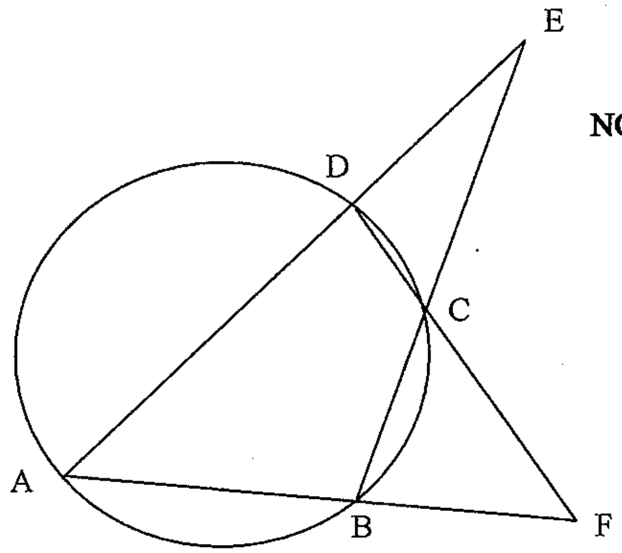


NOT TO SCALE

PR × QR = (

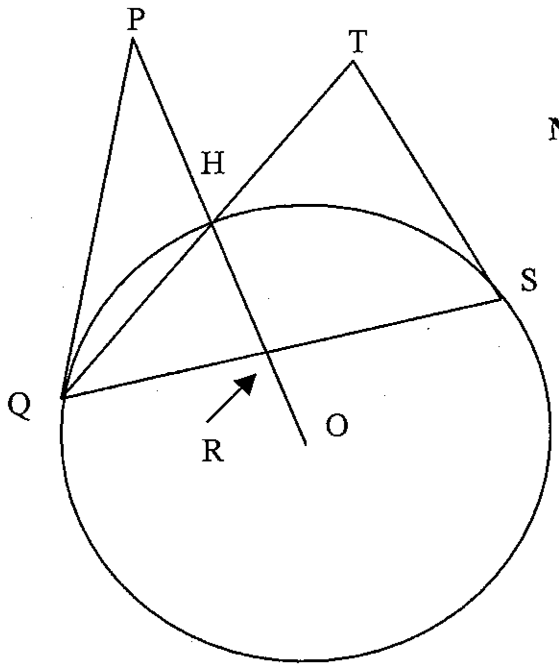
(2)

Question 7.



ABCD is a cyclic quadrilateral. The angles AEB and AFD are equal.
Prove that AC is a diameter of the circle. (4)

Question 8.



PQR is an isosceles triangle such that PQ = PR. PQ is a tangent at Q to the circle centre O. PR produced passes through O. QR produced meets the circle at S. The tangent to the circle at S meets QH produced at T.

Prove that angle HQR is 45^0. (4)

Question 1:

a) $a = \cancel{120} \; 60^\circ$
$b = 120^\circ$

b) $c = 20^\circ$

c) $j = \cancel{140^\circ} \; 70^\circ$
$h = 110^\circ$

d) $d = 50^\circ$
$e = 40^\circ$

e) $g = 40^\circ$
$k = 25^\circ$
$m = 65^\circ$
$s = 50^\circ$

f) $x = 35^\circ$
$y = 70^\circ$

Question 2

a) $\angle JLK = 90^\circ$ (angle in a semicircle is 90°)
$\theta + 80 + 90 = 180$ (angle sum of $\triangle JKL$ is 180°)
$\therefore \theta = 10$

b) $\alpha = 110^\circ$ (angle at the centre is twice the angle at the circumference standing on the same arc)

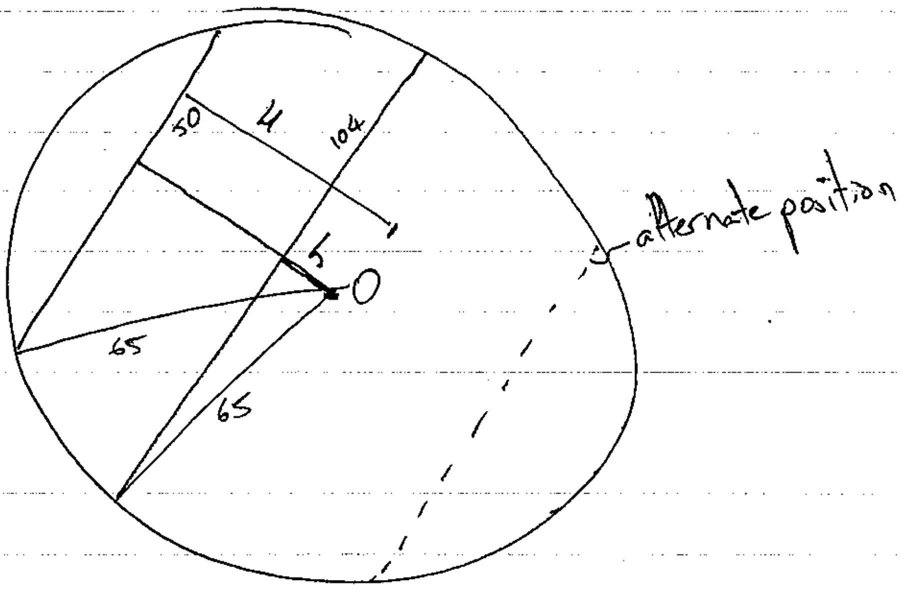
c) $\alpha = 40^\circ$ (angle at the centre is twice the angle at the circumference standing on the same arc)

d) $\alpha = 50^\circ$ (angles in the same segment are equal)

e) $\theta = 80^\circ$ (opposite $\angle$'s in a cyclic quadrilateral are supplementary)

f) $\beta = 110^\circ$ (angle between a chord & a tangent is equal to the angle in the alternate segment)

Question 3



$H^2 + 25^2 = 65^2$

$H^2 =$

$H =$

$52^2 + h^2 = 65^2$

$h^2 =$

$h =$

$\therefore$ distance is either

$H - h$ or $H + h$

$=$ $=$

Question 4

~~∠ABO=x~~ $AO = BO$ (equal radii)

$\therefore \triangle ABO$ is isosceles

$\therefore \angle ABO = x$ (base ∠'s in isosceles △'s are equal)

$\angle AOB = 180 - 2x$ (angle sum of $\triangle AOB$ is $180°$)

$\angle AOB = 2 \times \angle ADB$ (angle at the centre is twice the angle at the circumference standing on the same arc)

$180 - 2x = 2y$

Also $\angle ACB = 180 - \angle ADB$ (opposite ∠'s in a cyclic quad are supplementary)

$\therefore z = 180 - y$

$y = 180 - z$

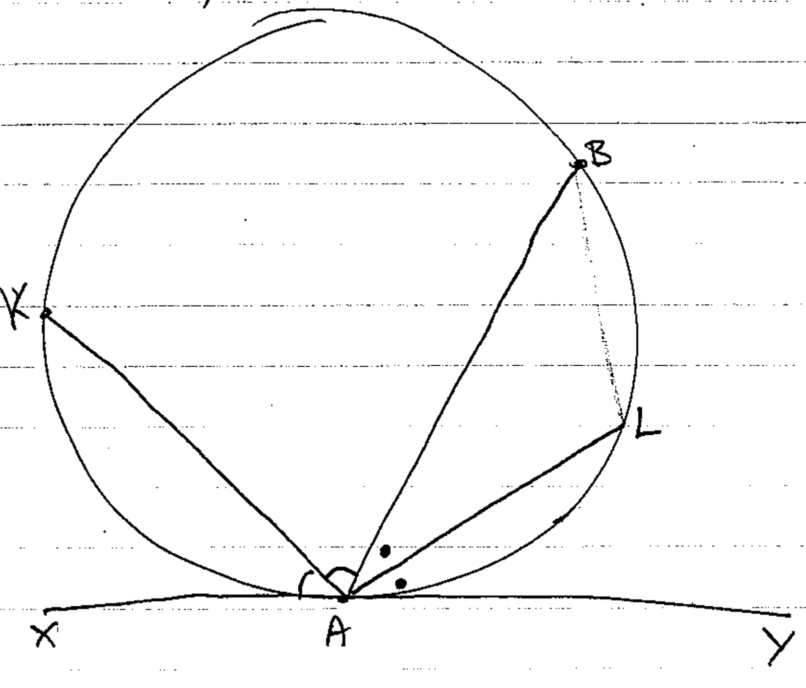
by substitution

$180 - 2x = y + 180 - z$

$z - y = 2x$

Question 5

a)



b) i) ~~∠ABL = ∠BAL =~~

~~(alg~~ $\angle BAL = \angle LAY$ (given)

$\angle ABL = \angle LAY$ (angle between a tangent & a chord is equal to the angle in the alternate segment)

$\therefore \triangle ABL$ is isosceles

$(\angle BAL = \angle ABL)$

$\therefore AL = BL$

ii) $\angle KAX = \angle BAK$ & $\angle BAL = \angle LAY$ (given)

~~$2\angle KAB + 2\angle B$~~

$2\angle KAB + 2\angle BAL = 180$ (adjacent supplementary)

$\therefore \angle KAB + \angle BAL = 90$

$\angle KAL = 90$

Since angle at the circumference is $90°$ KL must be a diameter.

Question 6

$PR \times QR = TR \times SR$ (ratio of the intercepts of a chord is maintained)

let $TS = x$

$$(7+5) \times 5 = (x+4) \times 4$$
$$60 = 4x + 16$$
$$4x = 44$$
$$x = 11$$

Question 7

$\angle ADF = 180 - \angle DAF - \angle AFD$ (angle sum of $\triangle AFD$ is $180°$)

similarly $\angle ABE = 180 - \angle DAF - \angle AEB$

since $\angle AEB = \angle AFD$ (given)

$\angle ABE = 180 - \angle DAF - \angle AFD$

$\therefore \angle ADC = \angle ABC$

$\angle ADC + \angle ABC = 180°$ (opposite $\angle$'s in a cyclic quad are supplementary)

$\therefore 2\angle ADC = 180°$

$\angle ADC = 90°$

$\therefore$ AC is a diameter (angle in a semi-circle is $90°$)

Question 8)

$\angle PQH = \angle HSQ$ (angle between chord..

$\angle HSQ = \frac{1}{2}\angle QOH$ (angle at the centre

$\angle PQO = 90°$ (angle between a radius & a tangent

Let $\angle HQR = x$

$\angle QPR = 90 - 2\angle HSQ$

$\angle QPR + 2\angle HRQ = 180$

$90 - 2\angle HSQ + 2\angle HRQ = 180$

$\angle HRQ = \cancel{90}\,45 + \angle HSQ$

$\angle HSQ + \angle HQR = 45 + \angle HSQ$

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2010 EXTENSION 1 MATHEMATICS

Preliminary Course Assessment Task 2

PART 2

Special Instructions

- Students are to hand in their papers in two bundles.
- PART A is to be in one bundle and PART B in another.

- **Students must start PART B on a new page.**

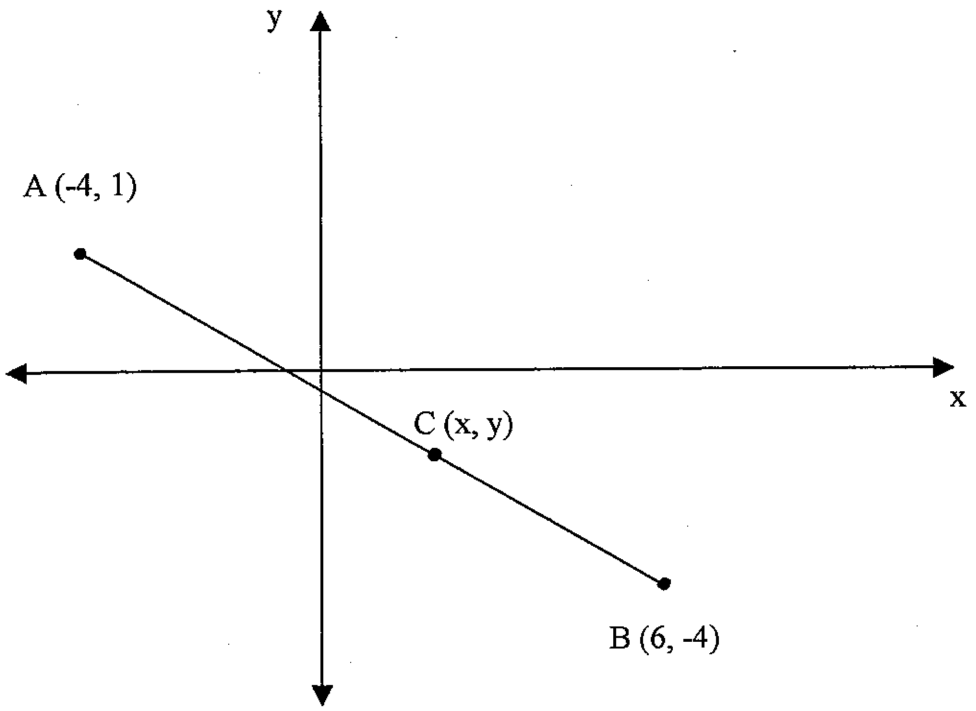
General Instructions

Time Allowed: 1 hour plus 5 minutes reading time

- Attempt all questions.
- Approved calculators may be used.
- Write using blue or black pen.
- Full marks may not be awarded where necessary working is not shown.

PART A.

1.



Given that 2AC = 3BC, find the coordinates of the point C. (3)

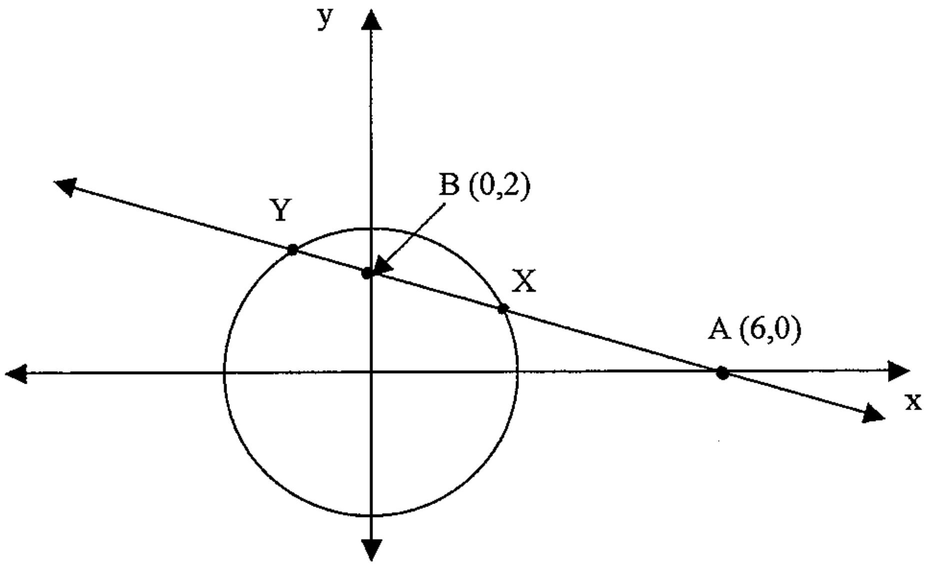
2. Find the coordinates of the point P, which divides the interval joining A (-3,6) and B (1,10) externally in the ratio 5 : 3. (3)

3. The point C (-1, -4) divides the interval AB internally in the ratio 2 : 1. If the coordinates of A are (3,2), find the coordinates of B. (3)

4. A (6,0) and B (0,2) are points on a number plane.

(a) Find the coordinates of the point C which divides the interval AB in the ratio k : 1. (1)

(b) Hence, or otherwise, find the ratio in which Y divides the interval AB, given that the line AB meets the circle $x^2+y^2=10$ at X and Y. (6)



PART B. START A NEW PAGE

1. Sketch the graph of $y=x(x+1)(x-1)^2$ (2)

2. Consider the polynomial $P(x)=2x^3+kx^2-18x-8$.

(i) Find the value of k for which $(x+2)$ is a factor of $P(x)$. (1)

(ii) For this value of k express $P(x)$ as a product of linear factors. (2)

3. Solve $x^3+2x^2-5x-6=0$. (3)

4. If α, β *and* γ are the roots of the equation $x^3-5x^2-3x+2=0$, find the value of

(i) $\alpha+\beta+\gamma$. (1)

(ii) $\alpha\beta\gamma$. (1)

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$. (2)

(iv) $\alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2$. (2)

5. The polynomial $P(x)=x^3-8x^2+kx-12$ has one root equal to the sum of the other two roots. Find the value of k. (3)

6. Two of the roots of $x^3-15x+4=0$ are reciprocals of one another. Find all three roots. (3)

7. When ax^3+bx^2+cx+d is divided by $x-\alpha$ the remainder is r_1, and when it is divided by $x+\alpha$ the remainder is r_2.

Prove that $\alpha=\dfrac{b(r_1-r_2)}{a(r_1+r_2)+2(bc-ad)}$. (4)

Part 2 Solutions Part A

1. If $2AC = 3BC$

$$\frac{AC}{BC} = \frac{3}{2}$$

$(-4, 1)$ $(6, -4)$

$3 : 2$

$$x = \frac{2(-4) + 3(6)}{5} \qquad y = \frac{2(1) + 3(-4)}{5}$$

$x = 2$ $y = -2$

$\therefore$ C is $(2, -2)$ 3

2. $(-3, 6)$ $(1, 10)$

$5 : -3$

$$x = \frac{-3(-3) + 5(1)}{2} \qquad y = \frac{-3(6) + 5(10)}{2}$$

$x = -2$ $y = 16$

$\therefore$ P is $(-2, 16)$ 3

3. $(3, 2)$ (x_1, y_1)

$2 : 1$

$$-1 = \frac{1(3) + 2(x_1)}{3} \qquad -4 = \frac{1(2) + 2(y_1)}{3}$$

$-3 = 3 + 2x_1$ $-12 = 2 + 2y_1$

$2x_1 = -6$ $2y_1 = -14$

$x_1 = -3$ $y_1 = -7$ 3

$\therefore$ B is $(-3, -7)$

4. a) $(6, 0)$ $(0, 2)$

$k : 1$

$$x = \frac{1(6) + k(0)}{k+1} \qquad y = \frac{1(0) + k(2)}{k+1}$$

$$= \frac{6}{k+1} \qquad = \frac{2k}{k+1}$$

k is $k+1$ $\left(\frac{6}{k+1}, \frac{2k}{k+1}\right)$ 1

b) Eqⁿ of AB is $y = 2 - \frac{x}{3}$ (1)

$x^2 + y^2 = 10$ — ①

$y = 2 - \frac{x}{3}$ — ②

$\therefore x^2 + \left(2 - \frac{x}{3}\right)^2 = 10$

$\therefore x^2 + 4 - \frac{4x}{3} + \frac{x^2}{9} = 10$

$9x^2 + 36 - 12x + x^2 = 90$

$10x^2 - 12x - 54 = 0$

$5x^2 - 6x - 27 = 0$

$5x \quad 9$

$x \quad -3$ (2)

$(5x + 9)(x - 3) = 0$

$x = -\frac{9}{5}$ or 3

$\therefore$ if $x = -\frac{9}{5}$, $y = 2 - \frac{-\frac{9}{5}}{3}$

$= \frac{13}{5}$ (1)

Y is $\left(-\frac{9}{5}, \frac{13}{5}\right)$

From a) $\frac{6}{k+1} = -\frac{9}{5}$

$\therefore 30 = -9k - 9$

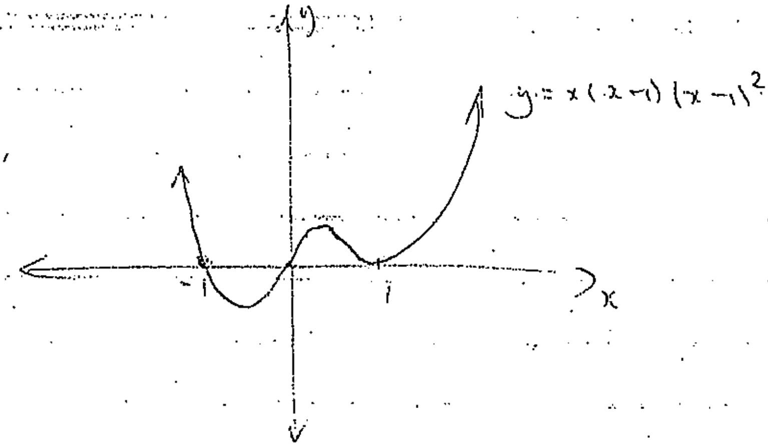
$-9k = -39$

$\therefore k = \frac{13}{3}$. (2)

$\therefore$ Y divides AB externally in the ratio 13 : 3. (1)

PART B

1.



(2)

2. (i) $P(x) = 2x^3 + kx^2 - 18x - 8$

$P(-2) = 2(-2)^3 + k(-2)^2 - 18(-2) - 8$

$0 = -16 + 4k + 36 - 8$

$4k = -12$

$k = -3$. (1)

(ii)

$$\begin{array}{r|l} & 2x^2 - 7x - 4 \\ \hline x+2 & 2x^3 - 3x^2 - 18x - 8 \\ & 2x^3 + 4x^2 \\ \hline & -7x^2 - 18x \\ & -7x^2 - 14x \\ \hline & -4x - 8 \\ & -4x - 8 \\ \hline & 0 \end{array}$$

$\therefore P(x) = (x+2)(2x^2 - 7x - 4)$ (2)

$= (x+2)(2x+1)(x-4)$.

3. Let $P(x) = x^3 + 2x^2 - 5x - 6$

$P(1) = 1 + 2 - 5 - 6$

$\neq 0$

$P(-1) = -1 + 2 + 5 - 6$

$= 0$

$\therefore (x+1)$ is a factor (1)

$$\begin{array}{r|l} & x^2 + x - 6 \\ \hline x+1 & x^3 + 2x^2 - 5x - 6 \\ & \underline{x^3 + x^2} \\ & \quad x^2 - 5x \\ & \quad \underline{x^2 + x} \\ & \qquad -6x - 6 \\ & \qquad \underline{-6x - 6} \end{array}$$

$\therefore f(x) = (x+1)(x+3)(x-2)$ (2)

$\therefore x = -1, -3$ or 2

4. (i) $\alpha + \beta + \gamma = -\frac{b}{a}$

$= 5$ (1)

(ii) $\alpha\beta\gamma = -\frac{d}{a}$

$= -2$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$

$= \frac{c/a}{-d/a}$

$= 3/2$ (2)

(iv) $([\alpha\beta + \beta\gamma] + \alpha\gamma)^2 = [\alpha\beta + \beta\gamma]^2 + 2[\alpha\beta + \beta\gamma]\alpha\gamma + (\alpha\gamma)^2$

$= \alpha^2\beta^2 + 2\alpha\beta^2\gamma + \beta^2\gamma^2 + 2\alpha^2\beta\gamma + 2\alpha\beta\gamma^2 + \alpha^2\gamma^2$

$= \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$\therefore \alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2 = (\alpha\beta + \beta\gamma + \alpha\gamma)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$= (-3)^2 - 2 \times -2(5)$

$= 9 + 20$

$= 29$

(2)

5. Let the roots be α, β, γ

$\therefore \alpha = \beta + \gamma$

Hence $\alpha + \alpha = \frac{-b}{a}$

$2\alpha = 8$

$\alpha = 4.$

$\therefore P(4) = 0$ 3

$(4)^3 - 8(4)^2 + 4k - 12 = 0$

$64 - 128 + 4k - 12 = 0$

$4k = 76$

$k = 19$

6. Let the roots be $\alpha, \beta, \frac{1}{\beta}$

Now $\alpha . \beta . \frac{1}{\beta} = -\frac{d}{a}$

$\alpha = -4.$ 1

Also $\alpha + \beta + \frac{1}{\beta} = -\frac{b}{a}$

$-4 + \beta + \frac{1}{\beta} = 0$

$\beta + \frac{1}{\beta} = +4$

$\beta^2 + 1 = +4\beta$ 2

$\beta^2 - 4\beta + 1 = 0$

$\beta = \frac{+4 \pm \sqrt{16-4}}{2} = \frac{+4 \pm 2\sqrt{3}}{2}$

$= +2 \pm \sqrt{3}$

$\therefore$ Roots are $-4, (+2 + \sqrt{3}), (+2 - \sqrt{3})$

7. $a\alpha^3 + b\alpha^2 + c\alpha + d = r_1$ — ①

$-a\alpha^3 + b\alpha^2 - c\alpha + d = r_2$ — ②

① + ② $\Rightarrow$ $2b\alpha^2 + 2d = r_1 + r_2$

$$\alpha^2 = \frac{r_1 + r_2 - 2d}{2b}$$

① − ② $\Rightarrow$ $2a\alpha^3 + 2c\alpha = r_1 - r_2$

$$2\alpha(a\alpha^2 + c) = r_1 - r_2$$

$$\alpha = \frac{r_1 - r_2}{2(a\alpha^2 + c)}$$

But $\alpha^2 = \dfrac{r_1 + r_2 - 2d}{2b}$

$$\text{i.e. } \alpha = \frac{r_1 - r_2}{2\left(a\left[\frac{r_1 + r_2 - 2d}{2b}\right] + c\right)}$$

$$= \frac{r_1 - r_2}{2\left(\frac{a[r_1 + r_2 - 2d] + 2bc}{2b}\right)}$$

$$= \frac{\cancel{2}b(r_1 - r_2)}{\cancel{2}(a(r_1 + r_2) - 2ad + 2bc)}$$

$$= \frac{b(r_1 - r_2)}{a(r_1 + r_2) - 2(bc - ad)}$$

4