Number: _____

Name: _____

Teacher: _____

GOSFORD HIGH SCHOOL

Preliminary HSC

Extension 1 Mathematics 2012

Assessment Task 2

TIME ALLOWED 80 minutes plus 5 minutes reading time

- Show all working for questions worth more than 1 mark.
- An approved calculator may be used

Marks

	Multiple choice	/5
	Question 1 Polynomials	/10
	Question 2 Ratio	/10
	Question 3 Polynomials	/10
	Question 4 Circle Geometry	/11
	Question 5 Polynomials	/10
Total		/56

Name:_____ Teacher:_____

Multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely, using a black pen.

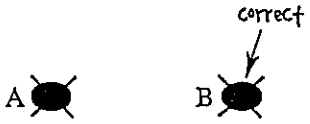
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A● B⊗ C○ D○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



Start here →

1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○
5. A○ B○ C○ D○

Section 1 Multiple Choice (5 Marks)

1. Which of the following is a factor of $x^6 + 6x^2 - 4x - 3$

A. $x+1$ B. $x-1$ C. $x+3$ D. $x-3$

2. Which of the following is a **NOT** a polynomial

A. $\frac{5}{x^2} + 3x$ B. $(x-2)(x-3)$ C. 3^{-1} D. x^{16}

3. Which of the following is a monic polynomial of degree 4?

A. $1 - x + x^2 - x^4$ B. $1 + 3x + 2x^2 + 4x^3$

C. $5x + x^4$ D. $4x$

4. If Q divides AB externally in the ratio 1: 3, then A divides BQ in the ratio

A. 3 : 1 B. $-1 : 3$ C. 1 : 2 D. 2 : 1

5. If $\alpha, \beta, and\ \gamma$ are the roots of $3x^3 - 5x + 4 = 0$, which of the following is true

(i) $\alpha + \beta + \gamma = \frac{5}{3}$

(ii) $\alpha\beta\gamma = \frac{-4}{3}$

A. (i) only B. (ii) only C. both (i) and (ii) D. neither (i) nor (ii)

Question 1 (10 Marks) Polynomials Start a New sheet of Paper **Marks**

a) Find the remainder when $F(x) = x^3 + 2x^2 - 5x + 3$ is divided by $x + 2$. **1**

b) Subtract $x^3 - 5x^2 + 3x$ from $x^4 - 2x^3 + 1$. **1**

c) Sketch the graph of $P(x) = x(x-3)(x+1)$ labelling all intercepts. **2**

(at least $\frac{1}{3}$ page sketch and rule your axes!)

d) (i) Divide $P(x) = x^3 + 4x^2 + 1$ by $A(x) = x - 3$ **2**

(ii) Express your answer in the form $P(x) = A(x).Q(x) + R(x)$ **1**

e) $P(x) = x^3 + ax^2 + bx - 18$. Find a and b if $(x+2)$ is a factor of $P(x)$ and -24 is the remainder when $P(x)$ is divided by $(x-1)$ **3**

Question 2 (10 Marks) Ratio Formula Start a New sheet of Paper

a) A is the point $(-3, 4)$ and B is the point $(2, -1)$. Find the coordinates of

(i) the point P which divides AB internally in the ratio 5: 3. **3**

(ii) the point Q which divides BA externally in the ratio 1: 2 **3**

b) (i) For the same points A and B above, write down an expression for P (x, y) in terms of k, which divides AB in the ratio k: 1. Answer in the form $x =$, and $y =$. **1**

(ii) The line $2x - 3y + 4 = 0$ cuts AB at the point D. Using the result from (i), find in what ratio D divides AB? **3**

Question 3 (10 Marks) Polynomials **Start a New sheet of Paper** **Marks**

a) Solve $3^{2x} - 4(3^x) + 3 = 0$ **3**

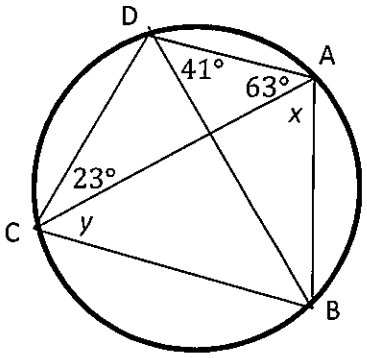
b) Solve $(3x^2 + 2x)^2 - 9(3x^2 + 2x) + 8 = 0$ **3**

c) Sketch the graph of $y = (x+1)^2(1-x)^3(x+2)$ **4**

labelling all x and y intercepts (at least $\frac{1}{3}$ page sketch and rule your axes!)

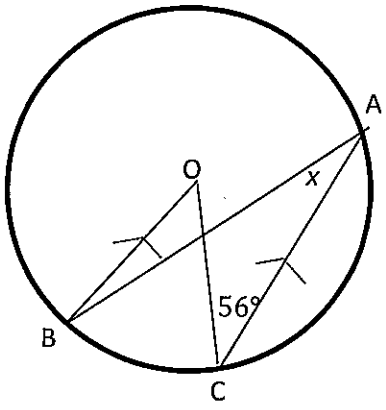
Question 4 (11 Marks) Circle Geometry. Start a New sheet of Paper
Copy or trace each diagram onto your writing pad.

a) Find the values of y and x giving all reasons **4**

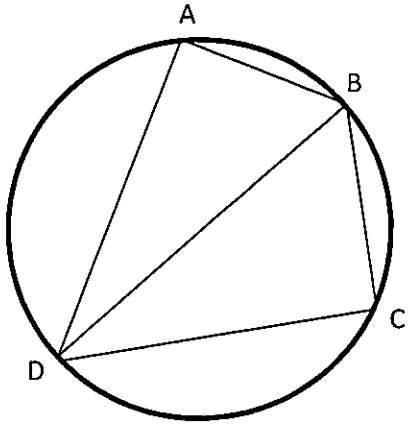


b) O is the centre of this circle. **2**

Find the value of x giving all reasons.



c)



A, B, C and D lie on a circle. BD bisects ∠ ABC and ∠ ADC.

Letting ∠ DBC = x and ∠ BDC = y

(i) Prove Δ ABC is isosceles **2**

(ii) Prove BD is a diameter **3**

You must give all reasons.

Hint: Join AC

Question 5 (10 Marks) Polynomials **Start a New sheet of Paper**

a) If $\alpha, \beta, and\ \gamma$ are the roots of $x^3 - x^2 + 4x - 5 = 0$ form the equation with roots $2\alpha, 2\beta, and\ 2\gamma$. **3**

b) $P(x)$ leaves a remainder of -6 when divided by $x - 1$ and 4 when divided by $x + 1$. Find the remainder when P(x) is divided by $(x^2 - 1)$. **2**

c) Two of the roots of $2x^4 - 5x^3 + cx^2 + 5x - 2 = 0$ are reciprocals of each other, while the remaining two roots are opposites of each other.

(i) Find the sum of the roots **1**

(ii) Find the product of the roots. **1**

(iii) Hence find the four roots and the value of c. **3**

Solutions to 2012 Extension 1 Preliminary HSC Assessment Task 2.

Multiple choice

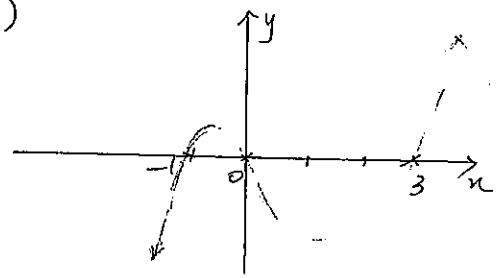
1. B 2. A 3. C 4. D 5. B

Question 1

a) $F(-2) = -8 + 8 + 10 + 3 = 13$

b) $x^4 - 2x^3 + 1 - x^3 + 5x^2 - 3x$
$= x^4 - 3x^3 + 5x^2 - 3x + 1$

c)



d) i)
$$x-3 \overline{)\, x^3 + 4x^2 + 0x + 1}$$
$x^3 - 3x^2$
$7x^2$
$7x^2 - 21x$
$21x + 1$
$21x - 63$
64

$\therefore x^3 + 4x^2 + 1 = (x-3)(x^2 + 7x + 21) + 64$

d) ii) $P(3) = 0 \Rightarrow -8 + 4a - 36 - 18 = 0$
or $2a - b = 13$ ——— ①

$P(1) = -24 \Rightarrow 1 + a + b - 18 = -24$
or $a + b = -7$ ——— ②

① + ②: $3a = 6$
$a = 2$

Sub into ②: $2 + b = -7$
$b = -9$

$\therefore a = 2, \quad b = -9$

Question 2

a) A $(x_1, y_1) = (-3, 4)$ B $(x_2, y_2) = (2, -1)$ $k : l = 5 : 3$

$x = \dfrac{kx_2 + lx_1}{k + l}$ $y = \dfrac{ky_2 + ly_1}{k + l}$

$x = \dfrac{10 - 9}{8}$ $y = \dfrac{-5 + 12}{8}$

P is $\left(\frac{1}{8}, \frac{7}{8}\right)$

Question 2 (continued)

B $(x_1, y_1) = (2, -1)$ A $(x_2, y_2) = (-3, 4)$ $k : l = -1 : 2$

$x = \dfrac{-1 \times -3 + 2 \times 2}{-1 + 2}$ $y = \dfrac{-1 \times 4 + 2 \times -1}{-1 + 2}$

$x = 7$ $y = -6$

Q is $(7, -6)$

b) i) $x = \dfrac{2k - 3}{k + 1}$ $y = \dfrac{-k + 4}{k + 1}$

ii) $2x - 3y + 4 = 0$

$2\left(\frac{2k-3}{k+1}\right) - 3\left(\frac{-k+4}{k+1}\right) = -4$

$4k - 6 + 3k - 12 = -4k - 4$

$11k = 14$

$k = \frac{14}{11}$

$\therefore$ D divides AB internally in ratio $14 : 11$.

Question 3

a) Put $u = 3^x$ $\therefore u^2 - 4u + 3 = 0$

$(u - 3)(u - 1) = 0$

$u = 3$ or 1

$\therefore 3^x = 3$ or $3^x = 1$

$x = 1$ or $x = 0$

b) Put $u = 3x^2 + 2x$

$\therefore u^2 - 9u + 8 = 0$

$(u - 8)(u - 1) = 0$

$u = 8$ or 1

$3x^2 + 2x - 8 = 0$ or $3x^2 + 2x - 1 = 0$

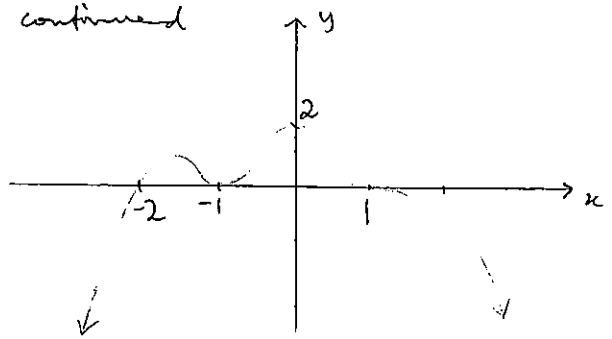
$(3x - 4)(x + 2) = 0$ $(3x - 1)(x + 1) = 0$

$3x \times -4$, $x \times +2$ $3x \times -1$, $x \times +1$

$x = \frac{4}{3}$ or -2 $x = \frac{1}{3}$ or -1

$\therefore x = \frac{4}{3}, -2, \frac{1}{3}$ or -1

Question 3 continued



Question 4 a) $y = 41°$ (Angles at circumference standing on same arc AB are equal)

$\angle CDB = 180 - (21 + 41 + 63) = 53°$ (angle sum of $\triangle = 180°$)

$x = 53°$ (Angles in same segment standing on arc BC are equal.)

b) $\angle BOC = 56°$ (Alternate $\angle$'s are equal $BO \parallel CA$)

$\angle BAC = x = \frac{1}{2} \times 56° = 28°$ (Angle at centre is twice $\angle$ at circumference standing on same arc BC)

c) i) $\angle ADB = \angle BDC = y$ (BD bisects $\angle ADC$ (given))

$\angle ACB = \angle ADB = y$ Angles at circumference standing on arc AB are equal.

$\angle CAB = \angle CDB = y$ (Angles at circumference standing on arc BC are equal)

$\therefore \angle CAB = \angle ACB = y$

$\therefore AB = BC$ (opposite equal angles)

$\therefore \triangle ABC$ is isosceles.

ii) $\angle DAC = \angle DBC = x$ (angles at circumference standing on arc DC are equal)

$\angle ABD = \angle DBC = x$ (DB bisects $\angle ABC$ (given))

$\therefore$ In $\triangle ABD$

$\angle BDA + \angle DAB + \angle ABD = 180°$ (angle sum of $\triangle ABD = 180°$)

$\therefore \quad y + (x+y) + x = 180°$

$2x + 2y = 180°$

$\therefore x + y = 90°$

But Angle in a semi-circle is a right angle

$\therefore BD$ is a diameter

Question 5

a) $x^3 - x^2 + 4x - 5 = 0$

$\alpha + \beta + \gamma = -\frac{b}{a} = 1$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = 4$

$\alpha\beta\gamma = -\frac{d}{a} = 5$

$\therefore 2\alpha + 2\beta + 2\gamma = 2 \times 1 = 2$

$2\alpha.2\beta + 2\beta.2\gamma + 2\gamma.2\alpha = 4 \times 4 = 16$

$2\alpha.2\beta.2\gamma = 8 \times 5 = 40.$

$\therefore$ Equation is $x^3 - 2x^2 + 16x - 40 = 0$

b) Let remainder be $ax + b$

$\therefore P(x) = (x+1)(x-1)Q(x) + ax + b$

$P(1) = -6 \therefore -6 = a + b$ —①

$P(-1) = 4 \therefore 4 = -a + b$ —②

① + ② $2b = -2$ ① − ② $2a = -10$

$b = -1$ and $a = -5$

c) i) Let roots be $\alpha, \frac{1}{\alpha}, \beta, -\beta$

$\therefore$ Sum of roots $= \alpha + \frac{1}{\alpha} = \frac{5}{2}$ —①

ii) Product of roots $= -\beta^2 = -\frac{2}{2}$

$\beta^2 = 1$ —②

iii) Solving ① $2\alpha^2 - 5\alpha + 2 = 0$

$(2\alpha - 1)(\alpha - 2) = 0$

$\alpha = \frac{1}{2}$ or 2

$2\alpha \quad -1$

$\alpha \quad -2$

Solving ② $\beta = \pm 1$

$\therefore$ The four roots are $2, \frac{1}{2}, +1, -1$

Sub any root to find c

eg Sub $x = 1$

$2 - 5 + c + 5 - 2 = 0$

$c = 0$