Name_____ Teacher_____

GOSFORD HIGH SCHOOL

2013

PRELIMINARY HSC

ASSESSMENT TASK 2

MATHEMATICS – EXTENSION 1

Duration- 75 minutes

Multiple choice	**4** questions worth **1 mark each.** (Answer this section on the **multiple choice response sheet provided)**	**/4**
Question 5	Polynomials	**/12**
Question 6	Circle Geometry	**/12**
Question 7	Inequalities, Counting Theory	**/12**
TOTAL		**/40**

ANSWER QUESTIONS 1 – 4 ON THE MULTIPLE CHOICE ANSWER SHEET PROVIDED.

Question 1.

The leading coefficient of $P(x) = 7x^2 - 5x^4 + x^3 + 3$ is:

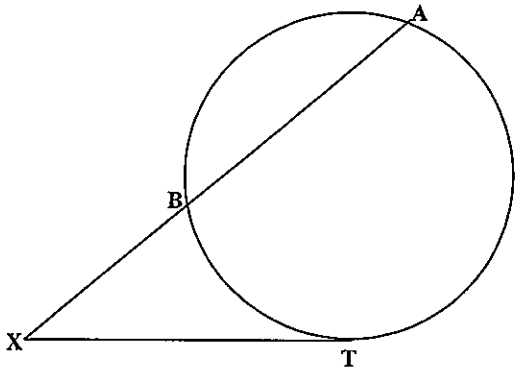
(A) 7 **(B)** -5 **(C)** 3 **(D)** 4

Question 2.

When $P(x) = 3x^3 + 4x^2 + 2x - 2$ is divided by $(x-1)$, the remainder is:

(A) -2 **(B)** 7 **(C)** -3 **(D)** 2

Question 3.



If XT is a tangent to the circle, which statement is true?

(A) $XT \times BX = AX$ **(B)** $AB^2 = AX \times XT$

(C) $XT^2 = AB \times BX$ **(D)** $XT^2 = XA \times XB$

Question 4.

${}^nC_r =$

(A) $\dfrac{n!}{r!(n-r)!}$ **(B)** $\dfrac{n!}{r!}$

(C) $\dfrac{{}^nP_r}{(n-r)!}$ **(D)** $\dfrac{r!}{(n-r)!}$

ANSWER QUESTIONS 5, 6 AND 7 ON YOUR OWN PAPER. START EACH QUESTION ON A NEW PAGE.

Question 5. (12 marks)

a) Given $P(x) = x^3 - 8x^2 + 9x + 18$

i) What is the degree of $P(x)$? **1**

ii) Show that $P(x)$ is divisible by $(x-3)$ and $(x+1)$. **2**

iii) Express $P(x)$ in terms of 3 linear factors. **2**

iv) Hence draw a neat sketch of $y = P(x)$, showing all x and y intercepts. **2**

b) Divide $P(x) = x^4 - 2x^2 + 5$ by $(x+2)$ and hence express $P(x)$ in the form $P(x) = (x+2)Q(x) + R(x)$ **3**

c) When the polynomial $P(x) = 2x^3 - x^2 + ax + b$ is divided by x the remainder is 7 and when it is divided by $x+1$ the remainder is -3. Find the values of a and b. **2**

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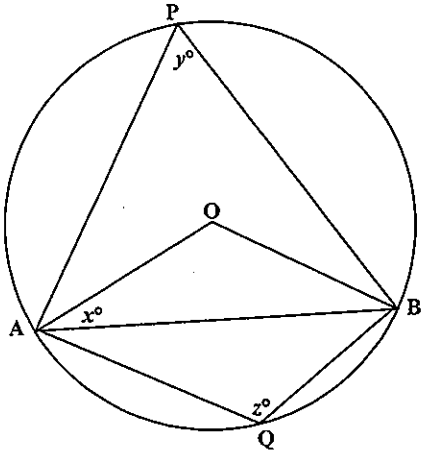
Question 6. (12 marks)

a)

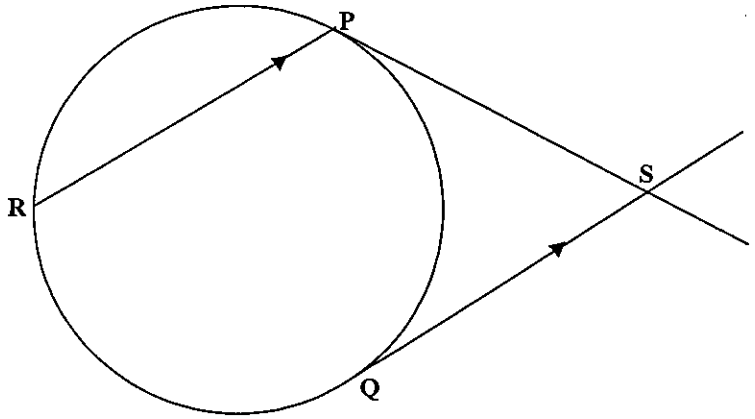


O is the centre of the circle. Prove that:

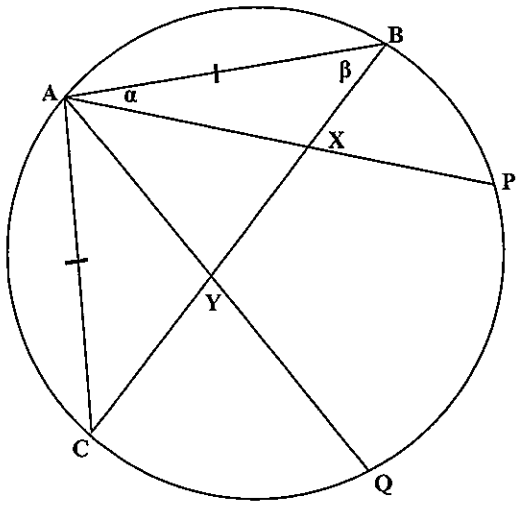
i) $x + y = 90°$ **2**

ii) $z - y = 2x$ **2**

b) In the diagram below, P and Q are points on a circle and the tangents to the circle at P and Q meet at S. R is a point on the circle so that the chord PR is parallel to QS. Prove that QP = QR. **2**



c)



i) State why $\angle AXC = \alpha + \beta$ **1**

ii) Prove that $\angle BQP = \alpha$ **1**

iii) Prove that $\angle BQA = \beta$ **2**

iv) Prove that PQYX is a cyclic quadrilateral **2**

START A NEW PAGE.

Question 7. (12 marks)

a) Solve $3(x-2)^2(x+1)>0$ **2**

b) Solve $\frac{x^2-1}{x} \geq 0$ **2**

c) Solve $\frac{2}{x-1} \leq \frac{1}{3}$ **2**

d) In how many ways can a team of 15 be selected from 17 available players? **1**

e) A committee of 3 women and 4 men is to be formed from 8 women and 7 men.

i) In how many ways can the committee be formed if there are no restrictions? **1**

ii) In how many ways can the committee be formed if one particular woman refuses to serve with one particular man? **2**

f) 5 boys and 4 girls form a queue. There are 2 brothers who want to stand together. The remaining 3 boys also wish to stand together and the 4 girls don't mind where they stand. In how many ways can the queue be formed? **2**

PRELIMINARY EXTENSION 1

ASSESSMENT TASK 2 2013

SOLUTIONS.

1) B

2) $P(1) = 3+4+2-2$
$= 7$ $\therefore$ B

3) D

4) A

5) a) $P(x) = x^3 - 8x^2 + 9x + 18$

i) 3

ii) $P(3) = 27 - 72 + 27 + 18$
$= 0$
$\therefore P(x)$ divisible by $(x-3)$

$P(-1) = -1 - 8 - 9 + 18$
$= 0$
$\therefore P(x)$ divisible by $(x+1)$

iii) $(x-3)(x+1) = x^2 - 2x - 3$

$$x^2-2x-3 \overline{)\, x^3-8x^2+9x+18}$$

$x-6$

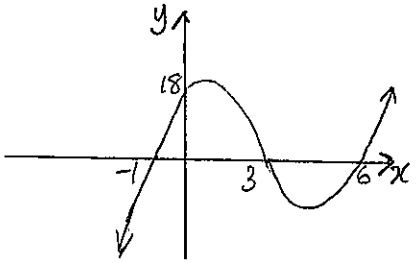
$x^3 - 2x^2 - 3x$

$-6x^2 + 12x + 18$

$-6x^2 + 12x + 18$

$\therefore P(x) = (x-3)(x+1)(x-6)$

iv)



b)

$$x+2 \overline{)\, x^4 - 2x^2 + 5}$$

$x^3 - 2x^2 + 2x - 4$

$x^4 + 2x^3$

$-2x^3 - 2x^2$

$-2x^3 - 4x^2$

$2x^2$

$2x^2 + 4x$

$-4x + 5$

$-4x - 8$

13

$\therefore P(x) = (x+2)(x^3 - 2x^2 + 2x - 4) + 13$

c) $P(0) = 7$ $\therefore b = 7$

$P(-1) = -3$

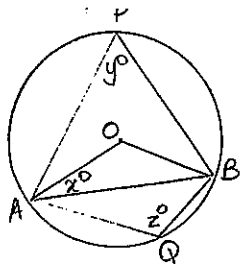
ie $-2 - 1 - a + b = -3$

$-a + b = 0$

$a = b$

$\therefore a = 7$

6) a)



i) $\angle AOB = 2y$ (angle at centre is twice angle at circumference, standing on same arc)

$OA = OB$ (equal radii)

$\therefore \triangle OAB$ is isosceles

$\therefore \angle OBA = \angle OAB = x$ (equal angles in isosceles $\triangle$)

now $x + x + 2y = 180^\circ$ (angle sum of $\triangle$)

$2x + 2y = 180^\circ$

$\therefore x + y = 90^\circ$

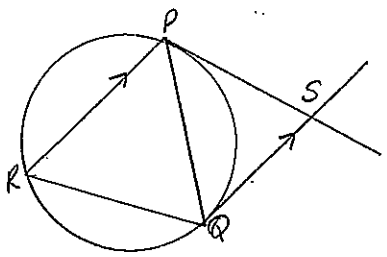
ii) $z + y = 180^\circ$ (opposite angles of cyclic quadrilateral are supplementary)

$z + y = 2(x+y)$ (from i))

$z + y = 2x + 2y$

$z - y = 2x$

b)



Prove $QP = QR$

$\angle PQS = \angle PRQ$ (angle in alternate segment theo

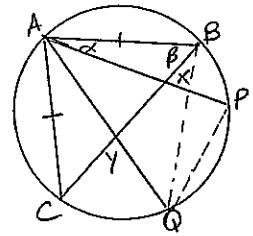
$\angle PQS = \angle RPQ$ (alternate angles $RP \parallel QS$)

$\therefore \angle PRQ = \angle RPQ$

$\therefore \triangle RPQ$ is isosceles

$\therefore QP = QR$

c)



i) $\angle AXC = \alpha + \beta$ (exterior angle of a $\triangle$ theorem)

ii) $\angle BQP = \alpha$ (angles at circumfere standing on same arc are e

iii) $AB = AC$ (given)

$\therefore \triangle ABC$ is isosceles

$\therefore \angle ACB = \angle ABC = \beta$

also

$\angle BQA = \angle BCA$ (angles at circumference standing on same arc are equal)

$\therefore \angle BQA = \beta$

iv)

$\angle PQY = \angle PQB + \angle BQA$

$= \alpha + \beta$ (from ii) + iii))

$\angle AXC = \alpha + \beta$ (from i))

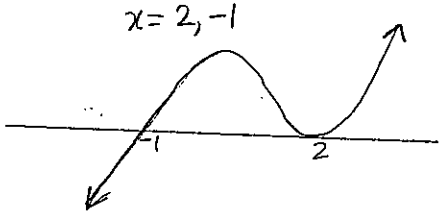
$\therefore$ PQYX is a cyclic quadrilateral (exterior angle equals interior opposite angle)

7)

a) $3(x-2)^2(x+1) > 0$

Solve $3(x-2)^2(x+1) = 0$

$x = 2, -1$



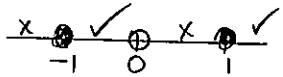
$-1 < x < 2$ or $x > 2$

b) $\frac{x^2-1}{x} \geq 0$

Solve $\frac{x^2-1}{x} = 0$

$x^2 - 1 = 0 \quad x \neq 0$

$x = \pm 1$



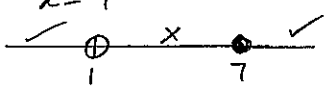
$-1 \leq x < 0$ or $x \geq 1$

c) $\frac{2}{x-1} \leq \frac{1}{3}$

Solve $\frac{2}{x-1} = \frac{1}{3}$.

$6 = x - 1, \quad x \neq 1$

$x = 7$



$x < 1$ or $x \geq 7$

d) ${}^{17}C_{15} = 136$

e) i) ${}^8C_3 \times {}^7C_4 = 1960$

ii) number of ways with both

$= {}^7C_2 \times {}^6C_3$

$= 420$

$\therefore$ number of ways without both

$= 1960 - 420$

$= 1540$

f) [2B] [3B] G G G G

$6! \times 2! \times 3! = 8640$

2013 PRELIMINARY MATHEMATICS EXTENSION 1

ASSESSMENT TASK 2

Student Name/Number:_____

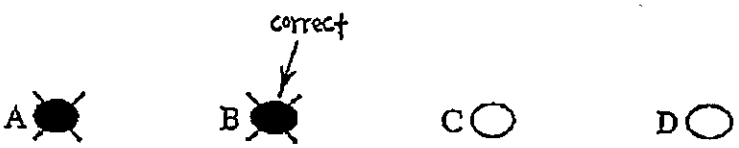
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 \div 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○