Student Name/Number .....

ASSESSMENT TASK 2 **June 2014**

MATHEMATICS EXTENSION 1

PRELIMINARY

General Instructions

- Reading Time – 5 minutes
- Working Time – 60 Minutes .
- Start each question on a new page.
- All necessary working should be shown in every question.

QUESTION	MARK
MC	
1 Algebra	
2 Circle geometry	
3 Circle geometry	
Total /40	

Student Name/Number:_____

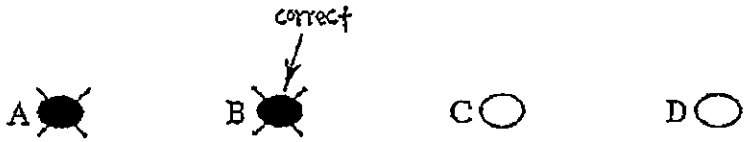
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○

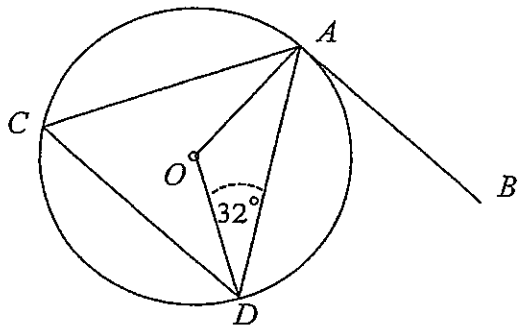
Instructions: Answer multiple choice on the multiple choice sheet

Question 1 (4 marks)

1 At a dinner party, the host, hostess and their six guests sit at a round table. In how many ways can they be arranged if the host and hostess are separated?

A) 720
B) 1440
C) 3600
D) 5040

2. *AB* is a tangent to the circle, centre *O*. $\angle ADO = 32^\circ$ Find the size of $\angle DAB$



A) 29°
B) 32°
C) 37°
D) 58°

3. What are the values of p such that $\frac{p+1}{p} \leq 1$

A) $p > 0$
B) $p < 0$
C) $p \leq 0$
D) $-1 \leq p < 0$

4 How many arrangements of the letters of the word OLYMPIC are possible if the C and the L are to be together in any order

A) 5!
B) 6!
C) 2x5!
D) 2x6!

Question 2 (13 marks)

(a) Find the value of n which satisfies the equation ${}^{n-2}P_2 = 156$ **2**

(b) Committees of 5 persons are to be drawn from a group of 7 men and 4 women.
How many of these committees include a majority of women? **2**

(c) Solve $|x^2 - 5| = 5x + 9$ **3**

(d) Solve simultaneously **2**

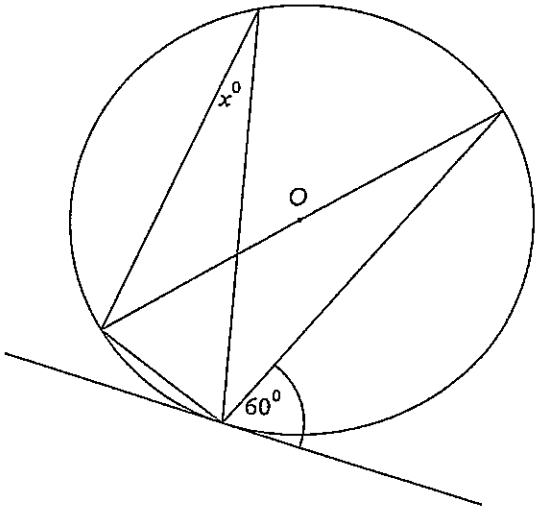
$$x^2 + y^2 = 100$$
$$3x - y = 10$$

(e) Simplify $36^{2-3a} \times 12^{3a} \div \left(9^a\right)^2$ **2**

(f) Find all values for x that satisfy $\frac{5}{x-4} \leq x$ **2**

QUESTION 3 Answer on a new page (12 Marks)

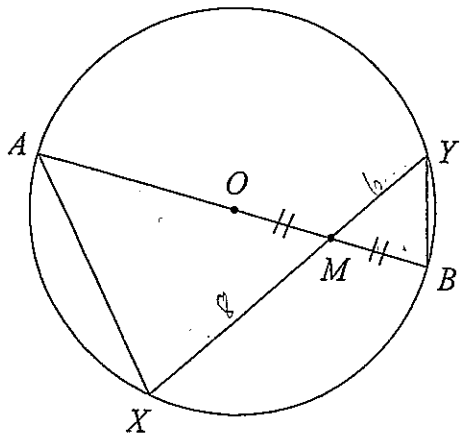
(a) Find the value of x: 2



(b) AB is a chord of a circle and CAD is a tangent to the circle at the point A. 2

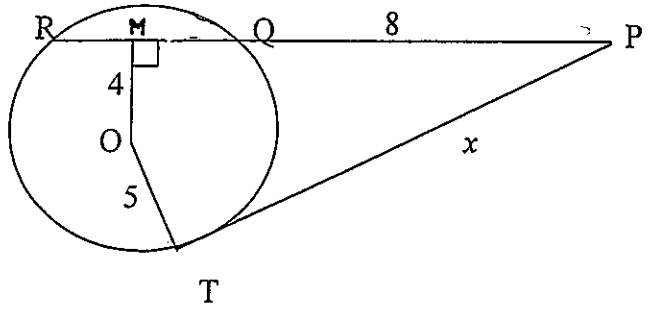
The bisector of $< BAC$ meets the circle again at P and the bisector of $< BAD$ meets the circle again at Q. Draw a careful diagram showing this information.

(c) In the diagram below, AB is a diameter of a circle, whose centre is the point O. The chord XY passes through M, the mid-point of OB. AX and BY are joined.



If $XM = 8$ cm and $YM = 6$ cm, find the length of the radius of the circle. 2

(d)

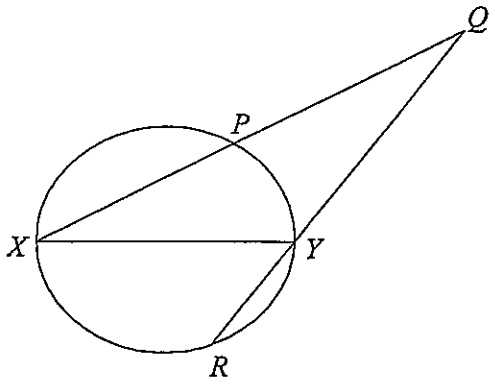


2

PT is a tangent to the circle, centre O. OM is perpendicular to the secant RQ.

QP = 8. Find the value of x, the length of PT

(e)



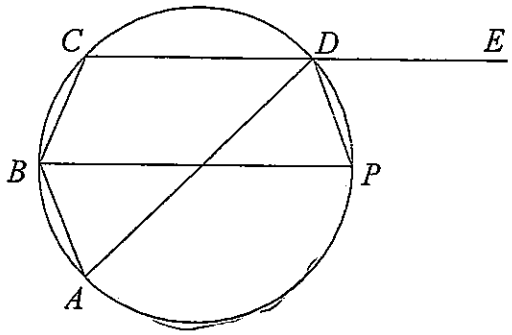
XY is the diameter of the circle $XPYR$. XP and RY are produced to Q. Given that $\angle PXY = 38°$ and $\angle PQY = 27°$, find the size of the following angles giving reasons:

(ii) $\angle XYP$ **2**

(iii) $\angle YPR$ **2**

QUESTION 4 (11 Marks)

(a)



In the diagram above *ABCD* is a cyclic quadrilateral. *CD* is produced to *E*. *P* is a point on the circle through *A, B, C, D* such that $\angle ABP = \angle PBC$.

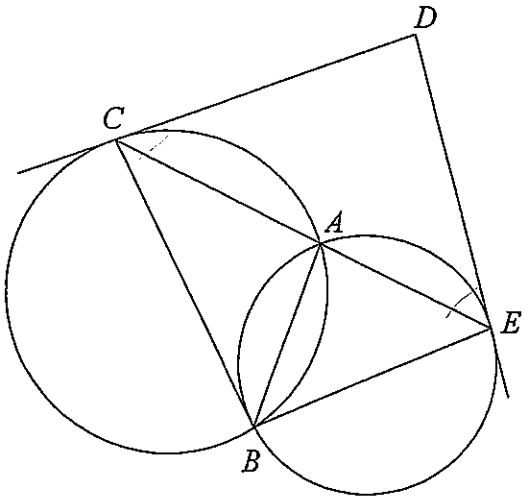
(i) Copy the diagram showing the above information.

(ii) Explain why $\angle ABP = \angle ADP$. 1

(iii) Show that PD bisects $\angle ADE$. 2

(iv) If, in addition, $\angle BAP = 90°$ and $\angle APD = 90°$, explain where the centre of the circle is located. 1

(b)



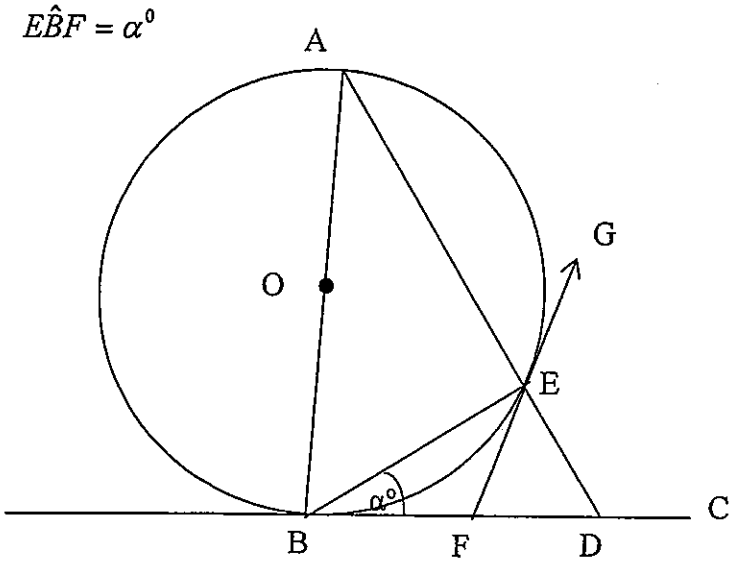
Two circles intersect at *A* and *B*. *CAE* is a straight line where *C* is a point on the first circle and *E* is a point on the second circle. The tangent at *C* to the first circle and the tangent at *E* to the second circle meet at *D*.

(i)Copy the diagram.

(ii) Prove that *BCDE* is a cyclic quadrilateral 3

(c) In the diagram, AB is the diameter of the circle, centre O and BC is tangential to the circle at B. The line AED intersects the circle at E and intersects BC at D. The tangent to the circle at E intersects BC at F.

$E\hat{B}F = \alpha^0$



i) Copy the diagram

ii) Prove that $F\hat{E}D = (90 - \alpha)^\circ$ **2**

iii) Prove F is the midpoint of BD **2**

QUESTION 1.

Round table
no restrictions $(n-1)!$

$7! = 5040$

sit both together

$6! = 720 \times 2$ (different ways.)

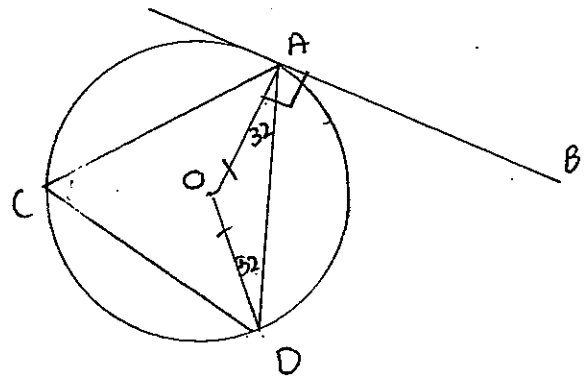
$= 1440$

∴ sitting apart

$5040 - 1440 = 3600$

∴ (C)

2)



$\angle AOD = 116°$

$\angle OAD = 32$

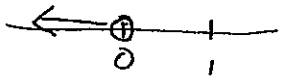
$\angle DAB = 90 - 32$

$= 58°$

(D)

3) $p \neq 0$

test $p = 1$



$\frac{2}{1} \leq 1$ false

$p < 0$

(B)

4) CL _ _ _ _

$5! \times 6$ and C&L switch

$2 \times 6!$ (D)

QUESTION 2.

a)

$${}^{n}P_{r} = \frac{n!}{(n-r)!} \qquad {}^{n-2}P_{2} = \frac{(n-2)!}{(n-2-2)!}$$

$$\frac{(n-2)(n-3)(n-4)!}{(n-4)!}$$

$(n-2)(n-3) = 156$

$n^2 - 5n + 6 = 156$

$n^2 - 5n - 150 = 0$

$(n-15)(n+10) = 0$ $\quad n = 15$

b) 4 women + 3 women

${}^{7}C_{1} \times {}^{4}C_{4} + {}^{7}C_{2} \times {}^{4}C_{3}$

$7 \times 1 + 21 \times 4$

$7 + 84 = 91$

c) $x^2 - 5 = 5x + 9$ or $x^2 - 5 = -5x - 9$

$x^2 - 5x - 14 = 0$ or $x^2 + 5x + 4 = 0$

$(x-7)(x+2) = 0$ $\quad (x+4)(x+1) = 0$

$x = 7, \; x = -2$ $\quad x = -4$ or -1

test solutions.

$|49 - 5| = 35 + 9$ (7) true ✓

$|4 - 5| = 10 + 9$ (−2)

$1 = -1$

False

$|16 - 5| = -20 + 9$ (−4)

$11 = -11$

False

$|1 - 5| = -5 + 9$ (−1)

$4 = 4$

true

Solutions $x = 7$ and -1.

d) $3x - 10 = y$ sub in (1)

$x^2 + (3x-10)^2 = 100$

$x^2 + 9x^2 - 60x + 100 = 100$

$10x^2 - 60x = 0$

$10x(x-6)=0$

$x=0 \quad x=6$

$y=-10 \quad y=8$

e) $\dfrac{q^{2-3a} \times 4^{2-3a} \times 4^{3a} \times 3^{3a}}{q^{29}}$

$= \dfrac{3^{4-6a} \times 2^{4-6a} \times 2^{6a} \times 3^{3a}}{q^{4a}}$

$= 2^4 \times 3^{4-3a-4a}$

$= 2^4 \times 3^{4-7a}$

f) $5 \leq x^2 - 4x \qquad x \neq 4$

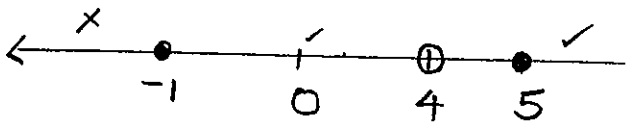
solve equation

$5 = x^2 - 4x$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1)=0$

$x = 5$ or -1

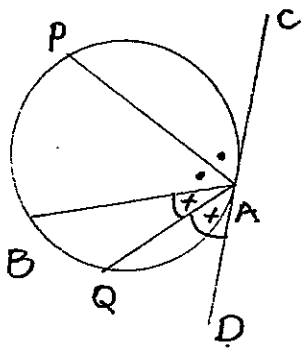


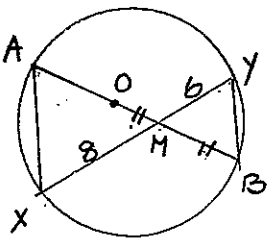
$-1 \leq x < 4$ and $x \geq 5$

QUESTION 3

a) $x = 30$

b)





Ratios let $MB = x$

$\therefore AM = 3x$.

$6 \times 8 = x \times 3x$

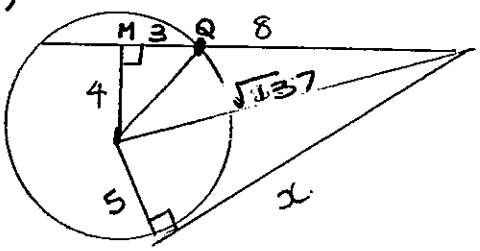
$48 = 3x^2$

$16 = x^2$

$x = 4$

radius = 8 cm

d)



$OQ = 5$

$\therefore QM = 3$

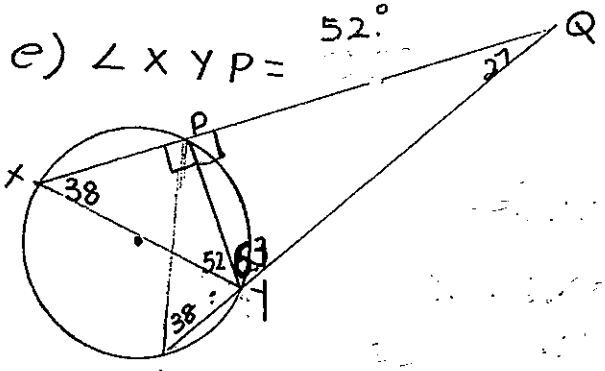
$11^2 + 4^2 = OP^2 \qquad OP = \sqrt{137}$

$TR^2 = 137 - 25$

$= 112$

$TR = \sqrt{112}$

e) $\angle XYP =$ $52°$



$\angle YPR =$ ext $\angle PYQ - \angle PRY$

$= 63 - 38$

$= 25°$

QUESTION 4

a) ii) angles standing on the same arc

iii) $\angle EDA$ is the exterior angle of a cyclic quad

$\therefore \angle EDA = \angle CBA$

Let $\angle ABP = x$

since $\angle CBA = 2x$

$\angle EDA = 2x$

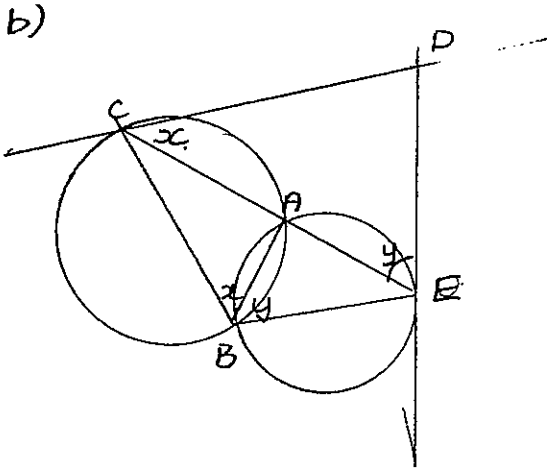
and $\angle ADP = x$ $\therefore \angle PDE = x$

$\therefore$ PD bisected $\angle ADE$

iv) at the intersection of BP and AD

---

b)



$\angle DCE = x = \angle CBA$

angle in alt seg.

similarly let $\angle DEC = y$

$\therefore \angle ABE = y$

now $\angle CDE = 180 - (x+y)$

angle sum of $\Delta = 180$

$\angle CBE = x+y$

since $\angle CBE$ and $\angle CDE$ are supplementary BCDE is cyclic quad

c) ii) since $BF = FE$

equal tangents to a circle

$\angle FBE = \alpha = \angle BEF$

(isosceles Δ base $\angle$ equal)

$\angle AEB = 90°$ angle in a semicircle is a rt $\angle$

$\angle BED = 90°$ supplementary adj

$\therefore \angle FED = 90 - \alpha$

iii) In ΔBED

$\angle BDE = 90 - \alpha$

($\angle$ sum $\Delta = 180°$)

$\therefore \angle FED = \angle FDE$ $(90-\alpha)$

$\therefore \Delta FED$ is isosceles Δ

$\therefore FE = FD$

since $BF = FE$

$BF = FD$ $\therefore$ F midpoint