NAME ______________________________

GOSFORD HIGH SCHOOL

2015

Preliminary

Higher School Certificate

MATHEMATICS

Extension 1

Assessment Task 2

General Instructions

- Reading time 5 minutes
- Working time 90 minutes
- Write using black or blue pen
- Board approved calculators may be used
- Use multiple choice answer sheet provided for Questions 1 to 7
- For Questions 8 to 10, show relevant mathematical reasoning and/or calculations
- Total marks 68

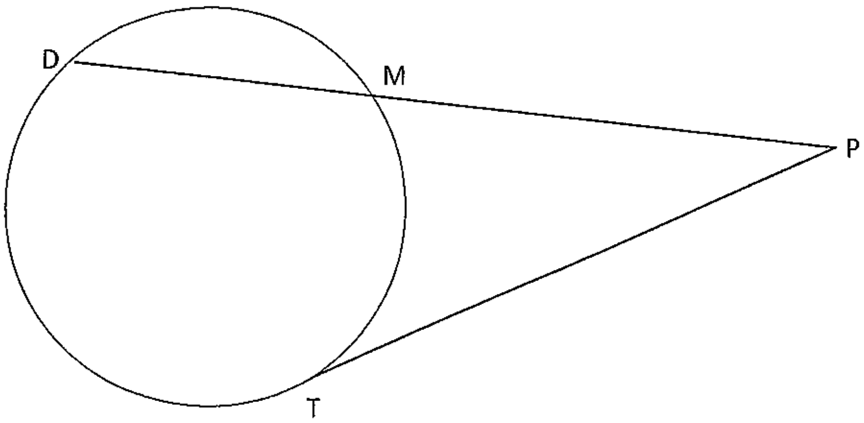
Question 1

$tan2A = ?$

(A) $\frac{2tanA}{1+tanA}$ (B) $\frac{2tanA}{1-tanA}$ (C) $\frac{2tanA}{1+tan^2A}$ (D) $\frac{2tanA}{1-tan^2A}$

Question 2

PT is a tangent to the circle and PD is a secant.

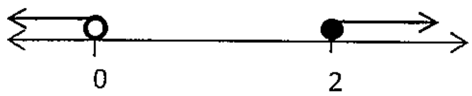


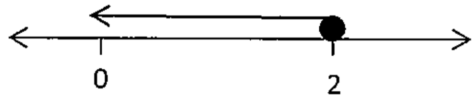
Therefore $PT^2 = ?$

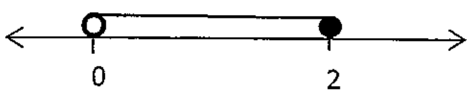
(A) $PM \times MD$ (B) $PM \times PD$ (C) $DM \times DP$ (D) $PM + MD$

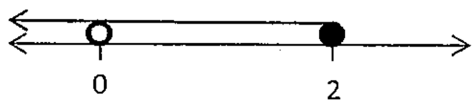
Question 3

Which of the following represents the graphical solution to the inequality $\frac{2}{x} \geq 1$

A) 

B) 

C) 

D) 

Question 4

A zookeeper has to accommodate 7 different animals in 7 enclosures, one in each enclosure. However, 2 of the animals are too large for 3 of the enclosures.
The number of different possible arrangements of the animals is :

(A) 5040 (B) 240 (C) 1440 (D) 720

Question 5

Three students write down different expressions for $sin^2 x$

Lena's answer states that $sin^2 x = 1 - cos^2 x$

Will's answer states that $sin^2 x = 4sin^2\left(\frac{x}{2}\right) \cdot cos^2\left(\frac{x}{2}\right)$

Abby's answer states that $sin^2 x = \frac{1}{2}(1 - cos2x)$

Which of the following statements is correct

A) Only Lena is correct

B) Only Lena and Will are correct

C) Only Lena and Abby are correct

D) All three students are correct

Question 6

For all values of X and Y

$$cosX \cdot cos(X+Y) + \sin X \cdot \sin(X+Y)$$

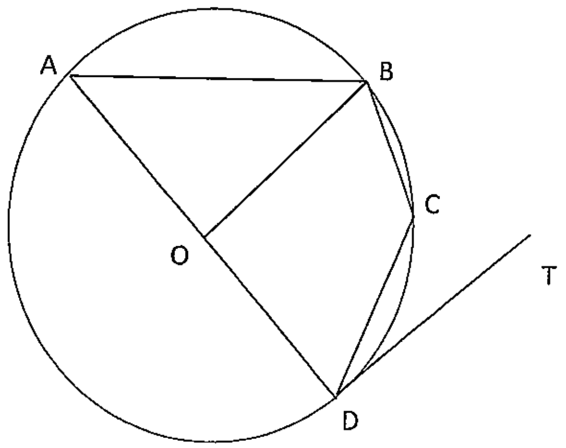
is equal to

(A) $cosX$ (B) $cosY$ (C) $-cosY$ (D) $\cos(2X+Y)$

Question 7

O is the centre of the circle below. A,B,C and D all lie on the circle. AD is a diameter, TD is a tangent , $\angle TDC = 38°$ and $\angle DCB = 108°$.

DIAGRAM NOT DRAWN TO SCALE



$\angle CBO = ?$

A) 56° B) 52° (C) 128° (D) 72°

Question 8 (19 marks)

(a) Simplify $\frac{9^n - 1}{3^n - 1}$ (2)

(b) Factorise $y^2 - x^2 + 2x - 1$ (2)

(c) Simplify $\frac{yx^{-1} - xy^{-1}}{x+y}$ (2)

(d) Solve $\frac{1}{x+2} < 1$ (3)

(e) A committee of three is to be formed from 4 men and 5 women.
Find the number of different selections are possible if :

(i) there are no other restrictions on selection (1)

(ii) there is to be at least one man on the committee? (2)

(f) In how many ways can the letters of the word READER be arranged if :

(i) there are no restrictions on the arrangements (1)

(ii) the two R's are together (1)

(g) In how many ways can 5 men and 5 women be arranged in a circle if

(i) there is no restrictions (1)

(ii) no two men are to sit next to each other (1)

(h) Solve $\frac{1}{(x-1)(x-3)} < -1$ (3)

Question 9 (22 marks)

(a) Find the exact value of $sin105°$ (2)

(b) Express $sin\,2x \cdot cotx - 1$ in simplest form (3)

(c) Simplify $\tan(x + 45°) \cdot \tan(x - 45°)$ (2)

(d) Solve $sin\alpha = cos2\alpha$ for $0° \leq \alpha \leq 360°$ (3)

(e) If $t = tan\left(\frac{\emptyset}{2}\right)$,

(i) express $sin\emptyset$ and $cos\emptyset$ in terms of t (2)

(ii) hence, solve $cos\emptyset + sin\emptyset + 1 = 0$ for $0° \leq \emptyset \leq 360$ (3)

(f) (i) Prove that $cos3x = 4cos^3x - 3cosx$ (3)

(ii) Explain why, if $x = 18°$, then $sin2x = cos3x$ (1)

(iii) With consideration to parts (i) and (ii) above ,

prove that $sin18° = \frac{\sqrt{5}-1}{4}$ (3)

Question 10 (20 marks)

(a) In the diagram, O is the centre of the circle and $\angle BOC = 136°$ as shown. Find $\angle BAC$, giving reasons. (2)

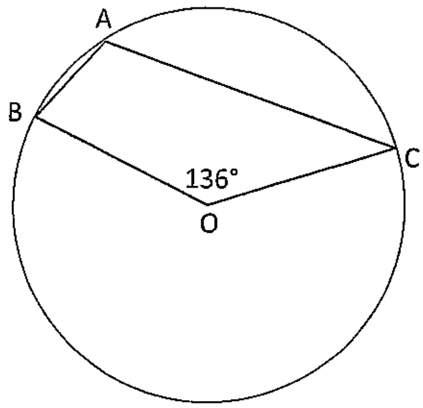


DIAGRAM NOT DRAWN TO SCALE

(b) Given that RT is a diameter, find x giving reasons. (3)

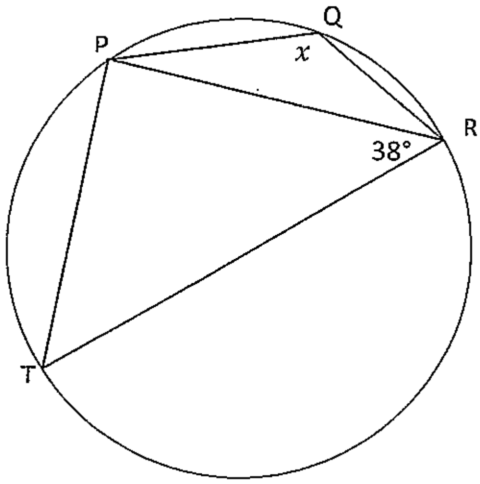
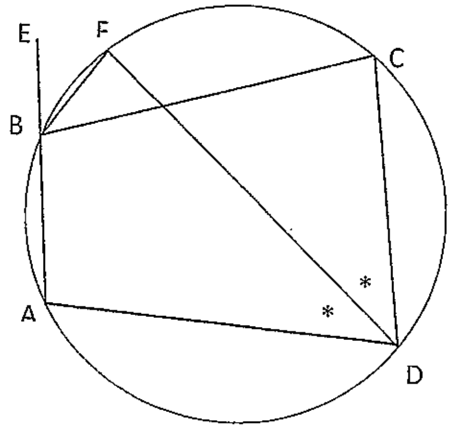


DIAGRAM NOT DRAWN TO SCALE

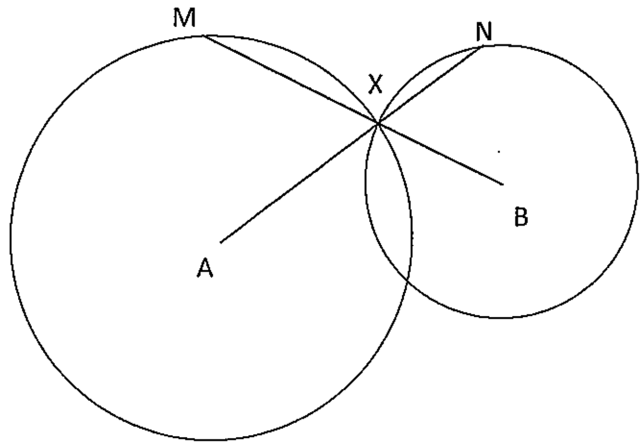
(c) A,B,C,D and F all lie on the given circle and AB has been produced to E.

FD bisects angle ADC.

Prove that FB bisects the angle EBC. (3)



(d) The two circles have centres A and B.
M and N are points on the circumferences of these respective circles.
AN and BM intersect at X, where X is one of the points of intersection of the two circles.

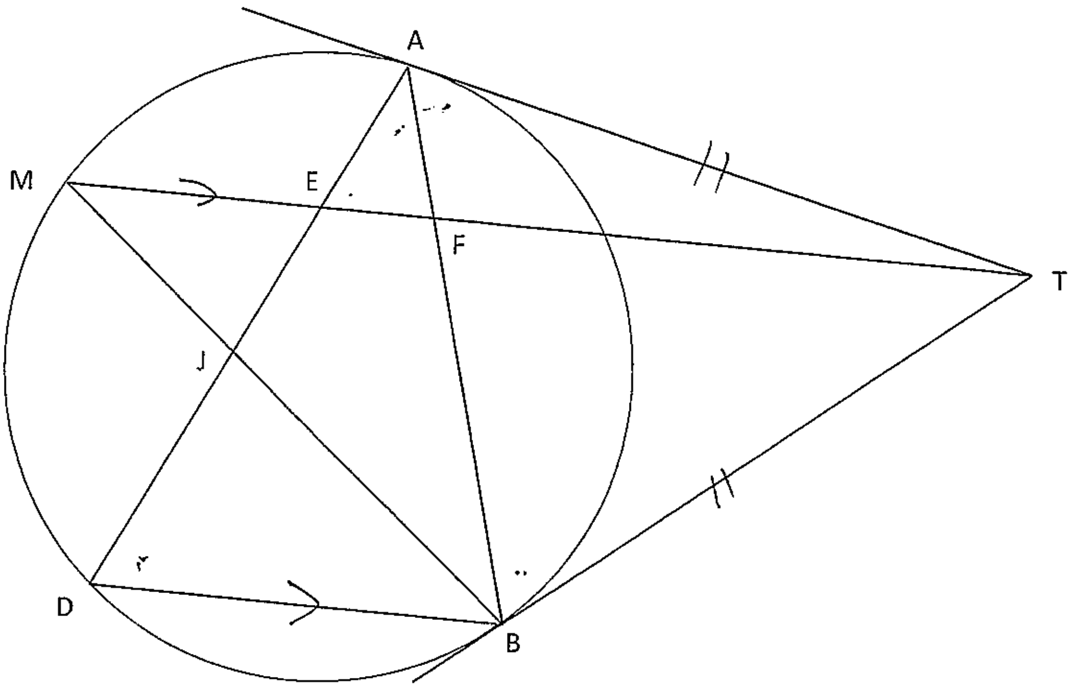


(i) Prove $\angle AMX = \angle AXM$ (1)

(ii) Prove $\angle AMX = \angle BNX$ (2)

(iii) State why A, B, N and M are concyclic. (1)

(e) In the diagram below TA and TB are tangents to the circle.
AB, BD, MB and DA are all chords.
MT is a secant which is parallel to the chord BD.



(i) Give reasons why triangle TAB is isosceles. (2)

(ii) State why $\angle TAB = \angle AET$ (2)

(iii) Prove ΔTAE is similar to ΔTFA (2)

(iv) Prove $TB^2 = TE \cdot TF$ (2)

End of Examination

NAME ________________________________

ANSWER SHEET FOR SECTION 1 - Multiple Choice

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D

GHS 2015 PRELIMINARY HSC X1 TASK 2 SOL

(1) D (2) B (3) C (4) C (5) D (6) B (7) A

QUESTION 8

(a) $\frac{9^n-1}{3^n-1} = \frac{(3^2)^n-1}{3^n-1}$

$= \frac{(3^n)^2-1}{3^n-1}$

$= \frac{(3^n-1)(3^n+1)}{3^n-1}$ 1 mark

$= 3^n+1$ 1 mark

b) $y^2 - x^2+2x-1 = y^2-(x^2-2x+1)$

$= y^2-(x-1)^2$ 1 mark

$= (y-x+1)(y+x-1)$ 1 mark

(c) $\frac{yx^{-1}-xy^{-1}}{x+y} = \frac{\frac{y}{x}-\frac{x}{y}}{x+y}$

$= \frac{y^2-x^2}{xy(x+y)}$ 1 mark

$= \frac{(y-x)(y+x)}{xy(x+y)}$ 1 mark

$= \frac{y-x}{xy}$ 1 mark

(e) (i) No. of selections $= {}^9C_3$ (1)

$=$

(ii) No. of selections $= {}^4C_1 \times {}^5C_2 + {}^4C_2 \times {}^5C_1 + {}^4C_3$

$=$ (2)

(f) (i) N°. of arrangements $= \frac{6!}{2!2!}$

$= 180$ (1)

(ii) N°. of arrangements $= \frac{1 \times 5!}{2!}$

$= 60$ (1)

(g) (i) N° of arrangements $= 9!$ (1)

$=$

(ii) N°. of arrangements $= 4!\,5!$

$=$ (1)

(h) Critical values when $x = 1, 3$ and when

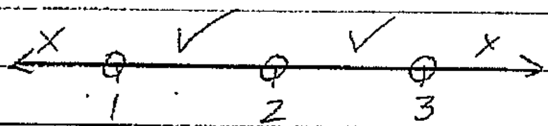
$\frac{1}{(x-1)(x-3)} = -1 \Longrightarrow -1 = (x-1)(x-3)$

$-1 = x^2 - 4x + 3$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$



Test $x < 1$; (say $x = 0$) ; $\frac{1}{3} < -1$, False

Test $1 < x < 2$; (say $x = \frac{3}{2}$) ; $-\frac{4}{3} < -1$; True.

Test $2 < x < 3$ (say $x = \frac{5}{2}$; $-\frac{4}{3} < -1$; True

Test $x > 3$ (say $x = 4$) ; $\frac{1}{3} < -1$; False

$\therefore 1 < x < 2$ or $2 < x < 3$

Question 9

(a) $\sin(105)^\circ = \sin(45+60)^\circ$

$= \sin 45 \cos 60 + \sin 60 \cos 45^\circ$ (1)

$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$ (1)

$= \frac{1+\sqrt{3}}{2\sqrt{2}}$

(b) $\sin 2x \cdot \cot x - 1 = 2 \sin x \cos x \cdot \frac{\cos x}{\sin x} - 1$ (1)

(1)

$= 2\cos^2 x - 1$

$= \cos 2x$ (1)

(c) $\tan(x+45^\circ) \cdot \tan(x-45^\circ) = \left(\frac{\tan x + \tan 45^\circ}{1 - \tan x \cdot \tan 45^\circ}\right) \cdot \left(\frac{\tan x - \tan 45^\circ}{1 + \tan x \tan 45}\right)$

$\frac{\tan x + 1}{1 - \tan x} \cdot \frac{\tan x - 1}{1 + \tan x}$ (1)

$= -1$ (1)

(d) $\sin \alpha = 1 - 2\sin^2 \alpha$ (1)

$2\sin^2 \alpha + \sin \alpha - 1 = 0$

$(2\sin \alpha - 1)(\sin \alpha + 1) = 0$ (1)

$\sin \alpha = \frac{1}{2}, -1$

$\therefore \alpha = 30^\circ, 150^\circ, 270^\circ$ (1)

(e) (i) $\sin \phi = \frac{2t}{1+t^2}$; $\cos \phi = \frac{1-t^2}{1+t^2}$ (2)

(ii) $\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} + 1 = 0$ Note $0 \le \tan\frac{\phi}{2} \le 180^0$

$\therefore 1-t^2+2t+1+t^2 = 0$ (1)

$2t+2 = 0$

$t = -1$ (1)

$\therefore \frac{\phi}{2} = 135^0$

$\therefore \phi = 270^0$ and 180^0 (inspection)

(f) (i) $\cos 3x = \cos(2x+x)$

$= \cos 2x \cos x - \sin 2x \sin x$ (1)

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \cdot \sin x$

$= 2\cos^3 x - \cos x - 2\cos x \cdot \sin^2 x$

$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$ (1)

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$ (1)

(f) (ii) $2x = 36^0$, $3x = 54^0$

$\sin 36^0 = \cos 54^0$ since $\sin\theta = \cos(90-\theta)$ (1)

(iii) $\sin(2 \times 18^0) = \cos(3 \times 18)^0$

$2\sin 18^0 \cos 18^0 = 4\cos^3 18 - 3\cos 18$

$2\sin 18^0 = 4\cos^2 18^0 - 3$

$3 + 2\sin 18^0 = 4(1-\sin^2 18^0)$

$= 4 - 4\sin^2 18^0$

$4\sin^2 18 + 2\sin 18^0 - 1 = 0$ (1)

$\sin 18^0 = \frac{-2 \pm \sqrt{4+16}}{8}$

$\sin 18^0 = \frac{-2 \pm \sqrt{20}}{8}$

$\sin 18^0 = \frac{-1 \pm \sqrt{5}}{4}$ (1)

But $\sin 18^0 > 0$ $\therefore \sin 18^0 = \frac{-1+\sqrt{5}}{4}$

(1)

Question 10

a) Reflex $\hat{BOC} = 360° - 136°$ (angles at a point form a revolution)
$= 224°$ (1)

$\therefore \hat{BAC} = \frac{1}{2} \times 224°$ (angle at the centre is twice the angle at the circumference standing on the same arc) (1)
$= 112°$

b) $\hat{TPR} = 90°$ (angle in a semi-circle is a right angle)

$\hat{PTR} = 180° - (90+38)°$ (angle sum of a triangle)
$= 52°$

$x = 180 - 52°$ (opposite angles of a cyclic quadrilateral are supplementary)
$= 128°$

c) $\hat{EBF} = \hat{ADF}$ (exterior angle of a cyclic quadrilateral is equal to the opposite interior angle)

$\hat{FBC} = \hat{FDC}$ (angles at the circumference standing on the same arc are equal)

But $\hat{ADF} = \hat{FDC}$ (given FD bisects $\hat{ADC}$)

$\therefore \hat{EBF} = \hat{FBC}$ (equal to equal angles) c)(i) TA = TB (tangents from an external point are equal)

∴ $\triangle TAB$ is isosceles since triangle has two equal sides. (1)

(ii) $T\hat{A}B = A\hat{D}B$ (angle between a tangent and a chord is equal to the angle in the alternate segment)

$A\hat{D}B = A\hat{E}T$ (corresponding angles, TM // BD)

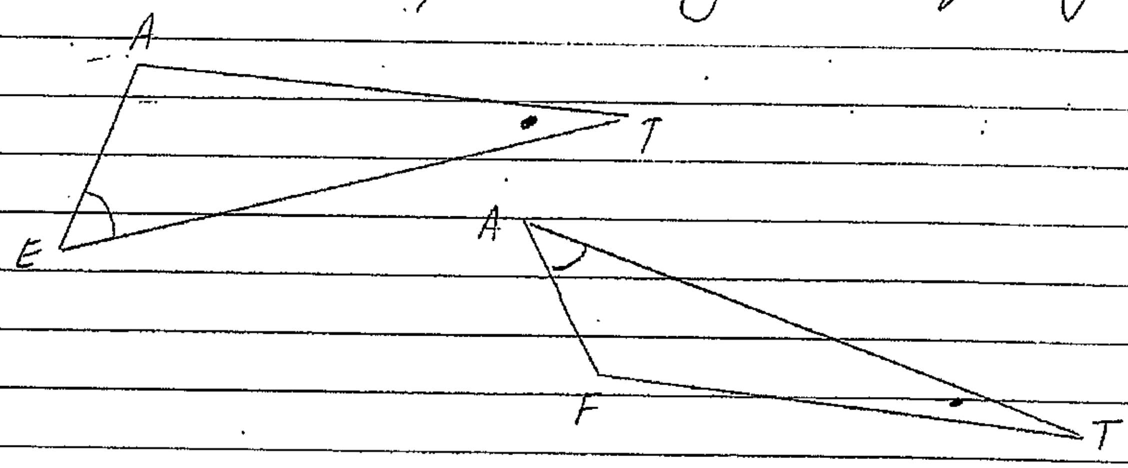
∴ $T\hat{A}B = A\hat{E}T$ (equal to equal angles) (2)

(iii) $A\hat{T}E = A\hat{T}F$ (common angle)

$T\hat{A}E = A\hat{E}T$ (proven in (ii)) (2)

∴ $\triangle TAE \parallel\!| \triangle TFA$ (triangles are equiangular)

(iv)



$\frac{TA}{TF} = \frac{TE}{TA}$ (corresponding angles of similar triangles are in proportion)

(2)

∴ $TA^2 = TE \cdot TF$

∴ $TB^2 = TE \cdot TF$ (since TA = TB)