2008 Yr 11 Ext 1 Half Yearly

Total marks – 83
Attempt Questions 1 – 6
Questions are NOT of equal value
Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your name.

Marks

Question 1 ***(14 marks)*** Use a *separate* piece of paper

a) Solve the following inequalities;

(i) $x^2 + 13x \geq 90$ — 2

(ii) $\dfrac{2}{x-3} \geq 4$ — 3

(ii) $\dfrac{1}{x} < \dfrac{1}{x+1}$ — 3

b) Solve the equation $|x-3| = 2x+1$ — 3

c) If $x = \dfrac{7-2\sqrt{2}}{7+2\sqrt{2}}$, find the value of $x + \dfrac{1}{x}$ — 3

Question 2 ***(13 marks)*** Use a *separate* piece of paper

a) (i) Sketch the graphs $y = 3x$ and $y = |2x-5|$ on the same number plane noting any intercepts with the axes and points of intersection. — 2

(ii) Hence solve $3x - |2x-5| \geq 0$ — 1

b) In your own words describe what is meant by "function" when referring to number plane graphs. — 2

c) (i) Sketch the function $y = |x+1| - |x-1|$ — 2

(ii) Determine whether the function is odd, even or neither, giving reasons for your answer. — 2

d) Given that $f(x) = \begin{cases} x^2 & , x > 2 \\ \dfrac{1}{x+1} & , -1 < x \leq 2 \\ 5 & , x \leq -1 \end{cases}$, find;

(i) $f(3) + f(-1)$ — 2

(ii) the domain of $f(x)$ — 1

(iii) the range of $f(x)$ — 1

Question 3 *(14 marks)* Use a *separate* piece of paper — *Marks*

a) If $\tan\theta = \dfrac{5}{12}$ and $\sin\theta < 0$, find the exact value of $\sec\theta$ — 2

b) Solve for θ, correct to the nearest degree where necessary, where $0^\circ \le \theta \le 360^\circ$

(i) $\cos^2\theta = \cos\theta$ — 3

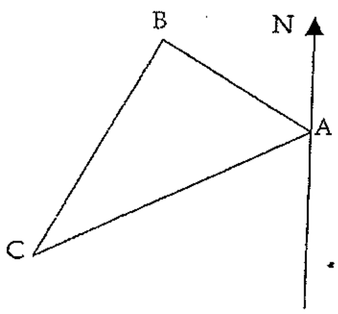
(ii) $\sqrt{3}\sin\theta - 3\cos\theta = 0$ — 3

(iii) $\sin 3\theta = \dfrac{1}{2}$ — 3

(iv) $4\tan\theta + 1 - \tan^2\theta = \sec^2\theta$ — 3

Question 4 *(13 marks)* Use a *separate* piece of paper

a) Factorise $(a^2 - b^2)^2 - (a-b)^4$ completely — 3



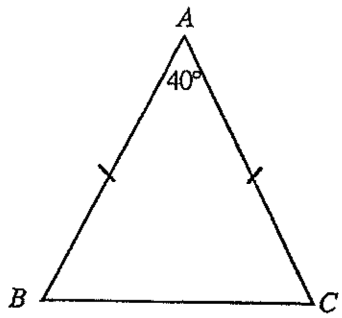
b) Boat B is 30 nautical miles from harbour A. Another boat C is 37 nautical miles from A and is sailing on a bearing of 245°. The distance between the boats is 34 nautical miles.

(i) Copy the diagram and indicate the given information — 1

(ii) Find the size of $\angle BCA$ — 2

(iii) Determine the bearing of boat B from boat C, correct to the nearest degree — 2

c) The vertical angle of an isosceles triangle is 40° and its area is $40\,\text{cm}^2$.



(i) Show that $AB = \sqrt{80\operatorname{cosec}40^\circ}$ — 2

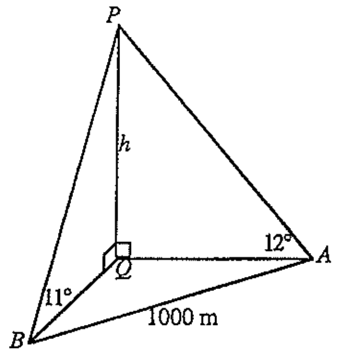
(ii) Hence, or otherwise, calculate the length of BC, correct to one decimal place — 3

Question 5 *(14 marks)* Use a *separate* piece of paper *Marks*

a) (i) On separate diagrams sketch the regions $y > \frac{1}{x}$ and $xy > 1$ *3*

(ii) Are both regions in (i) the same? In your own words, explain why this happens. *1*

b) The angle of elevation of a tower PQ of height h metres, at a point A due East of it, is 12°. From another point B, the bearing of the tower is 051°T and the angle of elevation is 11°. The points A and B are 1000 metres apart and on the same level as the base Q of the tower.



(i) Show that $\angle AQB = 141^\circ$ *2*

(ii) Consider ΔAPQ, show that $AQ = h\tan 78^\circ$ *2*

(iii) Find a similar expression for BQ *1*

(iv) Hence calculate h, correct to the nearest metre. *3*

c) Prove that $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{1+\tan\theta}{\sec\theta}$ is independent of θ *2*

GO TO THE NEXT PAGE FOR QUESTION 6

Question 6 *(15 marks)* Use a *separate* piece of paper *Marks*

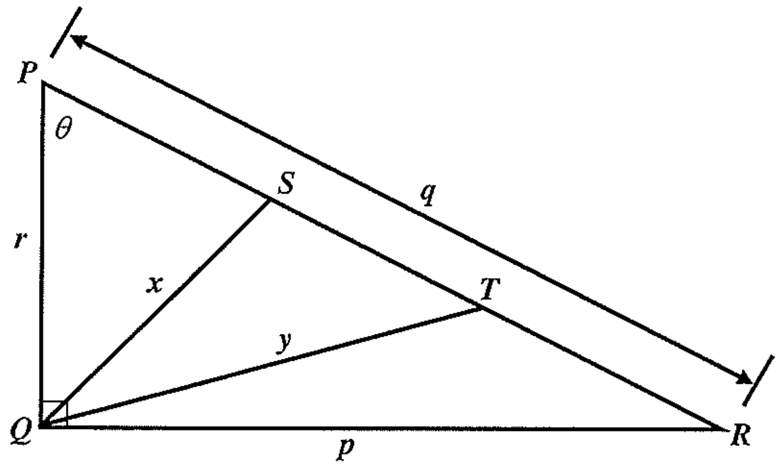
a) Find all the values of x and y, if $\frac{1}{x}+\frac{3}{y}=3$ and $\frac{1}{x}+y=1$ 3

b) If θ is acute and $\sin\theta=\frac{1}{\sqrt{3}}$

(i) Show that $\frac{\tan\theta}{1-\sec\theta}=-\sqrt{2}-\sqrt{3}$ 3

(ii) Find the value of this fraction when θ is obtuse. 2

c)



The right angled triangle PQR has its hypotenuse PR trisected at the points S and T, that is $PS = ST = TR$.

(i) Show that $\cos\theta=\frac{9r^2+q^2-9x^2}{6qr}$ 2

(ii) Show that $\sin\theta=\frac{9p^2+q^2-9y^2}{6pq}$ 2

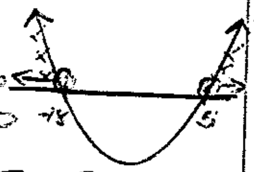
(iii) Hence, or otherwise, deduce that $5q^2=9\left(x^2+y^2\right)$ 3

Question 1 (14)

a) (i) $x^2 + 13x \geq 90$

$x^2 + 13x - 90 \geq 0$

$(x+18)(x-5) \geq 0$



$\underline{x \leq -18 \text{ or } x \geq 5}$ (2)

(ii) $\dfrac{2}{x-3} \geq 4$

$x - 3 \neq 0$

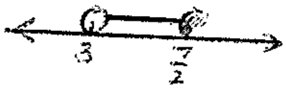
$x \neq 3$

$\dfrac{2}{x-3} = 4$

$2 = 4x - 12$

$4x = 14$

$x = \dfrac{7}{2}$



$\therefore\ \underline{3 < x \leq \frac{7}{2}}$ (3)

(iii) $\dfrac{1}{x} < \dfrac{1}{x+1}$

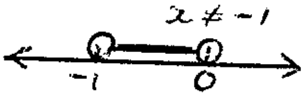
$x \neq 0,\ x+1 \neq 0$

$x \neq -1$

$\dfrac{1}{x} = \dfrac{1}{x+1}$

$x + 1 = x$

no solutions



$\therefore\ \underline{-1 < x < 0}$ (3)

b) $|x-3| = 2x+1$

$x - 3 = 2x + 1$ OR $-(x-3) = 2x+1$

$x = -4$ — not a solution

$-x + 3 = 2x + 1$

$3x = 2$

$x = \frac{2}{3}$

$\therefore\ \underline{x = \frac{2}{3}}$ (3)

c) $x + \dfrac{1}{x}$

$= \dfrac{7-2\sqrt{2}}{7+2\sqrt{2}} + \dfrac{7+2\sqrt{2}}{7-2\sqrt{2}}$

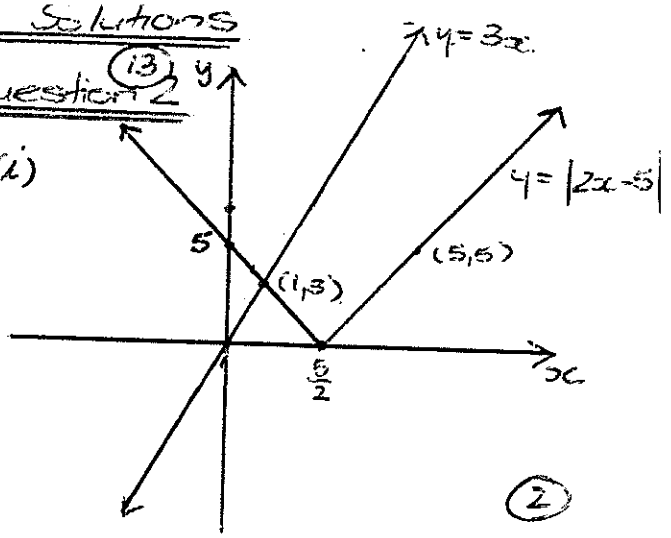
$= \dfrac{(7-2\sqrt{2})^2 + (7+2\sqrt{2})^2}{(7+2\sqrt{2})(7-2\sqrt{2})}$

$= \dfrac{2(7^2 + (2\sqrt{2})^2)}{7^2 - (2\sqrt{2})^2}$

$= \underline{\dfrac{114}{41}}$ (3)

Question 2 (13)

a) (i)



(2)

(ii) $3x - |2x-5| \geq 0$

$3x \geq |2x-5|$

$\underline{x \geq 1}$ (1)

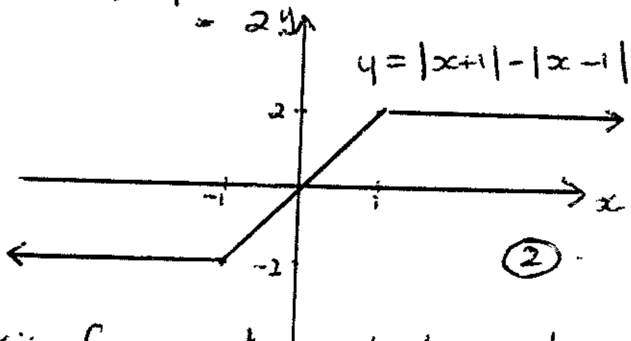
b) A function is a relation that has a maximum of one y value for every x value in its domain. (2)

c) $y = |x+1| - |x-1|$

$x < -1,\ y = -1 - x - (1-x)$
$= -2$

$-1 \leq x \leq 1,\ y = x + 1 - (1-x)$
$= 2x$

$x > 1,\ y = x + 1 - (x-1)$
$= 2$

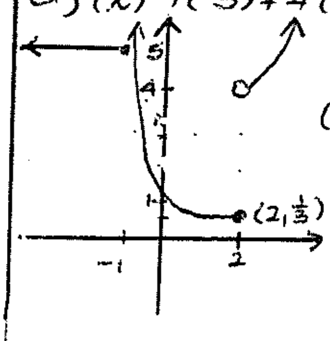


(2)

(ii) $f(-x) = |-x+1| - |-x-1|$
$= |-1||x-1| - |-1||x+1|$
$= |x-1| - |x+1|$
$= -f(x)$

$\therefore$ function is odd (2)

d) (i) $f(3) + f(-1) = 3^2 + 5$
$= \underline{14}$ (2)

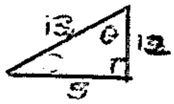


(ii) domain: all real x (1)

(iii) range: $y \geq \frac{1}{3}$ (1)

Question 3 (14)

a) $\tan\theta = \frac{5}{12}$, $\sin\theta < 0$ $\therefore$ Q3



$\underline{\sec\theta = -\frac{13}{12}}$ (2)

b)(i) $\cos^2\theta = \cos\theta$

$\cos^2\theta - \cos\theta = 0$

$\cos\theta(\cos\theta - 1) = 0$

$\cos\theta = 0$ or $\cos\theta = 1$

$\theta = 90^\circ, 270^\circ$ $\quad \theta = 0^\circ, 360^\circ$

$\underline{\theta = 0^\circ, 90^\circ, 270^\circ, 360^\circ}$ (3)

(ii) $\sqrt{3}\sin\theta - 3\cos\theta = 0$

$\sqrt{3}\sin\theta = 3\cos\theta$

$\tan\theta = \sqrt{3}$

Q1, 3

$\tan\alpha = \sqrt{3}$

$\alpha = 60^\circ$

$\therefore \theta = \alpha, 180+\alpha$

$\underline{\theta = 60^\circ, 240^\circ}$ (3)

(iii) $\sin 3\theta = \frac{1}{2}$

Q1, 2

$\sin\alpha = 30^\circ$

$3\theta = \alpha, 180-\alpha$ $\quad 0^\circ \le 3\theta \le 1080^\circ$

$3\theta = 30^\circ, 150^\circ, 390^\circ, 510^\circ, 750^\circ, 870^\circ$

(3)

$\underline{\theta = 10^\circ, 50^\circ, 130^\circ, 170^\circ, 250^\circ, 290^\circ}$

(iv) $4\tan\theta + 1 - \tan^2\theta = \sec^2\theta$

$4\tan\theta + 1 - \tan^2\theta = 1 + \tan^2\theta$

$2\tan^2\theta - 4\tan\theta = 0$

$2\tan\theta(\tan\theta - 2) = 0$

$\tan\theta = 0$ or $\tan\theta = 2$

$\theta = 0^\circ, 180^\circ, 360^\circ$

Q1, 3

$\tan\alpha = 2$

$\alpha = 63^\circ$

$\theta = \alpha, 180+\alpha$

$\theta = 63^\circ, 243^\circ$

$\underline{\theta = 0^\circ, 63^\circ, 180^\circ, 243^\circ, 360^\circ}$ (3)

Question 4

a) $(a^2-b^2)^2 - (a-b)^4$

$= (a+b)^2(a-b)^2 - (a-b)^4$

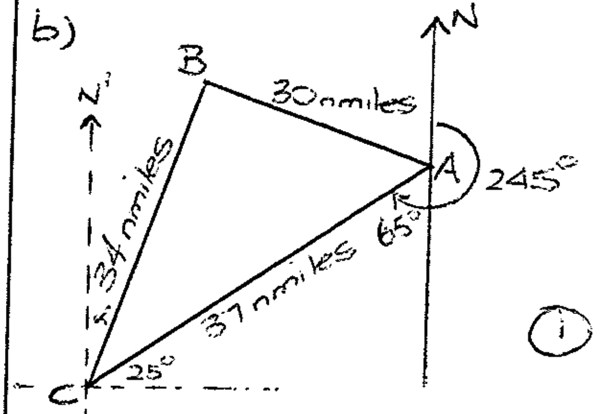
$= (a-b)^2[(a+b)^2 - (a-b)^2]$

$= (a-b)^2(a+b+a-b)(a+b-a+b)$

$= (a-b)^2(2a)(2b)$

$= \underline{4ab(a-b)^2}$ (3)

b)



(i)

(ii) $\cos C = \dfrac{34^2 + 37^2 - 30^2}{2 \times 34 \times 37}$

$\underline{C = 50^\circ}$ (2)

(iii) $\alpha + 30^\circ + 25^\circ = 90^\circ$

$\alpha = 35^\circ$

$\therefore$ $\underline{\text{bearing is } 035^\circ}$ (2)

c) $\frac{1}{2} \cdot AB \cdot AC \sin 40^\circ = 40$

$\frac{1}{2} AB^2 = \dfrac{40}{\sin 40^\circ}$

$AB^2 = 80\,\text{cosec}\,40^\circ$

$\underline{AB = \sqrt{80\,\text{cosec}\,40^\circ}}$ (2)

(ii)

$\dfrac{BC}{\sin 40^\circ} = \dfrac{AB}{\sin 70^\circ}$

$BC = \dfrac{AB \sin 40^\circ}{\sin 70^\circ}$

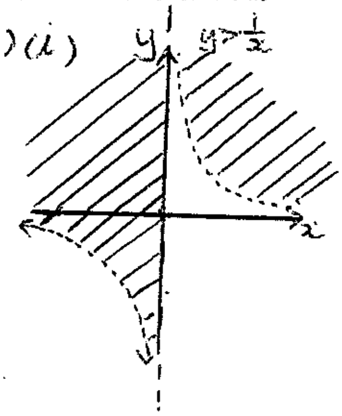
$= \dfrac{\sqrt{80\,\text{cosec}\,40^\circ} \cdot \sin 40^\circ}{\sin 70^\circ}$

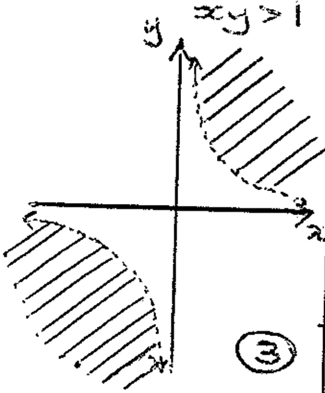
$= \dfrac{\sqrt{80 \sin 40^\circ}}{\sin 70^\circ}$

$= \underline{7.6\text{ cm}}$ (3)

Question 5 (14)

a) (i)

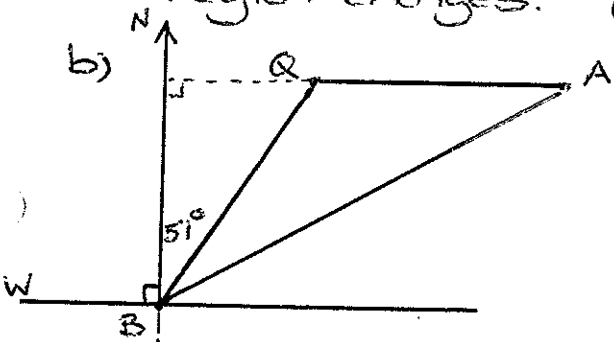




(3)

(ii) The regions are not the same.
They differ as to get $xy > 1$ you must multiply $y > \frac{1}{x}$ by x, but when $x < 0$ the inequality sign would change, thus the region changes. (1)

b)



$\angle WBQ = 141°$

$\therefore \angle AQB = \angle WBQ$ (alternate ∠'s =, AQ ∥ BW)

$\therefore \underline{\angle AQB = 141°}$ (2)

(ii) $\frac{AQ}{h} = \tan 78°$

$\underline{AQ = h\tan 78°}$ (2)

(iii) $\underline{BQ = h\tan 79°}$ (1)

(iv) $1000^2 = h^2\tan^2 78° + h^2\tan^2 79° - 2.h\tan 78.\tan 79.\cos 141°$

$1000^2 = h^2(\tan^2 78° + \tan^2 79° - 2\tan 78° \tan 79° \cos 141°)$

$$h^2 = \frac{1000^2}{\tan^2 78° + \tan^2 79° - 2\tan 78° \tan 79° \cos 141°}$$

$$h = \frac{1000}{\sqrt{\tan^2 78° + \tan^2 79° - 2\tan 78° \tan 79° \cos 141°}}$$

$\underline{h = 108\,m}$ (3)

c) $\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{1+\tan\theta}{\sec\theta}$

$$= \frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{1+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$= \sin\theta + \cos\theta - (\cos\theta + \sin\theta)$

$= \underline{0}$ (2)

Question 6

a) $\frac{1}{x} + \frac{3}{y} = 3$ (−)

$\frac{1}{x} + y = 1$

$\frac{3}{y} - y = 2$

$3 - y^2 = 2y$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$y = -3$ or $y = 1$

$y = 1$, $\frac{1}{x} + 1 = 1$

$\frac{1}{x} = 0$

no solutions

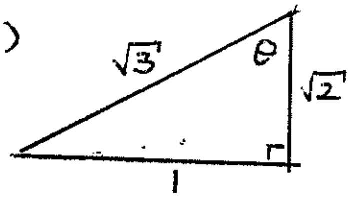
$y = -3$, $\frac{1}{x} - 3 = 1$

$\frac{1}{x} = 4$

$x = \frac{1}{4}$

$\therefore \underline{x = \frac{1}{4},\ y = -3}$ (3)

b)



$\tan\theta = \frac{1}{\sqrt{2}}$

$\sec\theta = \frac{\sqrt{3}}{\sqrt{2}}$

$$\frac{\tan\theta}{1-\sec\theta} = \frac{\frac{1}{\sqrt{2}}}{1-\frac{\sqrt{3}}{\sqrt{2}}}$$

$$= \frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$$

$$= \frac{\sqrt{2}+\sqrt{3}}{-1}$$

$= \underline{-\sqrt{2}-\sqrt{3}}$ (3)

(ii) if θ is obtuse $\tan\theta < 0$, $\sec\theta < 0$

$$\frac{\tan\theta}{1-\sec\theta} = \frac{-1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$= \underline{\sqrt{2}-\sqrt{3}}$ (2)

c) $\cos\theta = \dfrac{\frac{q^2}{9} + r^2 - x^2}{2 \times \frac{q}{3} \times r}$

$= \dfrac{q^2 + 9r^2 - 9x^2}{6qr}$ 2

(ii) In $\triangle QTR$, $\angle R = 90 - \theta$

$\cos(90-\theta) = \dfrac{\frac{q^2}{9} + p^2 - y^2}{2 \times \frac{q}{3} \times p}$

$= \dfrac{q^2 + 9p^2 - 9y^2}{6pq}$

But $\cos(90-\theta) = \sin\theta$

$\therefore \sin\theta = \dfrac{q^2 + 9p^2 - 9y^2}{6pq}$

(iii) $\cos\theta = \dfrac{r}{q}$

$\therefore \dfrac{r}{q} = \dfrac{q^2 + 9r^2 - 9x^2}{6qr}$

$6r^2 = q^2 + 9r^2 - 9x^2$

$9x^2 = q^2 + 3r^2$

$\sin\theta = \dfrac{p}{q}$

$\therefore \dfrac{p}{q} = \dfrac{q^2 + 9p^2 - 9y^2}{6pq}$

$6p^2 = q^2 + 9p^2 - 9y^2$

$9y^2 = q^2 + 3p^2$

$9(x^2 + y^2) = 2q^2 + 3p^2 + 3r^2$

But $p^2 + r^2 = q^2$ (Pythagoras)

$\therefore 9(x^2 + y^2) = 2q^2 + 3q^2$

$= 5q^2$ 3