BAULKHAM HILLS HIGH SCHOOL

HALF YEARLY EXAMINATION

2010

YEAR 11

MATHEMATICS
Extension 1

Time Allowed:

Two hours
(plus 5 mins reading time)

Instructions:

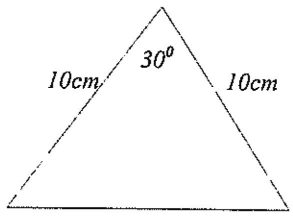
- Write in the answer booklets provided
- Do not write on this question paper
- Show all working
- Use black or blue pens only
- Start a new page for each question
- Write your name and your teacher's name at the top of each page
- Board approved calculators may be used
- Attach the cover sheet to the front and your question paper to the back of your answers

QUESTION 1 **Start on a new page**

		Marks
a)	Factorise $x^3 - 9xy^2 - x^2 + 9y^2$	2
b)	Show whether $f(x) = \frac{x}{x^2+1}$ is odd, even or neither.	2
c)	Show that $\frac{1}{4-\sqrt{3}} + \frac{1}{4+\sqrt{3}}$ is a rational number.	2
d)	Simplify $\frac{2x}{x^2-9} \div \frac{x^2}{x^2-2x-15}$	2
e)	Simplify $4^{2k+1} \div 2^{3k}$	2

QUESTION 2 **Start on a new page**

a) For the given triangle find:



NOT TO SCALE

		Marks
(i)	The area	1
(ii)	The perimeter (1 dec. pl.)	2

b) Solve $\frac{2x}{x+3} \geq 1$ **3**

c) Find the exact value of:

(i) $\tan 210°$ **2**

(ii) $cosec(-225°)$ **2**

QUESTION 3 Start on a new page

Marks

a) Solve for $0° \leq \theta \leq 360°$:

(i) $cos\theta = \frac{1}{\sqrt{2}}$ **2**

(ii) $sin2\theta = 1$ **2**

(iii) $3tan\theta = cot\theta$ **3**

b) Solve simultaneously $2^{2x+y} = 16$ and $2^{3x+4y} = 2$ **3**

QUESTION 4 Start on a new page

a) Solve simultaneously $4x^2 - y^2 = 35$ and $2x + y = 7$ **2**

b) Solve $6 + \frac{2}{x} = x$, expressing your answer in simplest surd form. **2**

c) The length of a rectangle is 6 cm longer than its width. If the dimensions are increased by 2 cm the area is increased by 44 cm^2. Find the dimensions of the original rectangle. **2**

d) Find the exact value of $tan\theta$ if $cos\theta = \frac{\sqrt{2}}{5}$ and $270^0 \leq \theta \leq 360^0$ **2**

e) Factorise fully $(t-2)^3 + (t+2)^3$ **2**

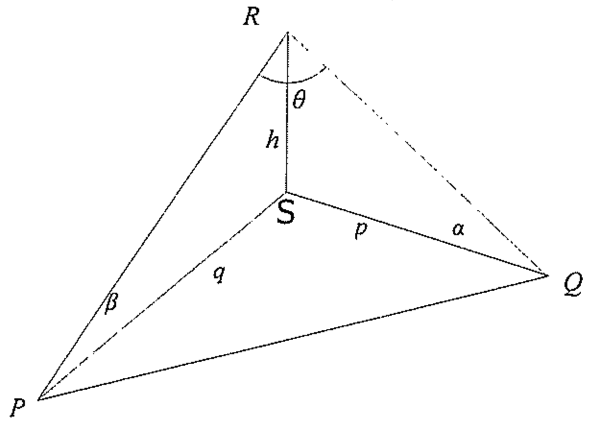
QUESTION 5 Start on a new page

a) (i) On the same set of axes sketch the graphs of $y = |x-1|$ and $y = |x+4|$. **2**

(ii) Use the graphs or otherwise to find the point(s) of intersection. **1**

(iii) Hence or otherwise solve $|x+4| \geq |x-1|$. **1**

b) A triangular pyramid as shown has QS = p, PS = q, RS = h, $\angle PRQ = \theta$, $\angle RQS = \alpha$ and $\angle RPS = \beta$. Also $\angle RSP = \angle PSQ = 90°$ and $\angle RSQ = 90°$



(i) Show that $p = h\cot\alpha$ and find a similar expression for q. **2**

(ii) Show that $\cos\theta = \frac{h^2}{\sqrt{(p^2+h^2)(q^2+h^2)}}$. **2**

(iii) Hence show that $\cos\theta = \sin\alpha\sin\beta$. **2**

QUESTION 6 Start on a new page

a) If $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(i) Find the value(s) of x for which $f(x) = g(x)$ **2**

(ii) Find $f[g(x)]$ in simplest form. **2**

b) State the domain for each of the following:

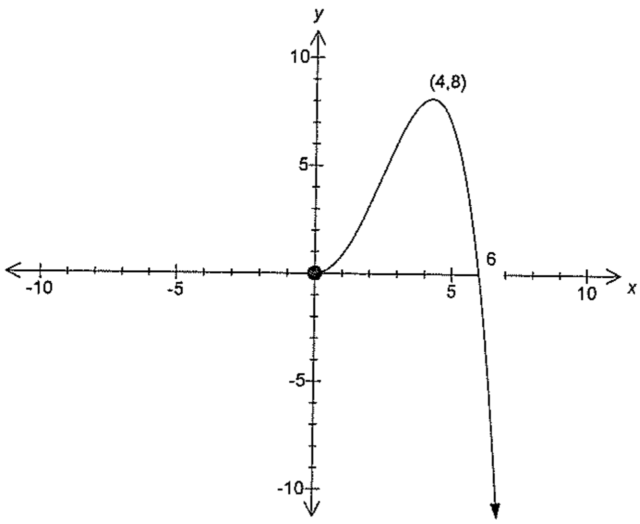
(i) $y = \sqrt{x^2 - 4}$ **2**

(ii) $y = \frac{1}{\sqrt{x+6}} + \frac{1}{|x|-5}$ **2**

c) Sketch the region defined by $y < \sqrt{9 - x^2}$ **2**

QUESTION 7 Start on a new page

a)



Above is a portion of an even function y = f(x).

(i) State all the x intercepts of the function. **1**

(ii) State the range of the function. **1**

(iii) For what values of x is f(x) > 0 **2**

b) Show that $\left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)^2 = \sec^2\theta - 2\tan\theta$ **3**

c) A hiker left camp A and walked 15 km on a bearing of N 32^0 E to B. He then turned and walked for 20 km to a point C due east of the camp. What was the bearing from B to C? **3**

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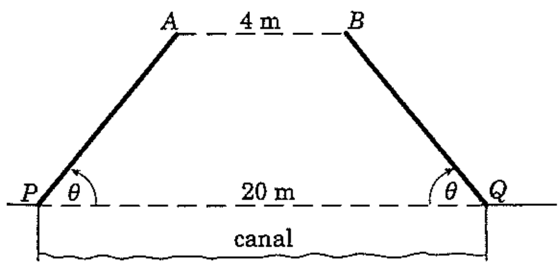
QUESTION 8 Start on a new page

a) Draw a neat sketch of:

(i) $y = \frac{x^2-1}{x-1}$ **2**

(ii) $y = |x^2 - 9|$ **2**

b)



The figure shows the side view of two ramps opened to allow barges to pass along a canal. When the equal arms of the ramps PA and QB are lowered, they meet exactly to form the straight pathway PQ which is 20 metres long. When the ramps PA and QB are raised through an angle θ as shown, the width AB is 4 metres wide. Calculate the size of angle θ. **3**

c) Prove $\frac{cos\theta - tan\theta sin\theta}{cos\theta + tan\theta sin\theta} = 1 - 2sin^2\theta$ **3**

END OF TEST

YEAR 11 HALF-YEARLY EXT 1 2010 – SOLUTIONS

Q1 [10]

(a) $x(x^2-9y^2)-1(x^2-9y^2)$

$= (x^2-9y^2)(x-1)$

$= (x-3y)(x+3y)(x-1)$ (2)

(b) $f(a)=\frac{a}{a^2+1}$

$f(-a)=\frac{-a}{(-a)^2+1}=\frac{-a}{a^2+1}=-f(a)$ – 1

Since $f(-a)=-f(a)$, fn is odd 1 (2)

(c) $\frac{4+4\sqrt3+4-\sqrt3}{(4-\sqrt3)(4+\sqrt3)}=\frac{8}{16-3}=\frac{8}{13}$ (2)

(d) $\frac{2x}{(x-3)(x+3)}\times\frac{(x-5)(x+3)}{x^2}$

$=\frac{2(x-5)}{x(x-3)}=\frac{2x-10}{x^2-3x}$ (2)

(e) $(2^2)^{2k+1}\div 2^{3k}=2^{4k+2}\div 2^{3k}=2^{k+2}$ (2)

Q2 [10]

(a)(i) $A=\frac12 ab\sin C$

$=\frac12\times10\times10\times\sin30^\circ$

$=\frac12\times100\times\frac12$

$=25\text{ cm}^2$ (1)

(ii) $x^2=10^2+10^2-2\times10\times10\times\cos30^\circ$

$=200-200\frac{\sqrt3}{2}$

$=200-100\sqrt3$

$x \doteqdot \sqrt{26.8}\doteqdot 5.2$

$\therefore P\doteqdot 10+10+5.2$

$\doteqdot 25.2\text{ cm}$ (2)

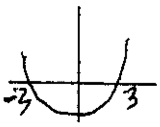
(b) $\frac{2x}{x+3}\times\frac{(x+3)^2}{1}\geq(x+3)^2$

$2x(x+3)\geq(x+3)^2$ – 1

$2x(x+3)-(x+3)^2\geq0$

$(x+3)[2x-(x+3)]\geq0$

$(x+3)(x-3)\geq0$



$\therefore x\leq-3$ or $x\geq3$ – 1

But $x\neq-3$

$\therefore x<-3$ or $x\geq3$ – 1 (3)

(c)(i) $\tan210^\circ=\tan(180^\circ+30^\circ)$

$=\tan30^\circ$

$=\frac{1}{\sqrt3}$ sign – 1 (2)

(ii) $\text{cosec}(-225^\circ)=\text{cosec }135^\circ$

$=\text{cosec }45^\circ$

$=\sqrt2$ (2)

Q3 [10]

(a)(i) $\theta=45^\circ, 315^\circ$ (2)

(ii) $2\theta=90^\circ, 450^\circ$

$\therefore \theta=45^\circ, 225^\circ$ (2)

(iii) $3\tan\theta=\frac{1}{\tan\theta}$

$\tan^2\theta=\frac13$ – 1

$\tan\theta=\pm\frac{1}{\sqrt3}$ / 2

$\theta=30^\circ,150^\circ,210^\circ,330^\circ$ (3)

(b) $2^{2x+y}=2^4$ and $2^{3x+4y}=2^1$ 1

$\therefore 2x+y=4$ ---(1)

$3x+4y=1$ ---(2) – 1

In (1) $y=4-2x$

$\therefore$ In (2) $3x+4(4-2x)=1$

$3x+16-8x=1$

$5x=15$

$x=3$

$\therefore y=4-6\ \therefore y=-2$ (3)

Q4 [10]

(a) $4x^2-y^2=35$ ---(1)

$y=7-2x$ ---(2)

In (1) $4x^2-(7-2x)^2=35$

$4x^2-49+28x-4x^2=35$

$28x=84$ (2)

$x=3, y=1$

(b) $6x+2=x^2$

$x^2-6x-2=0$

$x=\frac{6\pm\sqrt{36+8}}{2}$

$=\frac{6\pm\sqrt{44}}{2}=3\pm\sqrt{11}$ (2)

(c) $A=w(w+6)=w^2+6w$ --(1)

$A+44=(w+2)(w+8)$

or $A=w^2+10w-28$ --(2)

Eq (1) and (2) gives

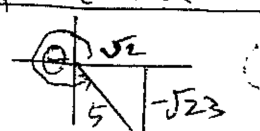
$w^2+10w-28=w^2+6w$

$4w=28$

$w=7$ (2)

$\therefore$ Length = 13cm, width = 7cm

(d) $\tan\theta=-\frac{\sqrt{23}}{\sqrt2}$ (2)

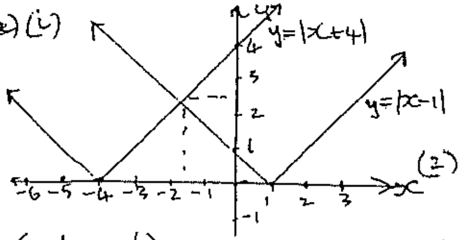


(e) $\text{Exp}=[(t-2)+(t+2)][(t-2)^2-(t-2)(t+2)+(t+2)^2]$

$=2t[t^2-4t+4-t^2+4+t^2+4t+4]$

$=2t(t^2+12)$ (2)

Q5(a)(i) [10]



(2)

(ii) $\left(-1\frac{1}{2}, 2\frac{1}{2}\right)$ (1)

(iii) $x \geq -1\frac{1}{2}$ (1)

(b)(i) In ΔQRS $\frac{p}{h} = \cot\alpha$

$\therefore p = h\cot\alpha$ – 1

In ΔPRS $\frac{q}{h} = \cot B$

$\therefore q = h\cot B$ – 1 (2)

(ii) In ΔPQS where $\angle PSQ = 90^\circ$

$PQ^2 = p^2 + q^2$

$\therefore PQ = \sqrt{p^2+q^2}$

In ΔPQR

$\cos\theta = \frac{PR^2 + RQ^2 - PQ^2}{2 \times PR \times RQ}$

$= \frac{(h^2+q^2)+(h^2+p^2)-(p^2+q^2)}{2\sqrt{h^2+q^2}\cdot\sqrt{p^2+h^2}}$

$= \frac{2h^2}{2\sqrt{p^2+h^2}\sqrt{q^2+h^2}}$

$\therefore \cos\theta = \frac{h^2}{\sqrt{p^2+h^2}\sqrt{q^2+h^2}}$ (2)

(iii) In ΔPRS, $\sin B = \frac{h}{\sqrt{h^2+q^2}}$

In ΔQRS, $\sin\alpha = \frac{h}{\sqrt{h^2+p^2}}$

$\therefore RHS = \sin\alpha\sin B$

$= \frac{h}{\sqrt{h^2+p^2}} \cdot \frac{h}{\sqrt{h^2+q^2}}$

$= \frac{h^2}{\sqrt{h^2+p^2}\cdot\sqrt{h^2+q^2}} = \cos\theta = RHS$

(2)

Q6.(a)(i) [10] If $f(x) = g(x)$

$1 - x^2 = 2x + 1$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = -2$ or 0 (2)

(ii) $f[g(x)] = 1 - (2x+1)^2$

$= 1 - 4x^2 - 4x - 1$

$= -4x^2 - 4x$ (2)

(b)(i) $y = \sqrt{x^2-4}$

since $x^2 - 4 \geq 0$

$x^2 \geq 4$

$\therefore x \leq 2$ or $x \geq 2$ (2)

(ii) $y = \frac{1}{\sqrt{x+6}} + \frac{1}{|x|-5}$

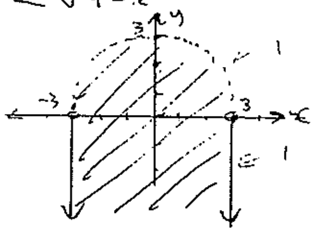
For $\frac{1}{\sqrt{x+6}}$, $x + 6 > 0$

$\therefore x > -6$

For $\frac{1}{|x|-5}$, $x \neq \pm 5$

Combining gives $x > -6, x \neq \pm 5$ (2)

(c) $y < \sqrt{9 - x^2}$



1

1

(2)

Q7.(a)(i) [10] $x = -6, 0, 6$ (1)

(ii) $y \leq 8$ (1)

(iii) $-6 < x < 0$, $0 < x < 6$ (2)

(b) $LHS = \frac{\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta}$

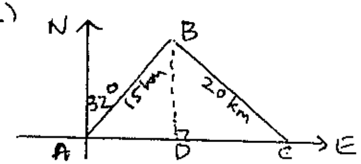
$= \frac{1 - 2\sin\theta\cos\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{2\sin\theta\cos\theta}{\cos^2\theta}$

$= \sec^2\theta - 2\tan\theta$

$= RHS$ (3)

(c)



Draw $BD \perp AE$

In ΔABD, $\angle ABD = 32^\circ$ (Alter. ∠'s, NA ∥ BD)

$\therefore \frac{BD}{15} = \cos 32^\circ$

$BD = 15\cos 32^\circ$

In ΔBDC

$\cos\angle DBC = \frac{15 \times \cos 32^\circ}{20}$

$= 0.6360$

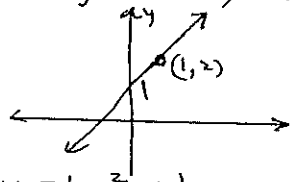
$\therefore \angle DBC = 50^\circ 30'$ – 1

$\therefore$ Bearing from B to C is $180^\circ - 50^\circ 30'$

$= 129^\circ 30'$T – 1 (3)

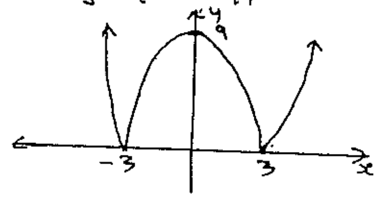
Q8(a)(i) [10] $y = \frac{(x-1)(x+1)}{x-1}$

$\therefore y = x + 1$, $x \neq 1$



(2)

(ii) $y = |x^2 - 9|$



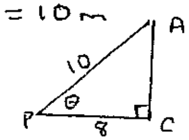
(2)

(b) $PA + BQ = 20$ – 1

$\therefore PA = 10$ m

$PC = \frac{1}{2}(20-4)$

$= 8$ m – 1



$\therefore \cos\theta = \frac{8}{10}$

$\therefore \theta = 36^\circ 52'$ – 1 (3)

c) $LHS = \frac{\cos\theta - \frac{\sin\theta}{\cos\theta} \times \sin\theta}{\cos\theta + \frac{\sin\theta}{\cos\theta} \times \sin\theta}$ – 1

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$ – 1

$= \frac{(1-\sin^2\theta) - \sin^2\theta}{1}$ – 1

$= 1 - 2\sin^2\theta$

$= RHS$ (3)