

Student Name:

BAULKHAM HILLS HIGH SCHOOL

Higher School Certificate

2011

Half-Yearly Examination

Mathematics

Extension 1

General Instructions

- Exam time – 1.5 hours and 5 mins reading time
- Start each question on a new page
- All necessary working should be shown, marks may be deducted for careless or badly arranged work.
- Write your student name at the top of each page of your answer booklet
- Board approved calculators may be used
- Write using black or blue pen

Total Marks: 72

Attempt ALL questions

Question 1 (9 marks)**Marks**

- a) Solve the inequality $x^2 - 5x < -6$ 2
- b) Factorise $x^3 - 4x + 5x^2y - 20y$ 2
- c) If $\cos \theta = \frac{-3}{7}$ and $\sin \theta > 0$, find an exact value for $\cot \theta$ 2
- d) Solve for $-90^\circ \leq x \leq 90^\circ$ to the nearest degree.
 $4 \sin^2 x - 1 = 0$ 3

Question 2 (9 marks) - Start a new page

- a) Express $4.0\dot{1}\dot{5}$ as a rational number showing all working. 2
- b) Factorise $8x^3 - (x - 1)^3$ 2
- c) Solve $\frac{4}{x - 3} \geq 1$ 3
- d) A committee of 3 men and 5 women is to be formed from a group of 7 men and 8 women. Write an expression for the number of ways this can be done. 2

Question 3 (9 marks) - Start a new page

- a) How many nine-letter arrangements can be made using the letters of the word ISOSCELES? 2
- b) Find the number of ways in which 3 girls and 4 boys can be seated in a row so that the girls are next to each other. 2
- c) Find the greatest possible domain of
- (i) $x = 9 - y^2$ 1
- (ii) $y = \frac{2\sqrt{x+3}}{x-7}$ 2
- d) Prove the identity 2
- $$\frac{\sin^3 x}{\cos x} + \sin x \cos x \equiv \tan x$$

Question 4 (9 marks) - Start a new page**Marks**a) Solve for $0^\circ \leq x \leq 360^\circ$ to the nearest degree.

(i) $\sin(x - 10^\circ) = 6 \cos(x - 10^\circ)$

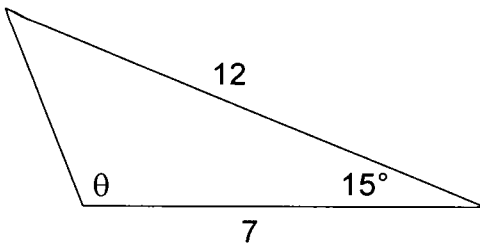
2

(ii) $\sin(540^\circ - x) \cos x + \sin(90^\circ - x) = 0$

3

b) Find θ to the nearest degree.

4

**Question 5 (9 marks) - Start a new page**

a) Solve the simultaneous equations

$$x + y = 2 \text{ and}$$

$$2x^2 + xy - y^2 = 8$$

2

b) Determine if the following functions are odd, even or neither (Show all working)

(i) $f(x) = x^3 - \sin x$

2

(ii) $f(x) = (x^3 - 2x)^{\frac{2}{3}}$

2

c) Solve for x

$\sqrt{4x - 4} + \sqrt[4]{x - 1} = 3$

3

Question 6 (9 marks) - Start a new page

a) Eight people are to be seated at a round table.

(i) How many seating arrangements are possible?

1

(ii) Two people, Shelley and Hong, refuse to sit next to each other.
How many seating arrangements are then possible?

2

b) (i) On the same axes sketch the graphs of $y = |x|$ and $y = 6 - x^2$, clearly labelling an points of intersection.

3

(ii) Hence solve $|x| \geq 6 - x^2$

1

c) The circle $x^2 + y^2 = 64$ and the parabola $y = ax^2 - b$ (where a and b are both positive) meet on both the x and y axes. Find a and b

2

Question 7 (9 marks) - Start a new page

Marks

a) Draw a neat sketch of

$$y = \frac{2x^2 - 50}{x - 5}$$

3

b) A four digit number is to be made from the digits 2, 3, 4, 5, 7 and 9. If no digit may be used more than once in the same number:

(i) how many four digit numbers can be made ?

1

(ii) how many ways can an odd four digit number can be made?

2

(iii) how many four digit numbers less than 3400 be made?

3

Question 8 (9 marks) - Start a new page

a) If $f(x) = 9^x$ and if $f(x + 2) = kf(x)$, find k

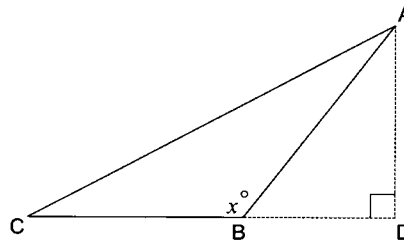
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b)

It is given that
 $3AD + BD = 2AB$

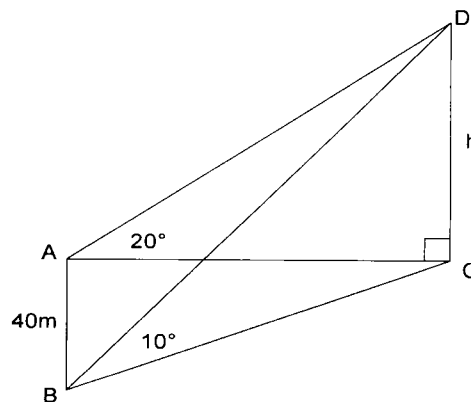
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Show that:
 $3 \sin x - \cos x = 2$



c)

The new Baulkham Hills High School pole, CD of height h metres, near the office, stands with base C on horizontal ground. A is a point on the ground due west of C and B is a point on the ground 40 metres due south of A. From A and B the angles of elevation to the top of the pole are 20° and 10° respectively.



(i) Show that $AC = \cot 20^\circ$ and find a similar expression for BC

2

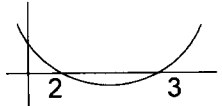
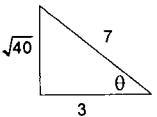
(ii) Show that $h = \frac{40}{\sqrt{\cot^2 10^\circ - \cot^2 20^\circ}}$

2

(iii) Hence find the height of the pole to the nearest metre.

1

Yr 11 Ext 1 HALF YEARLY 2011 SOLUTIONS

| Question 1 (9 marks) | Mk | Comments |
|--|----|---|
| a) $x^2 - 5x + 6 < 0$ $(x - 3)(x - 2) < 0$ $2 < x < 3$  | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Correctly factorised |
| b) $x^2 - 4x + 5x^2y - 20y$ $= [x(x^2 - 4) + 5y(x^2 - 4)]$ $= (x^2 - 4)(x + 5y)$ $= (x + 2)(x - 2)(x + 5y)$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Partially factorises |
| c)  in the 2 nd quadrant $\cot \theta = -\frac{7}{\sqrt{40}}$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Correctly identifies quadrant/sign Finds $\sqrt{40}$ |
| d) $\sin^2 x = \frac{1}{4}$ $\sin x = \pm \frac{1}{2}$ Acute $x = 30^\circ$ $x = \pm 30^\circ$ | 3 | <u>2 Mark</u> <ul style="list-style-type: none"> Correct solution over another domain containing at least 2 answers <u>1 Mark</u> <ul style="list-style-type: none"> Correct soln for $\sin x = \frac{1}{2}$ $\sin x = \pm \frac{1}{2}$ |
| Question 2 (9 marks) - Start a new page | | |
| a) $10x = 40. \dot{1}\dot{5}$ $1000x = 4015. \dot{1}\dot{5}$ $990x = 3975$ $x = \frac{3975}{990} = \frac{265}{66} = 4 \frac{1}{66}$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Uses a correct method Not simplified |
| b) $8x^3 - (x - 1)^3$ $= [2x - (x - 1)][4x^2 + 2x(x - 1) + (x - 1)^2]$ $= [x + 1][7x^2 - 4x + 1]$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Correctly factorises $a^3 - b^3$ Simplifies the factorisation Finds one correct factor |
| c) $\frac{4}{x - 3} \geq 1 \quad x \neq 3$ $4(x - 3) \geq (x - 3)^2$ $0 \geq (x - 3)^2 - 4(x - 3)$ $(x - 3)(x - 3 - 4) \leq 0$ $(x - 3)(x - 7) \leq 0 \text{ but } x \neq 3$ $3 < x \leq 7$ | 3 | <u>2 Marks</u> <ul style="list-style-type: none"> Bald answer Identifies 2 critical points via a correct method Correct conclusion to their critical points obtained using a correct method <u>1 Mark</u> <ul style="list-style-type: none"> Uses a correct method Acknowledges $x \neq 3$ <u>0 Mark</u> <ul style="list-style-type: none"> Solves like an eqn with no consideration of the denominator |
| d) ${}^7C_3 \times {}^8C_5 = 1960$ | 2 | Unsimplified answer is ok <u>1 Mark</u> <ul style="list-style-type: none"> 7C_3 or 8C_5 involved in answer Bald answer |

| | | | |
|--|--|------------|---|
| Question 3 (9 marks) - Start a new page | | | |
| a) | $\frac{9!}{3!2!} = 30240$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Correctly handles a multiple letter. |
| b) | $5! \times 3! = 720$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Correctly arranges all the people (eg.5!) |
| c) | (i) $x = 9 - y^2$ <u>Domain</u> : $x \leq 9$ Find the greatest possible domain of (ii) $y = \frac{2\sqrt{x+3}}{x-7}$ $x \neq 7$ $x + 3 \geq 0$ <u>Domain</u> : $x \geq -3$ <i>except</i> $x \neq 7$ | 1 2 | (i) (ii) <u>1 Mark</u> <ul style="list-style-type: none"> $x \neq 7$ $x \geq -3$ |
| d) | $LHS = \frac{\sin^3 x}{\cos x} + \frac{\sin x \cos^2 x}{\cos x}$ $= \frac{\sin x (\sin^2 x + \cos^2 x)}{\cos x}$ $= \frac{\sin x}{\cos x}$ $= \tan x$ $= RHS$ | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Uses $\sim \sin^2 x + \cos^2 x = 1$ $\sim \frac{\sin x}{\cos x} = \tan x$ to simplify the expression Expressed as a common factor |
| Question 4 (9 marks) - Start a new page | | | |
| a) | (i) $\sin(x - 10^\circ) = 6 \cos(x - 10^\circ)$ for $0^\circ \leq x \leq 360^\circ$ $\tan(x - 10^\circ) = 6$ Acute $(x - 10^\circ) = 80^\circ 32' = 81^\circ$ $x - 10^\circ = 81^\circ$ or 261° <u>$x = 91^\circ$ or 271°</u> (ii) $\sin(540^\circ - x) \cos x + \sin(90^\circ - x) = 0$ for $0^\circ \leq x \leq 360^\circ$ $\sin(180^\circ - x) \cos x + \cos x = 0$ $\cos x (\sin x + 1) = 0$ $\cos x = 0$ or $\sin x = -1$ $x = 90^\circ, 270^\circ$ | 2 3 | (i) <u>1 Mark</u> <ul style="list-style-type: none"> $\tan(x - 10^\circ) = 6$ Correctly finds one soln Finds the acute angle (ii) <u>2 Mark</u> <ul style="list-style-type: none"> Factorises to $\cos x (\sin x + 1) = 0$ Solves the eqn using a correct method <u>1 Mark</u> <ul style="list-style-type: none"> Transforms $\sin(540^\circ - x)$ or $\sin(90^\circ - x)$ Bald answer |
| b) | $c^2 = 12^2 + 7^2 - 2 \times 7 \times 12 \times \cos 15^\circ$ $c = 5.54 \dots$ $\frac{\sin \theta}{12} = \frac{\sin 15^\circ}{5.54}$ $\sin \theta = 0.56$ $\theta = 34^\circ$ or 146° (nearest degree) But θ is opposite the longest side $\therefore \theta \neq 34^\circ$ <i>ie.</i> $\theta = 146^\circ$ | 4 | <u>3 Mark</u> <ul style="list-style-type: none"> $\theta = 34^\circ$ <u>2 Mark</u> <ul style="list-style-type: none"> Uses the correct method to find θ, but gets the wrong acute angle <u>1 Mark</u> <ul style="list-style-type: none"> Finds the third side Considers θ is obtuse Uses Sine Rule correctly |

| Question 5 (9 marks) - Start a new page | | |
|---|---|---|
| <p>a) $x + y = 2$ -----(1) $(x + y)(2x - y) = 8$ -----(2)</p> <p>Sub (1) \rightarrow (2) $2x - y = 4$ -----(3) (3) + (1) $3x = 6$ <u>$x = 2$</u></p> <p>Sub $x \rightarrow$ (1) $y = 0$ <u>$\therefore x = 2, y = 0$</u></p> | 2 | <u>1 Mark</u> <ul style="list-style-type: none"> Finds x or y Substitutes to find the second value correctly |
| <p>b) (i) $f(x) = x^3 - \sin x$ $f(-x) = (-x)^3 - \sin(-x)$ $= -x^3 + \sin x$ $= -(x^3 - \sin x)$ $= -f(x)$ $\therefore f(x)$ is odd</p> <p>(ii) $f(x) = (x^3 - 2x)^{\frac{2}{3}} = \sqrt[3]{(x^3 - 2x)^2}$ $f(-x) = \sqrt[3]{[(-x)^3 - 2(-x)]^2}$ $= \sqrt[3]{(-x^3 + 2x)^2}$ $= \sqrt[3]{(x^3 - 2x)^2}$ $= f(x)$ $\therefore f(x)$ is even</p> | 2 | <p>(i) <u>1 Mark</u></p> <ul style="list-style-type: none"> Attempts to find $f(-x)$ <p>(ii) <u>1 Mark</u> Attempts to find $f(-x)$</p> |
| <p>c) $\sqrt{4x - 4} + \sqrt[4]{x - 1} = 3$ $2\sqrt{x - 1} + \sqrt[4]{x - 1} = 3$ $2(x - 1)^{\frac{1}{2}} + (x - 1)^{\frac{1}{4}} - 3 = 0$</p> <p>Let $(x - 1)^{\frac{1}{4}} = a$ $2a^2 + a - 3 = 0$ $(2a + 3)(a - 1) = 0$ $a = -\frac{3}{2}$ or $a = 1$</p> <p>$\sqrt{x - 1} = -\frac{3}{2}$ or $\sqrt{x - 1} = 1$ No soln or $x - 1 = 1$</p> <p><u>$\therefore x = 2$</u></p> | 3 | <u>2 Marks</u> <ul style="list-style-type: none"> Reduces it to a quadratic <u>1 Mark</u> <ul style="list-style-type: none"> Simplifies $\sqrt{4x - 4}$ Bald answer <u>0 marks</u> <ul style="list-style-type: none"> Simplified question to $(4x - 4)^2 + (x - 1) = 3$ [or equivalent] |

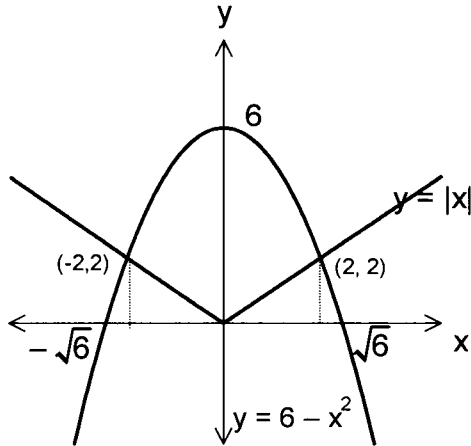
Question 6 (9 marks) - Start a new page

a) (i) $7! = 5040$

(ii) sit together = $6! \times 2 = 1440$
 Do not sit together = $7! - 6! \times 2 = 3600$

1 (i) Unsimplified answer is ok
 (ii) 1 Mark
 • Correctly subtracts for total
 • Finds *sit together* = $6! \times 2$

b) (i)



Points of intersection

When $x > 0$

$$x = 6 - x^2$$

$$x^2 + x - 6 = 0$$

$$(x - 2)(x + 3) = 0$$

$$x = 2 \text{ or } x = -3 \text{ (not in domain)}$$

When $x < 0$

by symmetry $x = -2 \text{ or } x = 3 \text{ (not in domain)}$

\therefore points of intersection are $(2, 2)$ and $(-2, 2)$

(ii) $\therefore x < -2 \text{ or } x > 2$

3 (i) 2 Marks
 • 2 correct graphs and important info
 • 1 correct graph with pts of intersection
1 Mark
 • One correct graph
 • Finds and shows the points of intersection.

1

(ii)

c) Parabola $y = a(x - 8)(x + 8)$
 $y = ax^2 - 64a$ ----(1)

y - int when $x = 0$

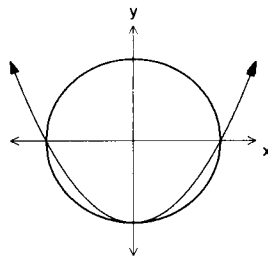
$$-64a = -8$$

$$a = \frac{1}{8}$$

Sub $a \rightarrow$ (1)

$$\therefore y = \frac{1}{8}x^2 - 8$$

$$\therefore a = \frac{1}{8}, b = 8$$



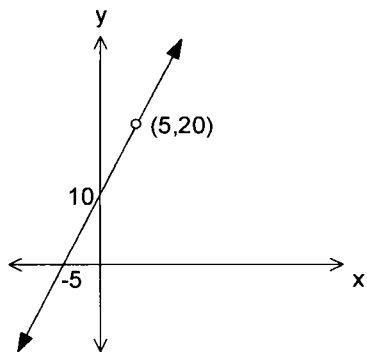
2 1 Mark
 • Finds a or b

Question 7 (9 marks) - Start a new page

a)
$$y = \frac{2x^2 - 50}{x - 5}$$

$$y = \frac{2(x + 5)(x - 5)}{x - 5}$$

$$\therefore y = 2(x + 5) \text{ where } x \neq 5$$
 It is discontinuous at (5, 20)



- 3** 2 Marks
- Draws $y = x + 5$
 - Establishes function is equivalent to $y = x + 5$
- 1 Mark
- Identifies $x = 5$ is discontinuous

- b) (i) $6 \times 5 \times 4 \times 3 = 360$
- (ii) $4 \times 5 \times 4 \times 3 = 240$
- (iii) Less than 3000 = $1 \times 5 \times 4 \times 3 = 60$
 Between 3000 and 3400 = $1 \times 1 \times 4 \times 3 = 12$
 Total = 72

- 1** (i) unsimplified is ok
- 2** (ii) 1 Mark
- Considers last digit is odd
- 3** (iii) 2 Mark
- Considers both cases (starting with 32__ and 2__)
- 1 Mark
- Considers one of the above cases

Question 8 (9 marks) - Start a new page

a) $f(x) = 9^x$
 $f(x+2) = 9^{x+2}$
 $f(x+2) = kf(x)$
 $9^{x+2} = k \times 9^x$
 $k = \frac{9^{x+2}}{9^x}$
 $k = 9^2$
 $k = 81$

2

1 Mark

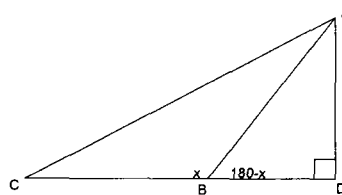
- Correctly finds $f(x+2)$
- Correctly finds k from their $f(x+2)$

b) $\angle ABD = 180^\circ - x$ (straight line)
 $\sin(180^\circ - x) = \frac{AD}{AB}$
 $\sin x = \frac{AD}{AB}$
 $\cos(180^\circ - x) = \frac{BD}{AB}$
 $-\cos x = \frac{BD}{AB}$
 $\cos x = -\frac{BD}{AB}$

given that $3AD + BD = 2AB$ ----- (1)

Show that: $3 \sin x - \cos x = 2$

$LHS = 3 \frac{AD}{AB} - \frac{BD}{AB}$
 $= \frac{3AD + BD}{AB}$
 $= \frac{2AB}{AB}$ ~ from (1)
 $= 2$
 $= RHS$



2

1 Mark

- Identifies $\sin(180^\circ - x) = \frac{AD}{AB}$ and $\cos(180^\circ - x) = \frac{BD}{AB}$
- Substitutes $\sin(180 - x) = \sin x$ or $\cos(180 - x) = -\cos x$

c) i) $\tan 20^\circ = \frac{h}{AC}$
 $AC = h \cot 10^\circ$

$\tan 10^\circ = \frac{h}{BC}$
 $BC = h \cot 10^\circ$

2

(i) 1 Mark

- Shows AC
- Writes a correct expression for BC

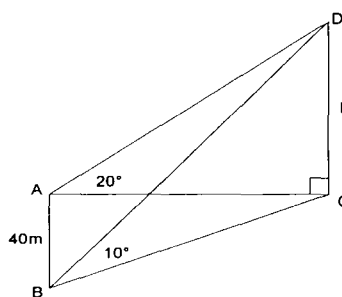
(ii) Show that $h = \frac{40}{\sqrt{\cot^2 10^\circ - \cot^2 20^\circ}}$

$40^2 = BC^2 - AC^2$ (by Pythag)
 $40^2 = h^2 \cot^2 20^\circ - h^2 \cot^2 10^\circ$
 $40^2 = h^2(\cot^2 20^\circ - \cot^2 10^\circ)$

$h^2 = \frac{40^2}{\cot^2 20^\circ - \cot^2 10^\circ}$

$h = \frac{40}{\sqrt{\cot^2 20^\circ - \cot^2 10^\circ}}$

(iii) $h = 6.347 \dots$
 $h = 6.35m$



2

(ii) 1 Mark

- Uses Pythagoras in an attempt to show h or equivalent

1

(iii)