BAULKHAM HILLS HIGH SCHOOL

2012

YEAR 11 HALF YEARLY EXAMINATIONS

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour 30 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 6 – 11
- Marks may be deducted for careless or badly arranged work

Total marks – 57

Section I Pages 2 – 3

5 marks

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

Section II Pages 4 – 6

52 marks

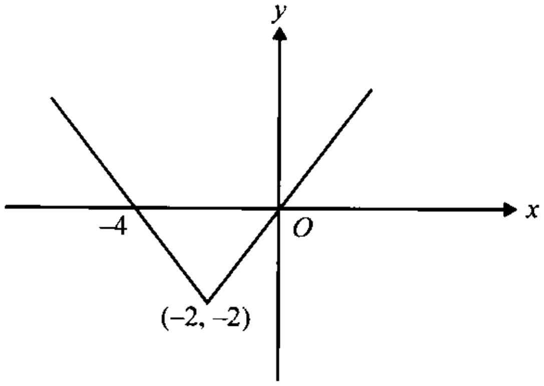
- Attempt Questions 6 – 11
- Allow about 1 hour 20 minutes for this section

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5

1



What is the equation of the function whose graph is shown above?

(A) $y = |x - 2| + 2$

(B) $y = |x + 2| - 2$

(C) $y = |2 - x| - 2$

(D) $y = |2 + x| + 2$

2 $2x^2 + 4x + 7$ is expressed in the form $2(x + p)^2 + q$.

What is the value of q?

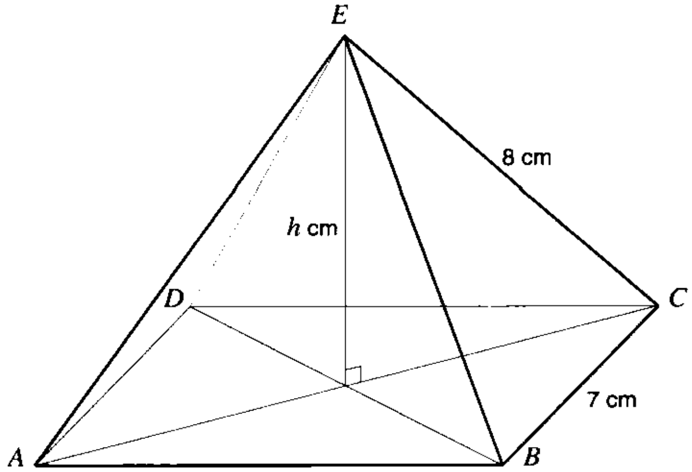
(A) 11

(B) 9

(C) 7

(D) 5

3 For the square-based pyramid, *ABCDE*, shown below, the sides of the base are 7 cm and the slant edges are 8 cm in length.



The vertical height, h cm, of this pyramid is closest to;

(A) 10.6 cm

(B) 7.2 cm

(C) 6.3 cm

(D) 3.9 cm

4 Determine the number of solutions for $(a\sin x + a)(b\cos x - c) = 0$ for $0 \le x \le 360$, if $0 < a < b < c$.

(A) 1

(B) 2

(C) 3

(D) 4

5 The function f satisfies the functional equation $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ where x and y are non-zero real numbers.

A possible rule for the function is;

(A) $f(x) = \frac{1}{x}$

(B) $f(x) = 2^x$

(C) $f(x) = 2x$

(D) $f(x) = \sin 2x$

END OF SECTION I

Section II

52 marks
Attempt Questions 6 – 11
Allow about 1 hour 20 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Marks

<u>Question 6</u> ***(6 marks)*** Use a *separate* answer sheet

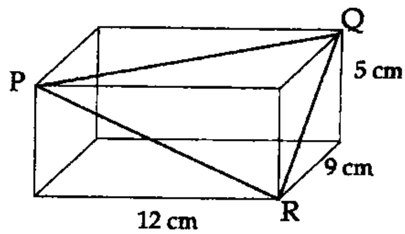
Solve the following inequalities;

(i) $\dfrac{5}{x-1} \geq 3$ *3*

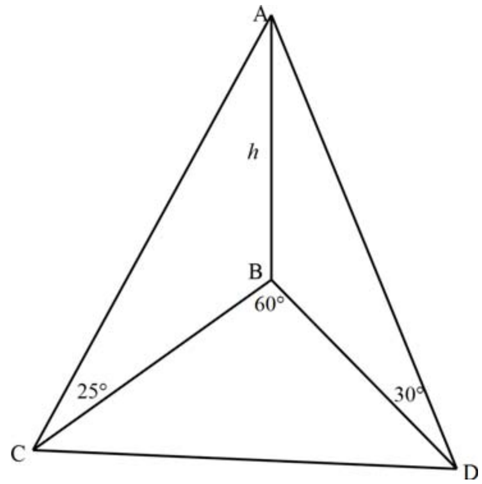
(ii) $x^3 \geq 6x - x^2$ *3*

<u>Question 7</u> ***(9 marks)*** Use a *separate* answer sheet

a) For this rectangular prism, find the size of $\angle PQR$, correct to the nearest degree. *3*



b) A vertical tower AB stands on level ground with A being the top. Points C and D are on the same level ground as point B. The angle of elevation of A from C is 25° and the angle of elevation of A from D is 30° and $\angle CBD = 60^\circ$. Let the height of the tower be *h*.



(i) Show that $BC = h\tan 65^\circ$ *1*

(ii) Show that $CD^2 = h^2\tan^2 65^\circ + h^2\tan^2 60^\circ - 2h^2\tan 65^\circ\tan 60^\circ\cos 60^\circ$ *3*

(iii) If the distance CD = 50 metres, find the height, *h*, correct to 1 decimal place. *2*

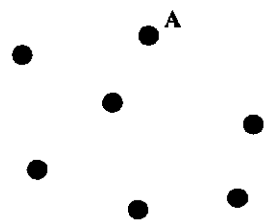
Marks

<u>Question 8</u> ***(8 marks)*** Use a *separate* answer sheet

a) Factorise $(a+b)^4-(a-b)^4$ completely. *3*

b) Suppose that $x+\frac{1}{x}=3$, evaluate $x^2+\frac{1}{x^2}$, without finding x. *2*

c) There are seven points on a plane so no three points lie on the same straight line.



(i) How many triangles can be formed using these points? *1*

(ii) How many quadrilaterals can be formed if the quadrilateral must contain the point A? *2*

<u>Question 9</u> ***(13 marks)*** Use a *separate* answer sheet

a) Solve for θ, correct to the nearest degree where necessary, where $0^\circ \le \theta \le 360^\circ$

(i) $\cos^2\left(\theta - 60^\circ\right) = \frac{1}{2}$ *3*

(ii) $4\sin\theta\cos\theta + 1 = 2\left(\sin\theta + \cos\theta\right)$ *3*

b) In a conference room there is a round table surrounded by ten equally spaced chairs. The group attending the conference consists of seven women and three men.

(i) How many seating arrangements are possible if there are no restrictions? *1*

(ii) How many seating arrangements are possible if the three men sit together? *2*

Two of the ten people were elected to attend the conference. What is the probability that they;

(iii) sit next to each other? *2*

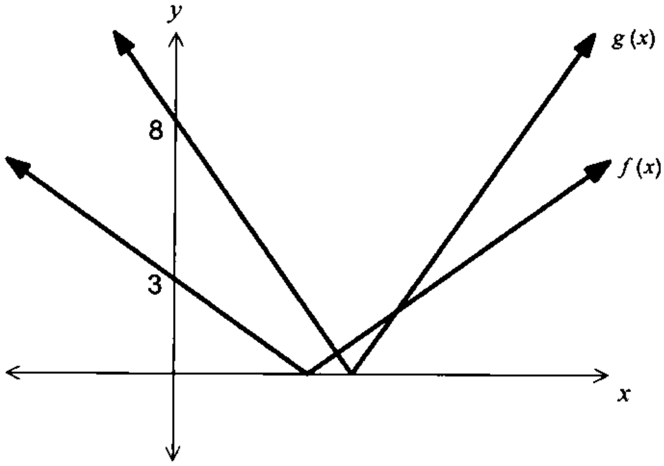
(iv) sit opposite each other? *2*

Marks

<u>Question 10</u> ***(7 marks)*** Use a *separate* answer sheet

a) Out of thirty consecutive integers, in how many ways can three numbers be selected whose sum is odd? *2*

b) The two absolute value functions $f(x) = |ax - b|$ and $g(x) = |cx - d|$ are drawn on the graph below, where a, b, c and d are positive.



(i) From the graph deduce the values of b and d. *1*

(ii) The points of intersection are $x = \frac{11}{3}$ and $x = 5$. *3*

Deterrmine the values of a and c.

(iii) Hence, or otherwise, solve $|ax - b| < |cx - d|$. *1*

<u>Question 11</u> ***(9 marks)*** Use a *separate* answer sheet

a) Let $g(x) = \sqrt{x - 1}$ and $h(x) = 10 - x^2$. Find the domain of $y = g(h(x))$ *2*

b) The letters of the name **NAGESWARAN** are arranged to form new words. How many arrangements are possible if;

(i) all ten letters are used? *2*

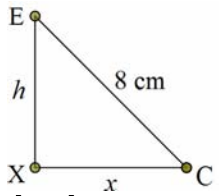
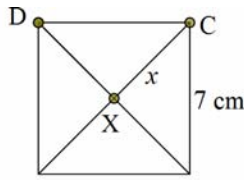
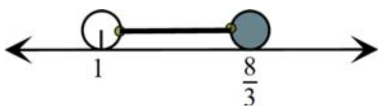
(ii) only nine letters are used? *2*

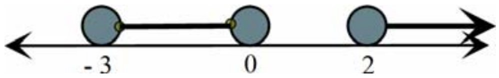
c) Solve the following simultaneous equations; *3*

$$x^2 + xy + x = 4$$
$$y^2 + xy + y = 2$$

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 11 EXTENSION HALF YEARLY 2012 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. B - $y = \|x + 2\| - 2$	**1**	
2. D - $2x^2 + 4x + 7 = 2(x + 1)^2 + 5$ $\therefore \quad q = 5$	**1**	
3.  $x^2 + x^2 = 7^2$ $2x^2 = 49$ $x^2 = \frac{49}{2}$ $x = \frac{49}{\sqrt{2}}$ $h^2 + x^2 = 8^2$ $h^2 = 64 - \frac{49}{2}$ $h^2 = 39.5$ $h = 6.28$ **C –** 6.3 cm	**1**	
4. $(a\sin x + a)(b\cos x - c) = 0$ $\sin x = -1$ or $\cos x = \frac{c}{b}$ $x = 270^\circ$ or no real solutions as $\frac{c}{b} > 1 \quad (b < c)$ $\therefore$ **A –** 1 solution	**1**	
5. C - $f(x) = 2x$ $f\left(\frac{x+y}{2}\right) = 2\left(\frac{x+y}{2}\right)$ $= x + y$ $\frac{f(x) + f(y)}{2} = \frac{2x + 2y}{2}$ $= x + y$	**1**	
SECTION II		
6 (i) $\frac{5}{x-1} \geq 3$ $x - 1 \neq 0$ $x \neq 1$ $\frac{5}{x-1} = 3$ $5 = 3x - 3$ $3x = 8$ $x = \frac{8}{3}$  $1 < x \leq \frac{8}{3}$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.

Solution	Marks	Comments
6 (ii) $x^3 \geq 6x - x^2$ $x^3 + x^2 - 6x \geq 0$ $x\left(x^2 + x - 6\right) \geq 0$ $x(x-2)(x+3) \geq 0$  $-3 \leq x \leq 0$ and $x \geq 2$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the three correct critical points via a correct method • Correct conclusion to their three critical points obtained using a correct method **1 mark** • Uses a correct method
7 a) $PQ^2 = 12^2 + 9^2$ $= 225$ $PQ = 15$ $PR^2 = 12^2 + 5^2$ $= 169$ $PR = 13$ $QR^2 = 5^2 + 9^2$ $= 106$ $QR = \sqrt{106}$ $\cos \angle PQR = \dfrac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR)}$ $= \dfrac{225 + 106 - 169}{2 \times 15 \times \sqrt{106}}$ $= 0.524494366\ldots$ $\angle PQR = 58.36578957\ldots$ $= 58^\circ$ (to nearest degree)	**3**	**3 marks** • Correct solution **2 marks** • Correctly substitutes into cosine rule **1 mark** • Attempts to use the cosine rule • Correctly finds the three required sides ***Note: no rounding penalty***
7 b) (i) $\dfrac{BC}{h} = \cot 25$ $BC = h\cot 25$ $BC = h\tan 65$	**1**	**1 mark** • Correct working in order to establish result
7 b) (ii) Similarly $BD = h\tan 60$ $CD^2 = BC^2 + BD^2 - 2 \times BC \times BD \times \cos \angle CBD$ $= h^2\tan^2 65 + h^2\tan^2 60 - 2(h\tan 65)(h\tan 60)\cos 60$ $= h^2\tan^2 65 + h^2\tan^2 60 - 2h^2\tan 65\tan 60\cos 60$	**3**	**3 marks** • Correctly establishes result *Note: substitution step **must** be shown* **2 marks** • Correctly substitutes into cosine rule • Uses cosine rule to establish result, without showing substitution step. **1 mark** • States a correct expression for BD • Attempts to use cosine rule
7 b) (iii) $CD^2 = h^2\left(\tan^2 65 + \tan^2 60 - 2\tan 65\tan 60\cos 60\right)$ $h^2 = \dfrac{CD^2}{\tan^2 65 + \tan^2 60 - 2\tan 65\tan 60\cos 60}$ $h = \dfrac{CD}{\sqrt{\tan^2 65 + \tan^2 60 - 2\tan 65\tan 60\cos 60}}$ $= \dfrac{50}{\sqrt{\tan^2 65 + \tan^2 60 - 2\tan 65\tan 60\cos 60}}$ $= 25.36889807\ldots$ $= 25.4$ metres (correct to 1 dp)	**2**	**2 marks** • Correct answer **1 mark** • Correctly makes h or h^2 the subject ***Note: no rounding penalty***

Solution	Marks	Comments
8 a) $(a+b)^4-(a-b)^4$ $=\left[(a+b)^2+(a-b)^2\right]\left[(a+b)^2-(a-b)^2\right]$ $=\left(2a^2+2b^2\right)(4ab)$ $=8ab\left(a^2+b^2\right)$	**3**	**3 marks** • Correct solution **2 marks** • Significant progress towards correct solution **1 mark** • Recognises the problem as difference of two squares
8 b) $\left(x+\frac{1}{x}\right)^2=x^2+2(x)\left(\frac{1}{x}\right)+\frac{1}{x^2}$ $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$ $=3^2-2$ $=7$	**2**	**2 marks** • Correct solution **1 mark** • Bald answer • Establishes a result for $x^2+\frac{1}{x^2}$
8 c) (i) Triangles $={}^7C_3$ $=35$	**1**	**1 mark** • ${}^7C_3$
8 c) (ii) Quadrilaterals $=1\times{}^6C_3$ $=20$	**2**	**2 marks** • Correct answer **1 mark** • Evidence of taking into account that A is included.
9 a) (i) $\cos^2(\theta-60)=\frac{1}{2}$ $0\le\theta\le 360$ $-60\le(\theta-60)\le 300$ $\cos(\theta-60)=\pm\frac{1}{\sqrt{2}}$ Quadrants 1,2,3 & 4 $\cos\alpha=\frac{1}{\sqrt{2}}$ $\alpha=45$ $\theta-60=-45,\ 45,\ 135,\ 225$ $\theta=15,\ 105,\ 195,\ 285$	**3**	**3 marks** • Correct solution **2 marks** • Correct solution for $\cos(\theta-60)=\frac{1}{\sqrt{2}}$ • Four correct solutions found ignoring domain **1 mark** • Finds an answer in all four quadrants • Calculates correct principal angle.
9 a) (ii) $4\sin\theta\cos\theta+1=2\left(\sin\theta+\cos\theta\right)$ $4\sin\theta\cos\theta-2\sin\theta-\cos\theta+1=0$ $2\sin\theta\left(2\cos\theta-1\right)-1\left(\cos\theta-1\right)=0$ $\left(2\cos\theta-1\right)\left(2\sin\theta-1\right)=0$ $\cos\theta=\frac{1}{2}$ or $\sin\theta=\frac{1}{2}$ Quadrants 1 & 4 Quadrants 1 & 2 $\cos\alpha=\frac{1}{2}$ $\sin\alpha=\frac{1}{2}$ $\alpha=60$ $\alpha=30$ $\theta=60,\ 300$ $\theta=30,\ 150$ $\theta=30,\ 60,\ 150,\ 300$	**3**	**3 marks** • Correct solution **2 marks** • Significant progress towards correct solution **1 mark** • Finds two correct answers via a valid method • Rewrites equation as $\left(2\cos\theta-1\right)\left(2\sin\theta-1\right)=0$
9 b) (i) Arrangements $=9!$ $=362880$	**1**	**1 mark** • $9!$
9 b) (ii) Arrangements $=3!7!$ $=30240$	**2**	**2 marks** • $3!7!$ **1 mark** • Accounting for group being in a circle • Calculating the ways or organising the men

Solution	Marks	Comments
9 b) (iii) P(next to each other) $= \frac{2! \times 8!}{9!}$ **or** $= \frac{2}{9}$ If one sits down then there are two chairs out of the nine that are the other can sit in $\therefore$ P(next to each other) $= \frac{2}{9}$	**2**	**2 marks** • Correct answer **1 mark** • Progress towards correct answer *Note: keep in mind their answer for b) (i)*
9 b) (iv) P(next to each other) $= \frac{1 \times 1 \times 8!}{9!}$ **or** $= \frac{1}{9}$ If one sits down then there is only one seat opposite $\therefore$ P(next to each other) $= \frac{1}{9}$	**2**	**2 marks** • Correct answer **1 mark** • Progress towards correct answer *Note: keep in mind their answer for b) (i)*
10 a) # Ways = 2 evens and 1 odd + 3 odds $= {}^{15}C_2 \times {}^{15}C_1 + {}^{15}C_3$ $= 105 \times 15 + 455$ $= 2030$	**2**	**2 marks** • Correct answer **1 mark** • Clearly considers different cases • 1 case correct
10 b) (i) $b = 3$ and $d = 8$	**1**	**1 mark** • Two correct answers
10 b) (ii) $\lvert ax - 3\rvert = \begin{cases} ax - 3 & x \geq \frac{3}{a} \\ 3 - ax & x < \frac{3}{a} \end{cases}$ $\lvert cx - d\rvert = \begin{cases} cx - 8 & x \geq \frac{8}{c} \\ 8 - cx & x < \frac{8}{c} \end{cases}$ From the graph it can be concluded that $\frac{3}{a} < \frac{11}{3} < \frac{8}{c}$ and $\frac{3}{a} < \frac{8}{c} < 5$ $\therefore\ 5a - 3 = 5c - 8$ $5a - 5c = -5$ $a - c = -1$ $\frac{11a}{3} - 3 = 8 - \frac{11c}{3}$ $\frac{11}{3}(a + c) = 11$ $a + c = 3$ $a + c = 3$ $\underline{a - c = -1}$ $2a = 2$ $a = 1$, $\therefore c = 2$	**3**	**3 marks** • Correct solution **2 marks** • Bald answer • Correctly evaluates either a or b • Finds two different relationships between a and c **1 mark** • Progress towards finding a relationship between a and c
10 b) (iii) From the graph $\lvert ax - b\rvert < \lvert cx - d\rvert$ $x < \frac{11}{3}$ or $x > 5$	**1**	**1 mark** • Correct answer
11 a) $y = g(h(x))$ $= \sqrt{\left(10 - x^2\right) - 1}$ $= \sqrt{9 - x^2}$ Domain: $9 - x^2 \geq 0$ $x^2 \leq 9$ $-3 \leq x \leq 3$	**2**	**2 marks** • Correct answer **1 mark** • Finds $g(h(x))$ • Finds correct domain for the calculated function
11 b) (i) Arrangements $= \frac{10!}{3!2!}$ $= 302400$	**2**	**2 marks** • Correct solution **1 mark** • Takes into account at least one repeated letter

Solution	Marks	Comments
11 b) (ii) Arrangements = N not used + A not used + G,E,S,W or R not used $= \frac{9!}{3!} + \frac{9!}{2!2!} + 5 \times \frac{9!}{3!2!}$ $= 60480 + 90720 + 151200$ $= 302400$ **OR** 9 letter arrangements = 10 letter arrangements as you can chop off the last letter from all 10 letter arrangements and will be left with distinct 9 letter arrangements.	**2**	**2 marks** • Correct solution **1 mark** • Attempts to break the problem into different cases.
11 c) $x^2 + xy + x = 4 \longrightarrow x(x + y + 1) = 4$ $y^2 + xy + y = 2 \qquad y(y + x + 1) = 2$ $\frac{x}{y} = 2$ $x = 2y$ $y^2 + 2y^2 + y = 2$ $3y^2 + y - 2 = 0$ $(3y - 2)(y + 1) = 0$ $y = \frac{2}{3}, \ x = \frac{4}{3}$ or $y = -1, \ x = -2$	**3**	**3 marks** • Correct solution **2 marks** • Finds one set of correct answers • Eliminates a variable to produce a correct quadratic equation **1 mark** • Attempts to eliminate a variable