BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2013

YEAR 11 TASK 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour and 30 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 6-11
- Marks may be deducted for careless or badly arranged work

Total marks – 51
Exam consists of 5 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2 (5 marks)

- Attempt Question 1-5

<u>Section II</u> – Pages 3-5 (46 marks)

- Attempt questions 6-11

Answer all questions in the appropriate space in the Answer booklet provided.

Section I – Multiple Choice - 5 marks

1. $a + b\sqrt{2} = \frac{2}{3+\sqrt{2}}$ find a and b.

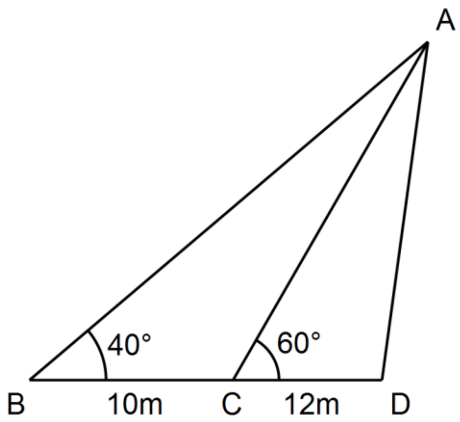
(A) $a = \frac{6}{7},\ b = -\frac{2}{7}$

(B) $a = \frac{6}{7}, b = \frac{2}{7}$

(C) $a = \frac{6}{5}, b = \frac{2}{5}$

(D) $a = \frac{6}{5}, b = -\frac{2}{5}$

2.



In the diagram $AC =$

(A) $12 \times \frac{\sin 20°}{\sin 40°}$

(B) $10 \times \frac{\sin 40°}{\sin 20°}$

(C) $12 \times \frac{\sin 40°}{\sin 20°}$

(D) $10 \times \frac{\sin 20°}{\sin 40°}$

3. If P divides AB externally in the ratio $5: 2$, then B divides PA in the ratio

(A) $3: 2$ (B) $3: -2$ (C) $2: 3$ (D) $2: -3$

4. Determine the number of solutions for
$\cos x - 4 \sec x = 0$ for $-180° \leq x \leq 180°$.

(A) 0 (B) 1 (C) 2 (D) 3

5. There are p triangles that can be formed from the 8 vertices of a given cube, and q of these triangles are equilateral. The values of p and q are:

(A) $p = 56,\ q = 6$

(B) $p = 20,\ q = 6$

(C) $p = 56,\ q = 8$

(D) $p = 20,\ q = 8$

End of Section 1

Section II – Extended Response

Attempt questions 6-11.
All necessary working should be shown in every question.

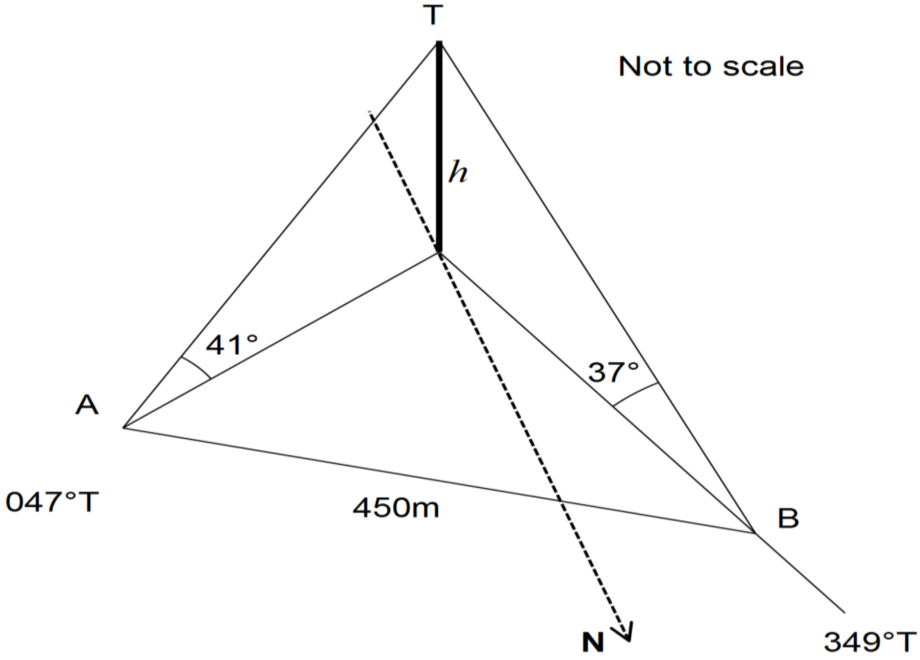
		Marks
Question 6 (8 marks)		
a)	Solve $\frac{4}{x-1} \geq 1$	3
b)	Simplify $\frac{a^n+a^{n-2}}{a^{n-1}}$	2
c)	Solve $2\cos^2 x - \sin x - 1 = 0$ for $0° \leq x \leq 360°$	3

Question 7 (9 marks)		
a)	The point $(6, k)$ is 8 units from the straight line, $3x + 4y + 2 = 0$, find k.	3
b)	Given the points $A(-5,1)$ and $B(11,9)$, find the point P which divides the interval AB externally in the ratio 5:3	3
c)	(i) Sketch $y = \|2x - 4\|$	1
	(ii) Find the values for c for which $\|2x - 4\| = x + c$ has two solutions.	2

Question 8 (8 marks)		
a)	A committee of 6 people is to be selected from 6 men and 4 women	
	(i) How many different committees can be formed?	1
	(ii) How many committees are possible if the men outnumber the women?	2
b)	If $4x^2 + 4xy + y^2 = 0$ find the value of $\frac{x-y}{x+y}$ where $x \neq y$	2
c)	Prove that $$\frac{1+\cos\theta}{1-\sin\theta} - \frac{1-\cos\theta}{1+\sin\theta} = 2\sec\theta\,(1+\tan\theta)$$	3

Question 9 (7 marks) | **Marks**

a) From the top of a tower T, two markers A and B can be seen on horizontal ground. **3**



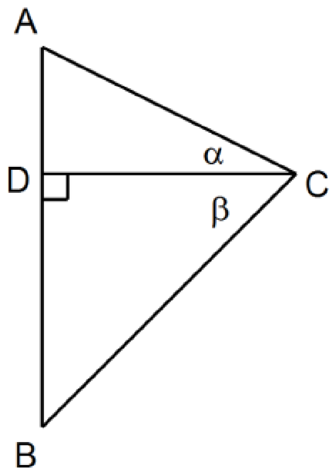
Marker A lies on a bearing of $047°T$ from the tower. Marker B has a bearing of $349°T$ from the tower.
The angles of elevation from of A and B to the top of the tower are $41°$ and $37°$ respectively. If AB is 450m, find h, the height of the tower, to the nearest metre.

b) (i) How many different arrangements of the letters of the word **PARALLEL** are there? **2**

(ii) How many of these arrangements begin and end with the letter **L**? **2**

Question 10 (6 marks)

a) **3**



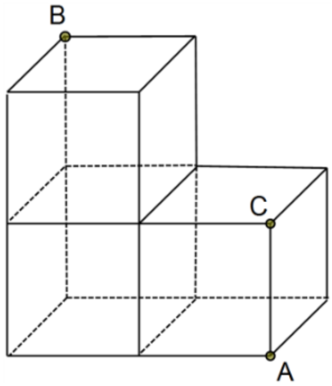
Given triangle ABC where D is a point on AB such that $CD \perp AB$, $\angle ACD = \alpha$ and $\angle DCB = \beta$.

Prove that

$$DB = \frac{AD\cos\alpha\ \sin\beta}{\cos\beta\sin\alpha}$$

b) Given x and y are rational, solve the following for x and y. **3**

$$xy + \sqrt{9x^2 + y^2} = 8 + 2\sqrt{13}$$

Question 11 (8 marks)			Marks
a)	(i)	Graph the function $y = 2\lvert x - 1\rvert + \lvert x - 4\rvert$	**3**
	(ii)	Use your graph to solve $2\lvert x - 1\rvert + \lvert x - 4\rvert \leq 6$	**1**
b)		 Three identical cubes of side 1cm are placed together as shown in the diagram.	
	(i)	Find the exact length of AB	**2**
	(ii)	Find $\angle ABC$ to the nearest degree.	**2**

End of Exam

	BANK	
9	Solve $\frac{12}{x^2-x} \leq 1$	**4**
3	If $\frac{4}{x-1} \geq 1$ then (A) $x < 1$ or $x \geq 5$ (B) $1 \leq x \leq 5$ (C) $1 < x \leq 5$ (D) $x \leq 1$ or $x \geq 5$	
7b	By graphing the function $y = \|x-2\| + \|x-4\|$ or otherwise solve $\|x-2\| + \|x-4\| = 6$	**3**
7a	Find the distance between the two straight lines $3x - 5y + 11 = 0$ and $5y - 3x + 6 = 0$	**3**
	A group of 12 people is to be seated at a long table with 6 seats each side. There are 4 people who wish to be on one side of the table and 3 people who wish to be on the other side. How many seating arrangements are there?	**3**
9b	The letters **AAA BB CC D EE FFF** are arranged in all possible ways. If one of the arrangements is choosen at random what is the probability that it starts with **ABCD**	**3**
	How many sets of 5 bands can be formed from 5 lead guitars, 5 bass guitars, 5 drummers and 5 pianists, if each band has to have at least one player of each instrument.	**3**
	If $\frac{12}{x^2-x} \geq 1$ then (A) $-3 \leq x \leq 0$ or $1 \leq x \leq 4$ (B) $-3 \leq x \leq 0$ (C) $-3 \leq x < 0$ or $1 < x \leq 4$ (D) $x \leq -3, 0 < x < 1$ or $x \geq 4$	

YEAR 11 EXT 1

ASSES 1 2013

1 EACH

(1) $a + b\sqrt{2} = \frac{2}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$

$= \frac{6 - 2\sqrt{2}}{9-2}$

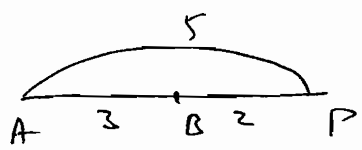
$= \frac{6}{7} - \frac{2\sqrt{2}}{7}$

$a = \frac{6}{7} \quad b = -\frac{2}{7}$ — (A)

(2) $\frac{AC}{\sin 40} = \frac{10}{\sin 20}$

$AC = \frac{10 \sin 40}{\sin 20}$ — (B)

(3)



PB : BA = 2:3 — (C)

(4) $\cos x - 4\sec x = 0$ (A)

$\cos x = \frac{4}{\cos x}$

$\cos^2 x = 4$

no soln

(5) $P = {}^8C_3 = 56.$ (C)

each diag → 2 Δs

∴ each face → 2×2 = 4 Δs

6 faces × 4 = 24 ÷3 (repetition

$q = 8$

11

MC 1 — A   4 — A
2 — B   5 — C
3 — C

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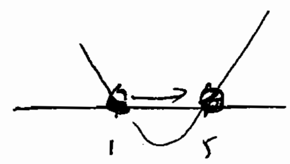
6 a) $\dfrac{4}{x-1} \geq 1 \qquad x \neq 1$ 1

$4(x-1) \geq (x-1)^2$

$(x-1)^2 - 4(x-1) \leq 0$ 1

$(x-1)(x-5) \leq 0$

$1 < x \leq 5$ 1



6 b) $\dfrac{a^n + a^{n-2}}{a^{n-1}}$

$= \dfrac{a^{n-2}(a^2+1)}{a^{n-1}}$ 1

$= \dfrac{a^2+1}{a}$ or $= a + \dfrac{1}{a}$ 1

6 c) $2\cos^2 x - \sin x - 1 = 0$

$2(1-\sin^2 x) - \sin x - 1 = 0$ 1

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or -1 1

$\therefore x = 30^\circ, 150^\circ$ or 270° 1

a) $D = \left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$= \left|\dfrac{3\times 6+4\times k+2}{\sqrt{3^2+4^2}}\right|$ ~~$=8$~~

$= \left|\dfrac{20+4k}{5}\right|$ 1

$\therefore \left|\dfrac{20+4k}{5}\right| = 8$

k = 5 or −15
2 marks.

$\therefore |20+4k| = 40$

$(5+k) = \mp 10$ 1 1

$k = 5$ or -15.

b) $A(-5,1)$ $B(11,9)$ $m:n = 5:-3$

$x = \dfrac{nx_1+mx_2}{m+n}$ $y = \dfrac{ny_1+my_2}{m+n}$ 1

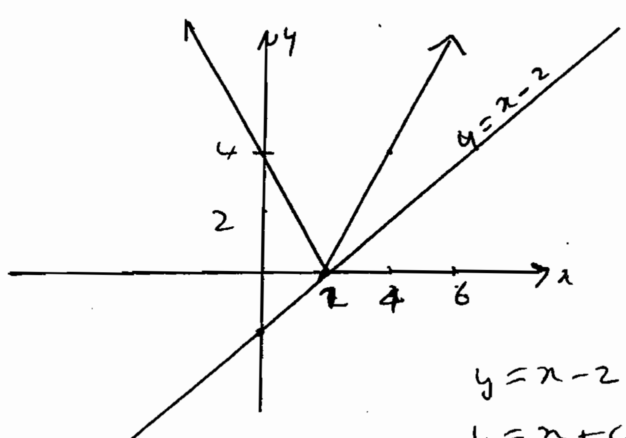
$= \dfrac{-3\times -5+5\times 11}{5-3}$ $= \dfrac{-3\times 1+5\times 9}{5-3}$

$= \dfrac{70}{2}$ $= \dfrac{42}{2}$

$= 35$ 1 $= 21$ 1

$\therefore$ P is point $(35, 21)$

c)



$y = x-2 \rightarrow$ one soln 1

$y = x+c$ $c > -2$ has 2 solns

2

8 a) i) $\quad {}^{10}C_6 = 210 \qquad 1$

ii) $\quad 4m\ 2w + 5m\ 1w + 6m$

$$= {}^6C_4 \cdot {}^4C_2 + {}^6C_5\, {}^4C_1 + {}^6C_6 \qquad 1$$
$$= 15 \times 6 + 6 \cdot 4 + 1$$
$$= 90 + 24 + 1$$
$$= 115 \qquad 1$$

b)
$$4x^2 + 4xy + y^2 = 0$$
$$2x + y = 0$$
$$y = -2x \qquad 1$$

$$\therefore \frac{x-y}{x+y} = \frac{x - -2x}{x - 2x}$$
$$= \frac{3x}{-x}$$
$$= -3 \qquad 1$$

c) $\text{LHS} = \dfrac{1+\cos\theta}{1-\sin\theta} - \dfrac{1-\cos\theta}{1+\sin\theta}$

$$= \frac{(1+\cos\theta)(1+\sin\theta) - (1-\cos\theta)(1-\sin\theta)}{1-\sin^2\theta} \qquad 1$$

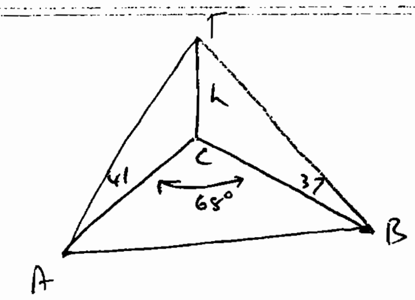
$$= \frac{1+\cos\theta+\sin\theta+\cos\theta\sin\theta - 1 + \sin\theta + \cos\theta - \sin\theta\cos\theta}{\cos^2\theta}$$

1 multiplied

$$= \frac{2}{\cos\theta}\left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right)$$

$$= \frac{2}{\cos\theta}\left(1 + \tan\theta\right) \qquad 1$$

$$= \text{RHS}$$



$\tan 41 = \dfrac{h}{AC}$ $\qquad\qquad$ $\tan 37 = \dfrac{h}{BC}$

$\therefore AC = h \tan 49^\circ$ $\qquad\qquad$ $BC = h \tan 53^\circ$ $\quad$ 1

$\angle ACB = 47 + 11 = 58^\circ.$

$AB^2 = AC^2 + BC^2 - 2.AC.BC.\cos\angle ACB.$

$\therefore\ 450^2 = h^2\tan^2 49^\circ + h^2\tan^2 53^\circ - 2.h^2\tan 49\tan 53\cos 58^\circ$ $\quad$ 1

$= h^2\left(\tan^2 49 + \tan^2 53^\circ - 2\tan 49\tan 53\cos 58^\circ\right)$

$\therefore\ h = \dfrac{450}{\sqrt{\tan^2 49 + \tan^2 53^\circ - 2\tan 49\tan 53\cos 58^\circ}}$

$= 372$ m. $\quad$ 1

b i) P A R C E $\qquad$ $No = \dfrac{8!}{2!.3!}$ 1 $\quad = 3360$ 1

A L

L

ii) L - - - - - - L $\rightarrow$ $\dfrac{6!}{2!}$ 1 $\quad = 360$ 1

(10) a)

in $\triangle ADC$ $\tan\alpha = \dfrac{AD}{DC}$

$\therefore DC = \dfrac{AD}{\tan\alpha}$

$= AD\dfrac{\cos\alpha}{\sin\alpha}$ 1

in $\triangle BDC$ $\tan\beta = \dfrac{DB}{DC}$

$\therefore DC = \dfrac{DB}{\tan\beta}$

$= \dfrac{DB\cos\beta}{\sin\beta}$ 1

$\therefore \dfrac{DB\cos\beta}{\sin\beta} = \dfrac{AD\cos\alpha}{\sin\alpha}$ 1

$DB = \dfrac{AD\cos\alpha\sin\beta}{\cos\beta\sin\alpha}$.

b) $xy = 8$ — (1)

$9x^2+y^2 = 52$ — (2) 1

Subs $y = \frac{8}{x} \rightarrow$ (2)

$\therefore 9x^2 + \dfrac{64}{x^2} = 52$

$\therefore 9x^4 - 52x^2 + 64 = 0$

$(9x^2-16)(x^2-4) = 0$ (or let $y = x^2$ etc)

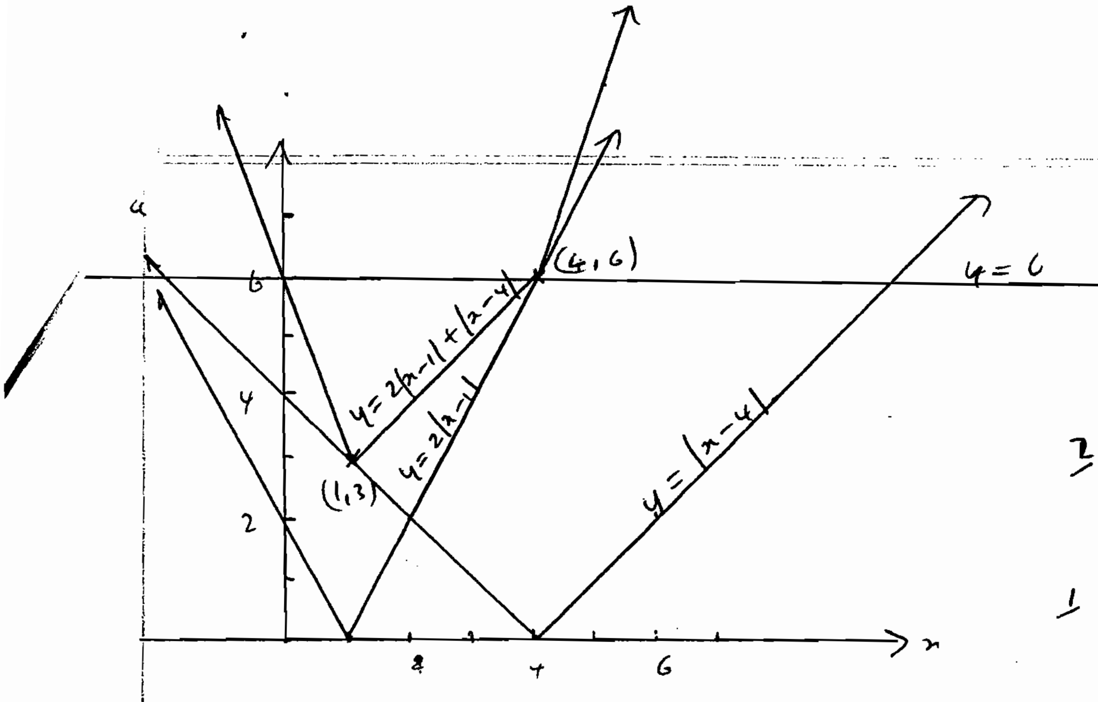
$\therefore x^2 = \frac{16}{9}$ or 4 1

$x = \mp\frac{4}{3}$ or ∓ 2

$\therefore y = \mp 6$ ∓ 4 from (1)

$\therefore$ solution $\left(\mp\frac{4}{3}, \mp 6\right)$ or $(\mp 2, \mp 4)$

1



2 correct shape

1 showing (0,6),(1,3),(4,6)

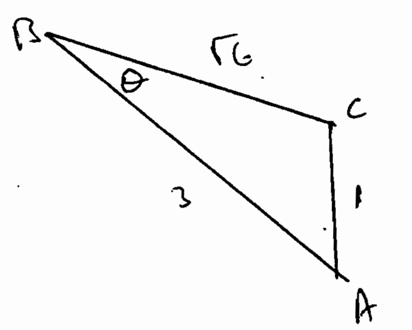
ii) $0 \le x \le 4$ 1

11 b $AB^2 = 2^2+2^2+1^2 = 9$ 1

$\therefore AB = 3$ 1

$BC^2 = 2^2+1^2+1^2 = 6$

$\therefore BC = \sqrt{6}$ 1



$$\cos \angle ABC = \frac{(\sqrt{6})^2 + 3^2 - 1}{2 \cdot 3\sqrt{6}}$$

$$= \frac{14}{6\sqrt{6}}$$

$\therefore \angle ABC = 17.7^\circ$ 1

$= 18^\circ$ to nearest degree