BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2014

YEAR 11 TASK 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 6-8
- Marks may be deducted for careless or badly arranged work

Total marks – 35
Exam consists of 6 pages.

This paper consists of TWO sections.

Section 1 – Pages 2-3 (5 marks)

- Attempt Questions 1-5

Section II – Pages 4-6 (30 marks)

- Attempt questions 6-8

Section I

5 marks

Attempt Questions 1-5

Allow about 6 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 5

---

1 If $x > y > 0$ and $z \neq 0$, the inequality which is NOT always correct is

(A) $x - z > y - z$

(B) $x^2 > y^2$

(C) $\frac{x}{z^2} > \frac{y}{z^2}$

(D) $xz > yz$

2 From a group of 7 men and 6 women, five people are to be selected to form a committee so that exactly 3 men are on the committee. In how many ways can it be done?

(A) 286 (B) 525 (C) 2520 (D) 6300

3 If $f(x) = x^2 - 7x + k$ and $f(k) = -9$ then $f(-1)$ equals

(A) -9

(B) 3

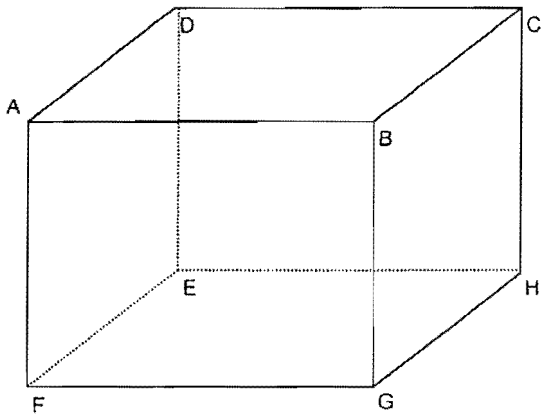
(C) 5

(D) 11

Section 1 continued on next page

4 *ABCDEFGH* is a cube.

What is the cosine of angle *CFH*?



(A) $\frac{\sqrt{2}}{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\sqrt{\frac{2}{3}}$

(D) $\frac{1}{\sqrt{2}}$

5 For the function $y = |3 - 2x| + |x + 4|$, which of the following equations represents the section of the graph for $x < -4$?

(A) $y = -x + 7$

(B) $y = x - 7$

(C) $y = -3x - 1$

(D) $y = 3x + 1$

End of Section I

Section II

30 marks
Attempt questions 6-8
Allow about 54 minutes for this section

Answer each question on the appropriate answer sheet. Each answer must show your name. Extra paper is available.
All necessary working should be shown in every question.

Marks

Question 6 (10 marks) Use the answer sheet for question 6

a) Simplify $\dfrac{x^n}{x^{n+1}-x^n}$ **1**

b) Six identical yellow discs and four identical blue discs are placed in a straight line.

i) How many arrangements are possible? **1**

ii) How many arrangements would have all the blue discs together? **1**

c) Solve $x+2 \leq \dfrac{4}{x-1}$ **3**

d) Prove that $\tan^2 A - \tan^2 B = \dfrac{\cos^2 B - \cos^2 A}{\cos^2 A \cos^2 B}$ **2**

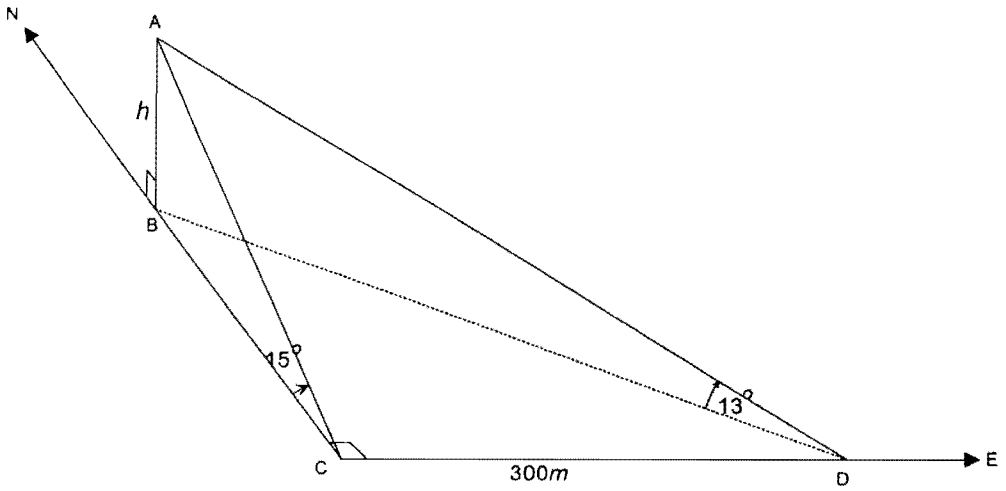
e) Find the number of ways in which 4 girls and 3 boys can be seated in a row so that no two girls are next to each other. **2**

End of question 6

Question 7 (10 marks) Start on the answer sheet for question 7

a) Solve the equation $\sin^2\theta + 2\sin\theta\cos\theta - 3\cos^2\theta = 0$ for $0 \leq \theta \leq 360^\circ$, giving your answer correct to the nearest degree. **3**

b) Bill, Caitlyn and David are all standing on level ground. Bill is standing at the base of a building of height h metres. Caitlyn is due south of Bill and David is 300m due east of Caitlyn. Anthony is standing at the top of the building.
From where Caitlyn is standing the angle of elevation to the top of the building is 15° and from David the angle of elevation to the top of the building is 13°.



Show that $h^2 = \dfrac{90000}{\cot^2 13^\circ - \cot^2 15^\circ}$ **2**

c) Let each different arrangement of all the letters of the word "DELETED" be considered a word.

i) How many words are possible? **1**

ii) In how many ways can the three E's be together? **1**

iii) How many words have two E's together and one separate? **2**

iv) In how many words are all the E's separated? **1**

End of question 7

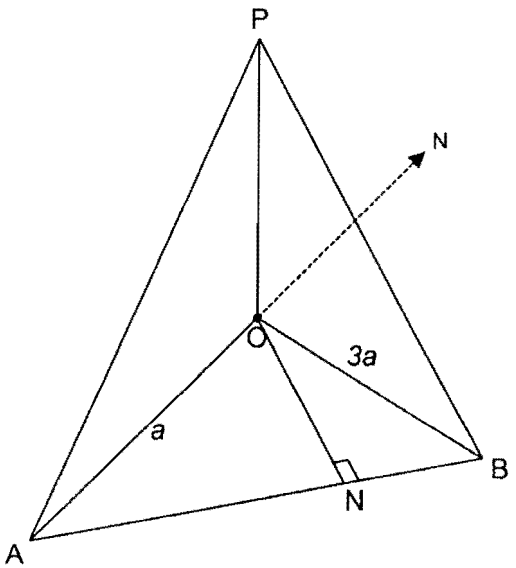
Question 8 (10 marks) Start on the answer sheet for question 8

a) Solve the following: **3**

$$|x + 1| > |2x - 2|$$

b) Find the number of seating arrangements in which eight people can sit around two identical circular tables (seating four on each table) if two particular people want to sit next to each other on a table, a second couple wish to sit at different tables and a third couple wish to sit at the same table (but not necessarily together). **2**

c) Adam is due south of a tower whose base is on level ground at O. Bob is on a bearing of $060^\circ T$ from O.
In triangle OAB, $\angle AOB = 120^\circ$, $OA = a$, $OB = 3a$ and ON is perpendicular to AB, meeting at N.



(i) By considering the area of ΔOAB in two different ways, show that the length of ON is given by $ON = \dfrac{3a\sqrt{3}}{2\sqrt{13}}$. **3**

(ii) The tower OP is perpendicular to the level ground OAB with $AP = a\sqrt{5}$. **2**
Find the angle between the plane containing PAB and the plane containing OAB.

End of paper

Yr 11 Mathematics Extension 1 2014 Half Yearly

1. D: If $x > y$
$xz > yz$ if $z > 0$

2. B: ${}^7C_3 \times {}^6C_2 = 525$

3. D: $f(k) = -9 = k^2 - 7k + k$
$k^2 - 6k + 9 = 0$
$(k-3)^2 = 0$
$k = 3$
$f(-1) = 1 + 7 + 3$
$= 11$

4. C: let each edge be x units
$FH = x\sqrt{2}$ pythagoras' Theorem $x^2 + x^2 = FH^2 \rightarrow FN = \sqrt{2x^2} = x\sqrt{2}$
$CF = x\sqrt{3}$ (" ") $CF^2 = x^2 + x^2 = CF = \sqrt{3x^2} = x\sqrt{3}$
$\therefore \cos(\angle CFH) = \frac{x\sqrt{2}}{x\sqrt{3}}$ (ΔCHF is right angled)
$= \sqrt{\frac{2}{3}}$

5. C: when $x < -4$ is: $y = |2x-3| + |x+4|$
$y = -(2x-3) - (x+4)$
$y = -3x - 1$

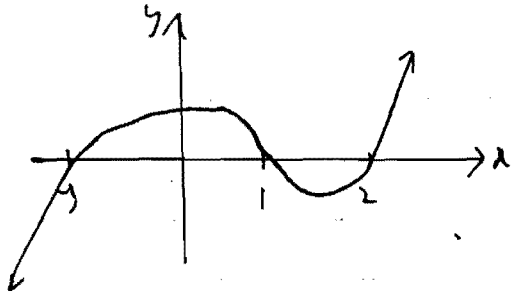
6 a) $\frac{x^{\wedge}}{x^{\wedge}(x-1)} = \frac{1}{x-1}$ ✓

b) i) Arrangements $= \frac{10!}{6!\,4!}$
$= 210$ } 1 ✓

ii) Blues together = 1 way
Arrangements = 7 $\left(\text{or } \frac{7!}{6!} = 7\right)$ ✓

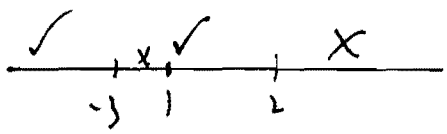
c) $\frac{4}{x-1} \geq x+2 \quad x \neq 1$ OR $x+2 = \frac{4}{x-1}$

$4(x-1) \geq (x+2)(x-1)^2$ ✓
$(x-1)\left[(x-1)(x+2)-4\right] \leq 0$
$(x-1)(x^2+x-6) \leq 0$
$(x-1)(x+3)(x-2) \leq 0$ ✓



$\therefore 1 < x \leq 2$ or $x \leq -3$ ✓

$(x+2)(x-1) = 4$
$x^2+2x-2 = 4$
$x^2+2x-6 = 0$
$(x+3)(x-2) = 0$ ✓



✓

$\therefore x \leq -3$ or $1 < x \leq 2$
✓

d) $RHS = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$

$= \frac{1}{\cos^2 A} - \frac{1}{\cos^2 B}$ ✓

$= \sec^2 A - \sec^2 B$

$= \tan^2 A + 1 - (\tan^2 B + 1)$ ✓

$= \tan^2 A - \tan^2 B$

$= LHS$

$\therefore \tan^2 A - \tan^2 B = \frac{\cos^2 B - \cos^2 A}{\cos^2 B \cos^2 A}$

6e) G B G B G B G

Girls seated in 4! ways ✓
Boys seated in 3! ways
} either correctly considered

$\therefore$ total $= 4! \times 3!$
$= 24 \times 6$
$= 144$ ways ✓

7a) $\sin^2\theta + 2\sin\theta\cos\theta - 3\cos^2\theta = 0$
$\tan^2\theta + 2\tan\theta - 3 = 0$ ✓
$(\tan\theta + 3)(\tan\theta - 1) = 0$
$\tan\theta = 1 \quad \tan\theta = -3$ ✓
$\theta = 45°, 225° \quad \theta = 180 - 72, 360 - 72$
$= 108°, 288°$
$\therefore \theta = 45°, 108°, 225°, 288°$ ✓

b) In $\triangle ABC$ $\cot 15° = \frac{BC}{h}$
$BC = h\cot 15°$

similarly in $\triangle ABD$
$BD = h\cot 13°$ ✓

In $\triangle BCD$
$DD^2 = BC^2 + CD^2$
$h^2\cot^2 13° = h^2\cot^2 15° + 300^2$
$h^2(\cot^2 13° - \cot^2 15°) = 90000$ ✓
$h^2 = \frac{90000}{\cot^2 13° - \cot^2 15°}$

7 c) i) No. of arrangements $= \frac{7!}{2!3!}$ ✓

$= 420$

ii) [EEE] D D L T

No of ways $= \frac{5!}{2!}$ ✓

$= 60$

iii) [EE] E D D L T

_ D _ D _ L _ T _ . in $\frac{4!}{2!} = 12$ ways ✓

[EE] and [E] in ${}^5P_2$ ways

total $= 12 \times 20$

$= 240$ ways ✓

iv) No of ways $= 420 - (60 + 240)$

$= 120$ ways. ✓

or _ D _ D _ L _ T _ in $\frac{4!}{2} = 12$ ways

3 E's inserted in ${}^5C_3 = 10$ ways.

$\therefore$ 120 ways.

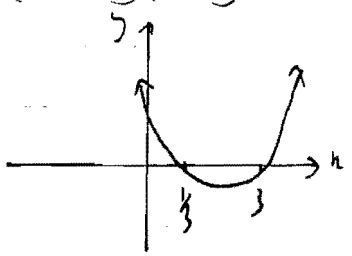
8a) $|x+1| > |2x-2|$

$(x+1)^2 > 4(x-1)^2$ ✓

$x^2+2x+1 > 4(x^2-2x+1)$

$0 > 3x^2 - 10x + 3$

$(3x-1)(x-3) < 0$ ✓



$\therefore \frac{1}{3} < x < 3$ ✓

b) 1st couple sits on either table in 2 ways

2 choices as to which of the second couple joins them and has choice of 2 seats

3rd couple has a choice of 1 tables and $^3P_2$ seats

last 2 people seated in 2 ways.

$\therefore$ total $= 2 \times 4 \times 6 \times 2$

$= 96$ ways

(2) correct solution

(1) some significant progress (eg 2 considerations of the solution)

c i) Area $\triangle OAB = \frac{1}{2} \times a \times 3a \times \sin 120°$

$= \frac{3a^2 \sin 60°}{2}$

$= \frac{3a^2\sqrt{3}}{4}$ ✓

Also using cosine rule, $AB^2 = a^2 + (3a)^2 - 2(a)(3a)\cos 120°$

$= 10a^2 + 6a^2 \cos 60°$

$= 10a^2 + 3a^2$

$AB^2 = 13a^2$

$AB = a\sqrt{13}$ ✓

8c i) cont'd

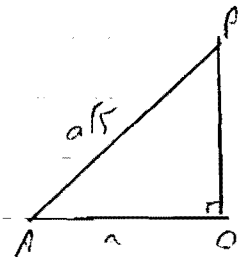
$\therefore$ Area $\Delta_{ONS} = \frac{1}{2} \times NS \times ON$

$$\frac{3a^2\sqrt{3}}{4} = \frac{1}{2} a\sqrt{3}\, ON$$

$$ON = \frac{6a^2\sqrt{3}}{4a\sqrt{13}}$$

$$ON = \frac{3a\sqrt{3}}{2\sqrt{13}} \quad \checkmark$$

c ii)

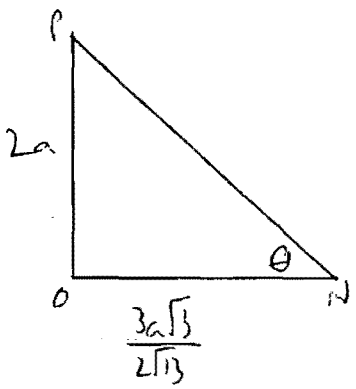


$$OP = \sqrt{(a\sqrt{5})^2 - a^2}$$

$$= \sqrt{4a^2}$$

$$OP = 2a \quad \checkmark$$

marks
(2) correct solution
(1) finds $OP = 2a$
(1) uses incorrect OP to calculate θ



$$\tan\theta = \frac{2a}{\frac{3a\sqrt{3}}{2\sqrt{13}}}$$

$$= \frac{4\sqrt{13}}{3\sqrt{3}}$$

$$\tan\theta = 2.7755\ldots$$

$$\theta = 70.1864\ldots$$

$\theta = 70^\circ$ (nearest degree)
$\theta = 70^\circ 11'$ (nearest minute)
or equivalent $\checkmark$