BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2015

YEAR 11 TASK 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 6-9
- Marks may be deducted for careless or badly arranged work

Total marks – 44
Exam consists of 5 pages.

This paper consists of TWO sections.

Section 1 – Page 2 (5 marks)

- Attempt Question 1-5

Section II – Pages 3-5 (39 marks)

- Attempt questions 6-9

Answer all questions in the appropriate space in the Answer booklet provided.

Section I – Multiple Choice - 5 marks

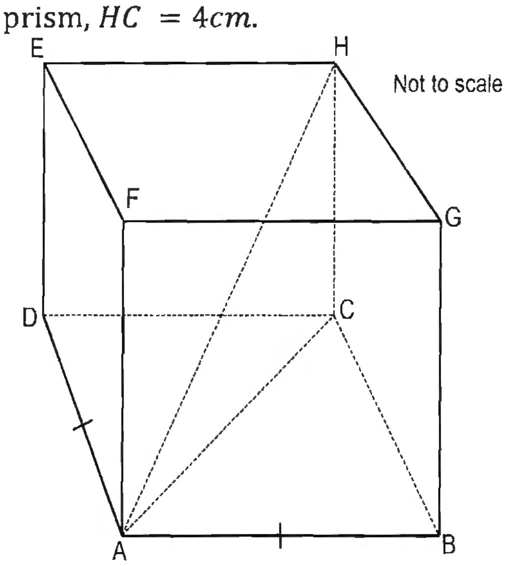
1. Solve for x,

$$\frac{2x+1}{1-x} \geq 1$$

(A) $0 \leq x < 1$ (B) $x \leq 0$ or $x > 1$

(C) $x > 0$ or $x > 1$ (D) $0 < x \leq 1$

2. A rectanglular prism with a square base ABCD, is shown below. The diagonal of the prism, $AH = 8cm$, the height of the prism, $HC = 4cm$.



The volume of this rectangular prism is

(A) $64\ cm^3$ (B) $96cm^3$

(C) $128cm^3$ (D) $192cm^3$

3. The domain of the function $f(x) = (4 - x^2)^{-\frac{1}{2}}$

(A) $x \leq -2$ *or* $x \geq 2$ (B) $x < -2$ *or* $x > 2$ (C) $-2 \leq x \leq 2$ (D) $-2 < x < 2$

4. If the equation $f(2x) - 2f(x) = 0$ is true for all real values of x, then $f(x)$ could be

(A) $\frac{x^2}{2}$ (B) $2x$

(C) $\sqrt{2x}$ (D) $x - 2$

5. Ten people are to be seated around a circular table. How many possible seating arrangements are there if two particular friends want to sit directly opposite each other?

(A) $2\times 8!$ (B) $2 \times 9!$ (C) $4! \times 4!$ (D) $8!$

End of Section 1

Section II – Extended Response

Attempt questions 6-9.
All necessary working should be shown in every question.

Question 6 (10 marks) **Marks**

a) Prove the identity 2

$$\frac{2\sin^3 x + 2\cos^3 x}{\sin^2 x + \sin x.\cos x} = 2\text{cosec}x - 2\cos x$$

b) Determine if the function 2
$f(x) = x^2 + \cos x$ is odd, even or neither. Show all working.

c) Solve 3

$$\frac{x^2+2}{x} \geq 2x - 1$$

d) A committee of 5 is to be chosen from 6 men and 8 women.
Find how many committees are possible, if

i) the committee will consist of 3 men and 2 women. 1

ii) there is at least one woman on the committee. 2

Question 7 (10marks)

a) (i) Sketch on the same number plane the graph of $y = \sqrt{3-x}$ and $y = |x-1|$. 2

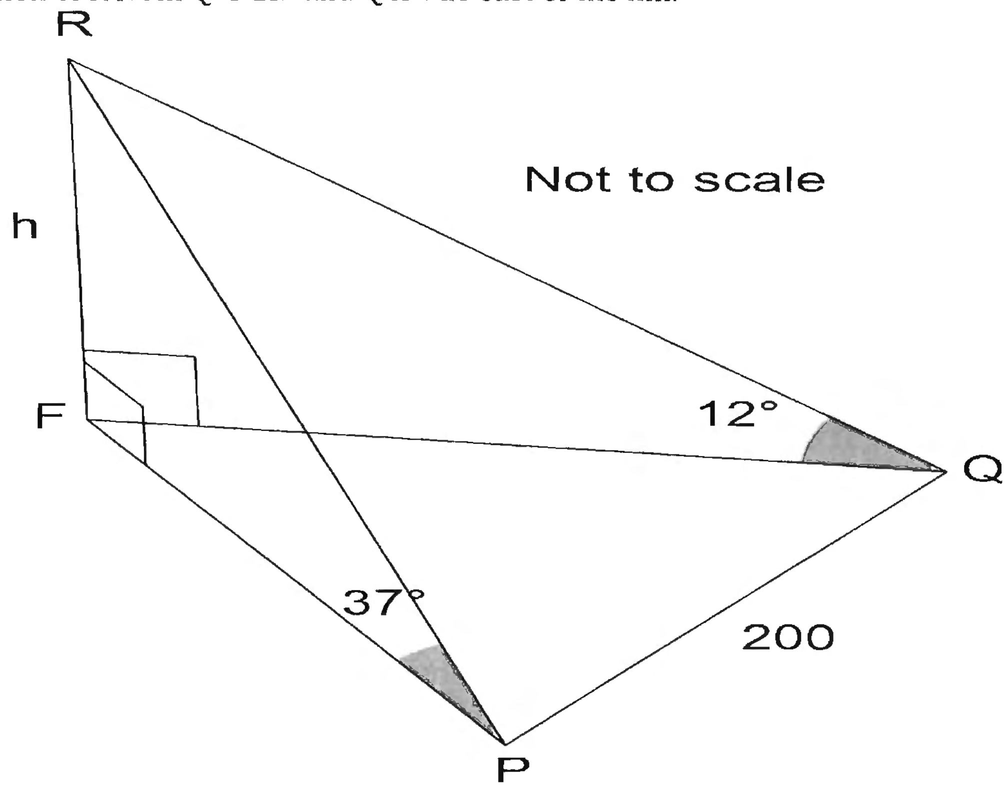
(ii) Hence or otherwise solve $\sqrt{3-x} \leq |x-1|$ 2

b) Solve the equation $3\cot\theta = tan\theta + 2$ for $0^\circ \leq \theta \leq 360^\circ$, giving your answer correct to the nearest minute. 3

c) Factorise completely $x^5 + x^2y^2(y-x) - y^5$ 3

Question 8 (8 marks)

a) A bushwalker walking on a horizontal straight road PQ observes that from his position P the bearing of a hill FR is 337° and he notices the peak R of the hill at an angle of elevation of 37°. After walking 200metres, he arrives at Q. The angle of elevation of R from Q is 12° and Q is due east of the hill.



i) Show that $FP = h\,tan53^{\circ}$ **1**

ii) By finding a similar expression for FQ, show that **2**

$200^2 = h^2\tan^2 53^{\circ} + h^2\tan^2 78^{\circ} - 2h^2 tan53^{\circ}\,tan78^{\circ}cos67^{\circ}$

iii) Hence find the height of the tower. **1**

b) At a dinner party, the host, hostess and their six guests sit at a round table. In how many ways can they be arranged, if the host and hostess are not sitting together? **2**

c) How many different numbers greater than 6000 can be formed with the digits 4, 5, 6, 7, 8 if no digit is used more than once? **2**

Question 9 (11 marks) **Marks**

a) (i) How many different arrangements of the letters of the word ISOSCELES are possible? **2**

(ii) How many of these arrangements have all S's together? **2**

(iii) How many of them have the letter S as the first and last letter? **2**

b) Given a function

$$y = \frac{x}{9 - x^2}$$

(i) Find all the asymptotes of the function. **2**

(ii) Determine whether the function is even, odd or neither. Justify your answer. **1**

(iii) Sketch the curve. **2**

End of Exam

Year 11 – Half-yearly 2015

Question 6

a) Prove $\dfrac{2\sin^3 x + 2\cos^3 x}{\sin^2 x + \sin x\cos x} = 2\operatorname{cosec} x - 2\cos x$

Proof: $LHS = \dfrac{2\sin^3 x + 2\cos^3 x}{\sin^2 x + \sin x\cos x} = \dfrac{2(\sin^3 x + \cos^3 x)}{\sin x(\sin x + \cos x)}$

(1) $= \dfrac{2\cancel{(\sin x + \cos x)}(\sin^2 x - \sin x\cos x + \cos^2 x)}{\sin x\cancel{(\sin x + \cos x)}}$

$= \dfrac{2(1 - \sin x\cdot\cos x)}{\sin x} = \dfrac{2 - 2\sin x\cos x}{\sin x}$

(1) $= \dfrac{2}{\sin x} - \dfrac{2\cancel{\sin x}\cdot\cos x}{\cancel{\sin x}} = \dfrac{2}{\sin x} - 2\cos x$

$= 2\operatorname{cosec} x - 2\cos x = RHS$ ∴ proven

not shown

b) $f(x) = x^2 + \cos x$

(1) $f(-x) = (-x)^2 + \cos(-x)$ but $\cos(-x) = \cos x$ even function

$= x^2 + \cos x$ (1)

$= f(x)$ ∴ even function

(if) ~~do $f(-x) = x^2 + \cos x$~~

but then state $f(x) \neq f(-x)$

and $-f(x) \neq f(-x)$ ∴ neither ∴ 1/2

c) $\dfrac{x^2+2}{x} \geq 2x - 1 \quad /\times x^2 \qquad x \neq 0$

$x(x^2+2) \geq x^2(2x-1)$

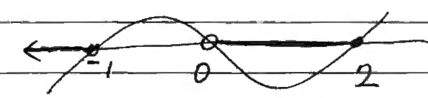
$0 \geq x^2(2x-1) - x(x^2+2)$ (1)

$0 \geq x[x(2x-1) - (x^2+2)]$

$0 \geq x[2x^2 - x - x^2 - 2]$

$0 \geq x[x^2 - x - 2]$

$0 \geq x[x-2][x+1]$ (1)



∴ $x \leq -1, \quad 0 < x \leq 2$ (1)

d) i) 6 men → pick 3 ∴ ${}^6C_3$

8 women → pick 2 ∴ ${}^8C_2$

∴ answer ${}^6C_3 \times {}^8C_2 = 560$ (1)

ii) At least one women chosen (1) for recognising all combos getting at least 1 correct

∴ 1w or 2w or 3w or 4w or 5w

$= {}^8C_1 \times {}^6C_4 + {}^8C_2 \times {}^6C_3 + {}^8C_3 \times {}^6C_2 + {}^8C_4 \times {}^6C_1 + {}^8C_5 = 1996$

OR All possible − No women on committee ∴ only men

$= {}^{14}C_5 - {}^6C_5$

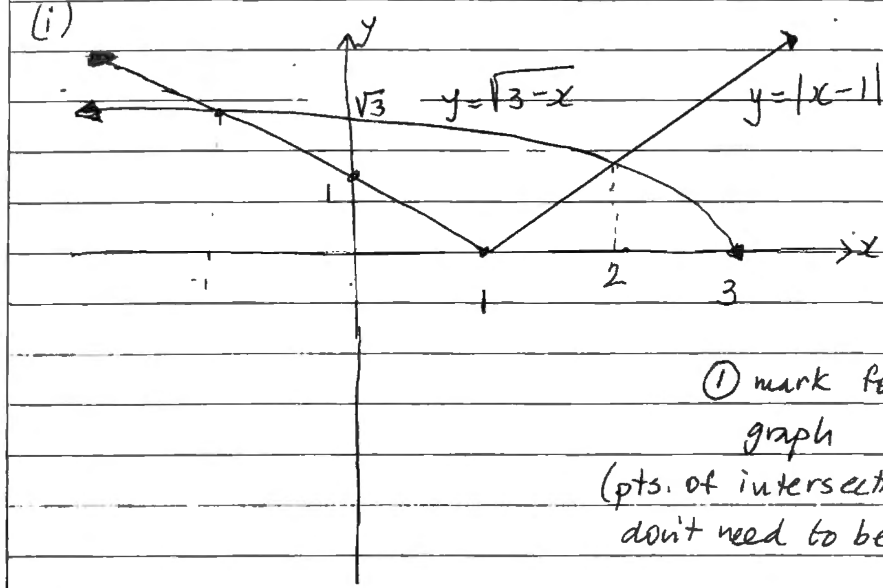
$= 2002 - 6 = 1996$

(if) ${}^{14}P_5 - {}^6P_5 = 239520$ 1 mark

Question 7

a) $y = \sqrt{3-x}$ $\qquad y = |x-1|$

(i)



① mark for each graph

(pts. of intersection don't need to be shown)

(ii) $\sqrt{3-x} = |x-1|$ first

$$3-x = (x-1)^2$$
$$0 = x^2-2x+1+x-3$$
$$0 = x^2-x-2 = (x-2)(x+1)$$
$$\therefore x=2 \qquad x=-1 \quad ①$$

$$\therefore \sqrt{3-x} \le |x-1|$$

① soln: $x \le -1$ or $2 \le x \le 3$

b) $3\cot\theta = \tan\theta + 2 \qquad 0^\circ \le \theta \le 360^\circ$

$$\frac{3}{\tan\theta} = \tan\theta + 2 \quad / \times \tan\theta$$
$$3 = \tan^2\theta + 2\tan\theta$$
$$0 = \tan^2\theta + 2\tan\theta - 3$$
$$0 = (\tan\theta+3)(\tan\theta-1)$$

for simplifying identities & creating quad. equation ①

$$\therefore \tan\theta = -3 \quad \text{or} \quad \tan\theta = 1$$

$\theta = 108^\circ 26', 288^\circ 26'$ ① $\qquad \theta = 45^\circ, 225^\circ$

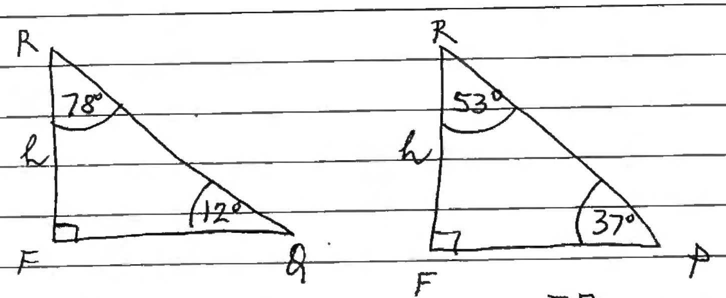
c) $x^5 + x^2y^2(y-x) - y^5$

$$= x^5 + x^2y^3 - x^3y^2 - y^5$$
$$= x^2(x^3+y^3) - y^2(x^3+y^3) \quad ①$$
$$= (x^2-y^2)(x^3+y^3) = (x-y)(x+y)(x+y)(x^2-xy+y^2)$$

① ①

or $= (x+y)^2(x-y)(x^2-xy+y^2)$

Question 8

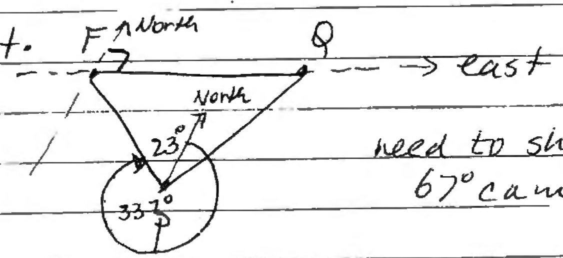


(1) (i) $\tan 53^\circ = \dfrac{FP}{h}$

need to show some evidence

need to show $FQ = h.\tan 78^\circ$ (1) $\therefore FP = h \cdot \tan 53^\circ$

(ii) cont.



need to show where 67° came from (1)

$\therefore \angle QFP = 180^\circ - 23^\circ - 90^\circ = 67^\circ$

$\therefore$ by cosine rule in ΔFQP

$$PQ^2 = FP^2 + FQ^2 - 2 \times FP \times FQ \times \cos 67^\circ$$

$$\therefore 200^2 = h^2 \cdot \tan^2 53^\circ + h^2 \cdot \tan^2 78^\circ - 2 \times h \tan 53^\circ \cdot h \cdot \tan 78^\circ \times \cos 67^\circ$$

$$\therefore 200^2 = h^2 \cdot \tan^2 53^\circ + h^2 \tan^2 78^\circ - 2h^2 \tan 53^\circ \tan 78^\circ \cdot \cos 67^\circ$$

$\therefore$ proven

a) iii)

$$200^2 = h^2\left[\tan^2 53^\circ + \tan^2 78^\circ - 2\tan 53^\circ \tan 78^\circ \cos 67^\circ\right]$$

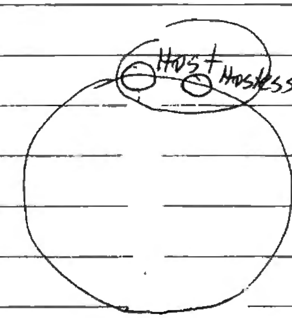
$$\therefore h^2 = \frac{200^2}{\tan^2 53^\circ + \tan^2 78^\circ - 2\tan 53^\circ \tan 78^\circ \cos 67^\circ}$$

$$\therefore h = \frac{200}{\sqrt{\tan^2 53^\circ + \tan^2 78^\circ - 2\tan 53^\circ \tan 78^\circ \cos 67^\circ}}$$

$\therefore h = 45.864$ (1) (3 d.p) [ignore rounding]

$\doteqdot 46\text{m}$

b)



Answer = All possibilities − Host Hostess together

$= (8-1)! - 2 \times (7-1)!$

$= 5040 - 1440 = 3600$

2- marks → correct solution with working

1- mark → well presented methods with incorrect conclusion

(c) 4 digit numbers – must start with 6, 7, 8 _ _ _ 3 more digits

$\therefore 3 \times {}^4P_3 = 72$ (1)

or

5 digit numbers $= 5! = {}^5P_5 = 120$

$\therefore$ Total $= 72 + 120 = 192$

2 marks – correct answer with working

1 mark – 4 digits or 5 digit numbers
Or coherant working – mistake obvious

Question 9

ISOSCELES

(a) (i) $\dfrac{9!}{3!\,2!} = 30240$

S repeats 3 times $3!$
E repeats 2 times $2!$

2-marks – correct solution with working

1- mark $= \dfrac{9!}{\text{wrong repetition}}$

ii) SSS $\underbrace{\_\_\_\_\_\_}_{7\text{ entities}}$ any remaining letter $\therefore \dfrac{7!}{2!\,(EE)} = 2520$

2- marks – correct solution with working

1- mark – seeing $7!$ in the working
or dividing by $2!$

iii) S $\underbrace{\_\_\_\_\_\_\_}_{7\text{ places} \therefore 7!}$ S
fixed fixed

$\dfrac{7!}{2!\,(EE)} = 2520$

2-marks – correct solution with working

1 mark – working correctly towards solution

(b) $y = \dfrac{x}{9-x^2}$

(i) vertical asymptotes $9-x^2 \neq 0$

$\therefore x = \pm 3$ (1)

horizontal asymptote

$x \to +\infty$ (pick a large number, sub. in)

$\therefore y \to 0^-$

if $x \to -\infty$ (pick a small number eg. $x=-100$, sub in)

$\therefore y \to 0^+$

$\therefore$ horizontal asymptote is $y = 0$ (1)

ii) $f(x) = y = \dfrac{x}{9-x^2}$

$f(-x) = \dfrac{-x}{9-(-x)^2} = \dfrac{-x}{9-x^2} = -f(x)$ (1)

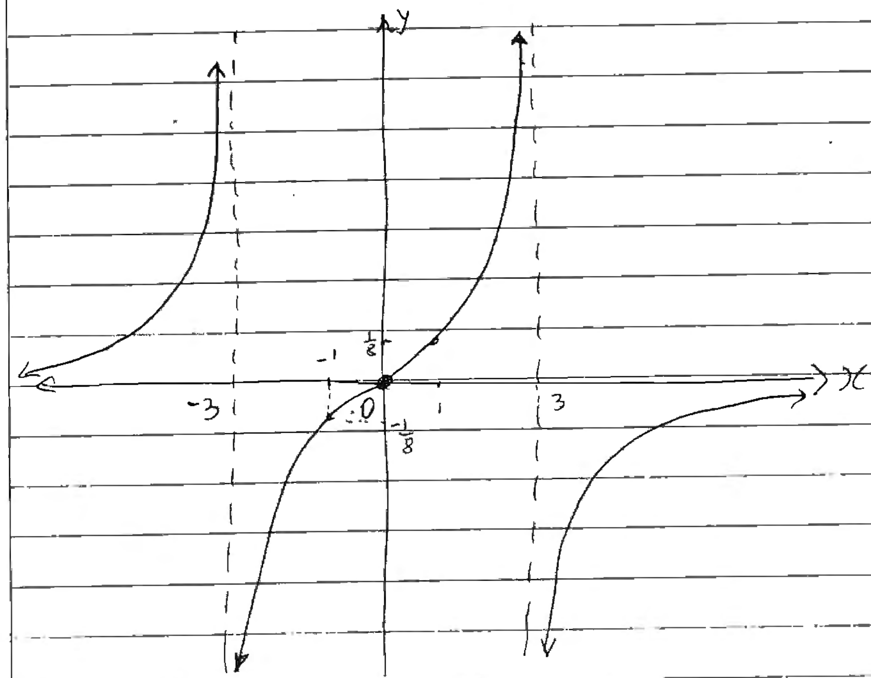
must show substitution

$\therefore$ odd function

iii)

x	-1	0	1
y	$\frac{-1}{8}$	0	$\frac{1}{8}$

- By using table of values and the property of the odd function drawing the shape



2 marks – correct shape & asymptotes & x-int.

1 mark – showing asymptotes correctly or odd function features

Multiple choice answers

1. A
2. B
3. D
4. B
5. D