BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 11

HALF YEARLY EXAMINATIONS

Mathematics Extension

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in
  Questions 6 – 8
- Marks may be deducted for careless or badly arranged work

Total marks – 50

Section I Pages 2 – 3

5 marks

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

Section II Pages 4 – 6

45 marks

- Attempt Questions 6 – 8
- Allow about 80 minutes for this section

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5

1 Which of the following is **FALSE** ?

(A) $\sin(90° - \alpha) = \cos\alpha$

(B) $\sin(90° + \alpha) = \cos\alpha$

(C) $\cos(90° - \alpha) = \sin\alpha$

(D) $\cos(90° + \alpha) = \sin\alpha$

2 Which expression is equal to ${}^{n}\mathbf{C}_{2}$?

(A) $\frac{n}{2}$

(B) $\frac{n^2 - n}{2}$

(C) $\frac{n^2 + n}{2}$

(D) n

3 How many solutions does the equation $x^{\frac{1}{3}} = |x - 2| - 3$ have ?

(A) 0

(B) 1

(C) 2

(D) 3

4 A test is administered with 15 questions.

Students are allowed to answer any 10 questions.

How many ways can a student get 9 out of 10 questions correct?

(A) 30030

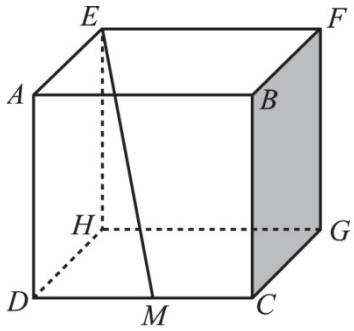
(B) 5005

(C) 3003

(D) 15

5 The cube below has sides four metres long.

M is the midpoint of *DC*.



The angle *EMH* is closest to

(A) 41.8°

(B) 48.2°

(C) 49.1°

(D) 54.7°

END OF SECTION I

Section II

45 marks
Attempt Questions 6 – 8
Allow about 80 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Marks

Question 6 ***(15 marks)*** Use a *separate* answer sheet

(a) Solve the inequation $\frac{x-2}{x+5} < 2$ *3*

(b) A class consists of 14 boys and 6 girls. *2*
4 students are selected. How many of these selections contain at least 2 girls?

(c) If $a = \frac{5-\sqrt{5}}{5+\sqrt{5}}$, evaluate, showing all working:

(i) $a + \frac{1}{a}$ *2*

(ii) $a^2 + \frac{1}{a^2}$ *2*

(d) 47271 is a five digit number whose digits sum to $4 + 7 + 2 + 7 + 1 = 21$. *3*

How many five digit numbers are there whose digits sum to 43?

(e) On a number plane, shade the region where (x,y) satisfies both of the inequalities *3*

$$y < \sqrt{16 - x^2} \quad \text{and} \quad y \leq x$$

Marks

Question 7 ***(15 marks)*** Use a *separate* answer sheet

(a) How many ways can 8 people be arranged in a line if:

(i) there are no restrictions? *1*

(ii) Lance and Vinuja must be together? *2*

(iii) Duvaraha must be at the beginning and Arpita must be at the end? *2*

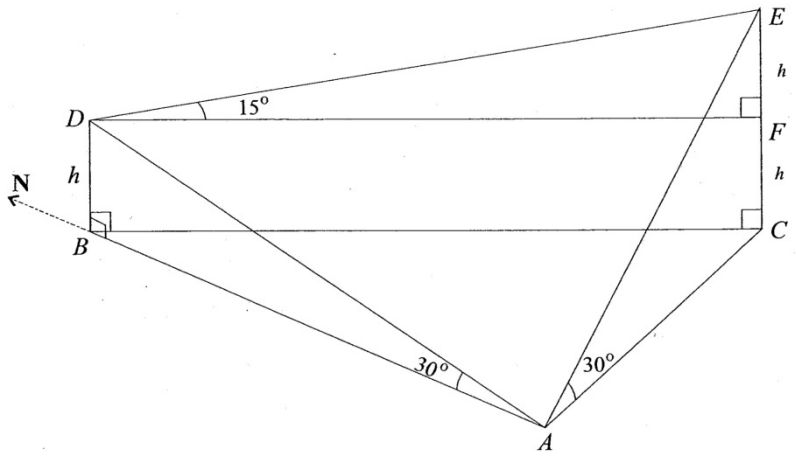
(b) Given that $3\sin^2 x + 2\sin x = 6\cos x + 9\sin x\cos x$ and $-90° < x < 90°$, find the possible values of $\tan x$. *3*

(c) How many distinct arrangements of all of the letters of the word **MISSISSIPPI**, are there if:

(i) there are no restrictions? *1*

(ii) all four I's do not come together? *2*

(d) The diagram below shows two vertical towers *BD* and *CE* of heights *h* and *2h* respectively, on a horizontal plane *ABC*.
Point *A* is due south of point *B*, and the angles of elevation of the tops of the towers from *A* are both $30°$.
The angle of elevation of *E* from *D* is $15°$



(i) Show that $BC = h\tan 75°$ *1*

(ii) Find similar expressions for both *AB* and *AC* *1*

(iii) Hence find the bearing of the taller tower from point *A* *2*

Marks

Question 8 ***(15 marks)*** Use a *separate* answer sheet

(a) 4 female and 4 male students are to be seated around a circular table. *2*
In how many ways can this be done if the males and females must alternate?

(b) Solve

(i) $\dfrac{x}{3x-4} \leq \dfrac{1}{x-1}$ *3*

(ii) $x^2 - |x| - 3 = 2x + |x|$ *3*

(c) In this question $f^2(x)$ denotes $f(f(x))$, $f^3(x)$ denotes $f(f(f(x)))$, and so on.

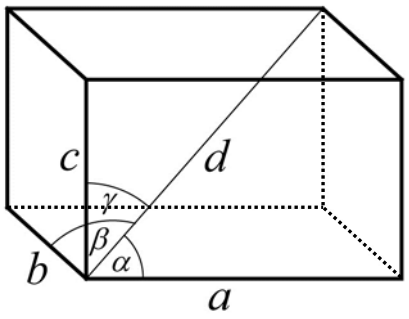
If $f(x) = \dfrac{x+\sqrt{3}}{1-x\sqrt{3}}$;

(i) find $f^2(x)$ in terms of x *1*

(ii) show that $f^3(x) = x$ *1*

(iii) find $f^{2016}(x)$ *1*

(d) A rectangular box has edges of lengths a, b and c units.
A diagonal of length d is drawn through the box between opposite corners as shown.
The three different angles between this diagonal and the three edges a, b and c of the box are labelled α, β and γ respectively.



(i) Express d in terms of a, b and c. *2*

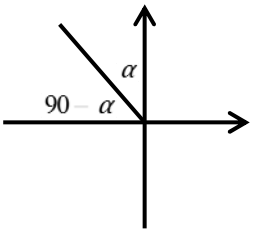
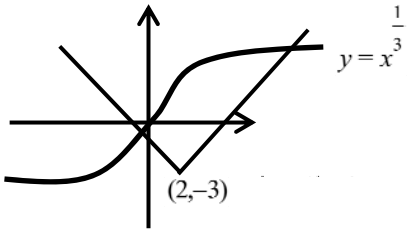
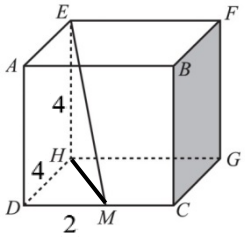
(ii) Hence, or otherwise, show that the angles α, β and γ satisfy the identity *2*

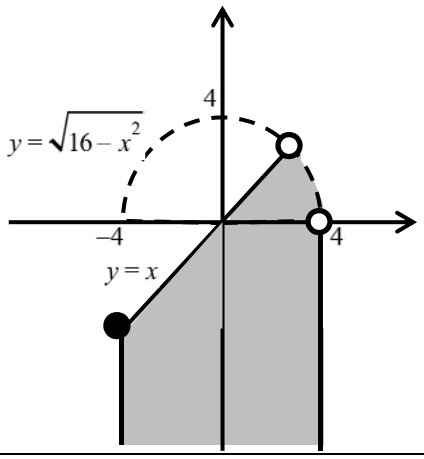
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

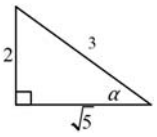
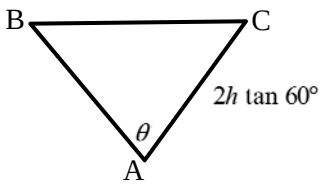
End of paper

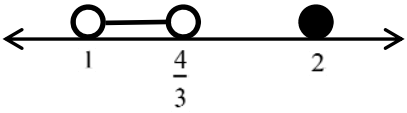
BAULKHAM HILLS HIGH SCHOOL

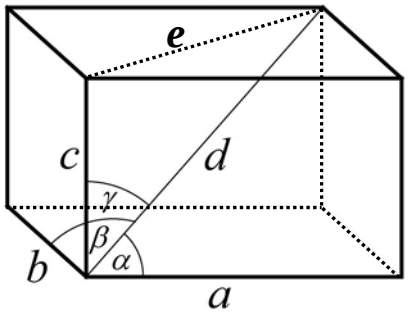
YEAR 11 MATHEMATICS EXTENSION HALF YEARLY EXAMINATION 2016 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. D - $\cos(90° + \alpha) = -\cos(90° - \alpha)$ $= -\sin\alpha$ $\neq \sin\alpha$ 	**1**	
2. B - ${}^{n}C_2 = \dfrac{n!}{2!(n-2)!}$ $= \dfrac{n(n-1)}{2}$ $= \dfrac{n^2 - n}{2}$	**1**	
3. C - 2 solutions 	**1**	
4. A - Ways $= {}^{15}C_{10} \times 10$ $= 3003 \times 10$ $= 30030$ *OR* let correct $= C$ wrong $= W$ blank $= B$ How many words can be made from CCCCCCCCCWBBBBB # Ways $= \dfrac{15!}{9!5!}$ $= 30030$	**1**	
5. A - $HM^2 = 2^2 + 4^2$ $HM = \sqrt{20}$ $\tan \angle EMH = \dfrac{4}{\sqrt{20}}$ $\angle EMH = 41.8°$ 	**1**	

Solution	Marks	Comments
SECTION II		
QUESTION 6		
6(a) (i) $\dfrac{x-2}{x+5} < 2$ $x+5 \neq 0$ $x \neq -5$ $x-2 = 2(x+5)$ $x-2 = 2x+10$ $x = -12$  $x < -12$ or $x > -5$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
6 (b) # selections $= {}^6C_2 \times {}^{14}C_2 + {}^6C_3 \times {}^{14}C_1 + {}^6C_4 \times {}^{14}C_0$ $= 1365 + 280 + 15$ $= 1660$	**2**	**2 marks** • Correct solution **1 mark** • Correctly calculates one case
6 (c) (i) $a + \dfrac{1}{a} = \dfrac{5-\sqrt{5}}{5+\sqrt{5}} + \dfrac{5+\sqrt{5}}{5-\sqrt{5}}$ $= \dfrac{(5-\sqrt{5})^2 + (5+\sqrt{5})^2}{(5+\sqrt{5})(5-\sqrt{5})}$ $= \dfrac{2\left(5^2 + (\sqrt{5})^2\right)}{5^2 - (\sqrt{5})^2}$ $= \dfrac{60}{20}$ $= 3$	**2**	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
6 (c) (ii) $a^2 + \dfrac{1}{a^2} = \left(a + \dfrac{1}{a}\right)^2 - 2 \times a \times \dfrac{1}{a}$ $= 3^2 - 2$ $= 7$	**2**	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
6(d) Only two possible combination of digits = 43 ***Case 1:*** $9+9+9+9+7 = 43$ Ways $= \dfrac{5!}{4!}$ $= 5$ ***Case 2:*** $9+9+9+8+8 = 43$ Ways $= \dfrac{5!}{3!2!}$ $= 10$ $\therefore$ total five digit numbers $= 5 + 10$ $= 15$	**3**	**3 marks** • Correctly solution **2 marks** • Identifies two correct cases • Correctly evaluates the number of possibilities in one case **1 mark** • Attempts to evaluate one of the correct cases
6(e)  	**3**	**3 marks** • Correct region making note of which boundaries and points of intersection are included **2 marks** • Both boundaries correct with only one region correct • Region correct, however boundary incorrectly identified **1 mark** • One boundary correctly identified

Solution	Marks	Comments
QUESTION 7		
7(a) (i) Ways $= 8!$ $= 40320$	**1**	**1 mark** • Correct answer
7(a) (ii) Ways $= 2! \times 7!$ $= 10080$	**2**	**2 marks** • Correct solution **1 mark** • Treats L & V as one object • Calculates # of arrangements of L & V
7(a) (iii) Ways $= 1 \times 1 \times 6!$ $= 720$	**2**	**2 marks** • Correct solution **1 mark** • Correctly handles restriction
7 (b) $3\sin^2 x + 2\sin x = 6\cos x + 9\sin x\cos x$ $3\sin^2 x + 2\sin x - 6\cos x - 9\sin x\cos x = 0$ $\sin x(3\sin x + 2) - 3\cos x(2 + 3\sin x) = 0$ $(3\sin x + 2)(\sin x - 3\cos x) = 0$  $\sin x = -\frac{2}{3}$ OR $\tan x = 3$ Quadrant 4 $\therefore \tan x = -\frac{2}{\sqrt{5}}$ $\therefore \tan x = -\frac{2}{\sqrt{5}}$ or $\tan x = 3$	**3**	**3 marks** • Correct solution **2 marks** • Finds one correct answer for tan$x$ **1 mark** • Correctly factorises the terms or equivalent merit
7(c) (i) Ways no restrictions $= \frac{11!}{4!4!2!}$ $= 34650$	**1**	**1 mark** • Correct answer
7(c) (ii) Ways I's together $= \frac{8!}{4!2!}$ $= 840$ Ways I's not together $= 34650 - 840$ $= 33510$	**2**	**2 marks** • Correct solution **1 mark** • Calculates # ways I's together • Uses the complementary event idea
7 (d) (i) $\frac{DF}{h} = \tan 75°$ $DF = h\tan 75°$ But BC=FD $\therefore BC = h\tan 75°$	**1**	**1 marks** • Correct solution
7 (d) (ii) $AB = h\tan 60°$ $AC = 2h\tan 60°$	**1**	**1 marks** • Correct answer
7 (d) (iii) $\cos\theta = \frac{h^2\tan^2 60° + 4h^2\tan^2 60° - h^2\tan^2 75°}{4h^2\tan^2 60°}$ $= \frac{5\tan^2 60° - \tan^2 75°}{4\tan^2 60°}$ $= \frac{15 - \tan^2 75°}{12}$ $= 0.89316.....$ $\theta = 84°53'$ $\therefore$ bearing of the tower is $N\ 85°\ E$  	**2**	**2 marks** • Correct solution **1 mark** • Correct substitution into the cosine rule

Solution	Marks	Comments
QUESTION 8		
8 (a) # Ways $= 3! \times 4!$ $= 144$	**2**	**2 marks** • Correct solution **1 mark** • Correctly deals with circle arrangement as opposed to line arrangement • Correctly deals with the alternation
8(b) (i) $\dfrac{x}{3x-4} \le \dfrac{1}{x-1}$ $3x-4 \neq 0 \quad x-1 \neq 0$ $x \neq \dfrac{4}{3} \quad x \neq 1$ $x(x-1) = 3x-4$ $x^2 - x = 3x-4$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $x = 2$  $1 < x < \dfrac{4}{3}$ or $x = 2$	**3**	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the three correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with both denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
8(b) (ii) $x^2 - \lvert x\rvert - 3 = 2x + \lvert x\rvert$ $2\lvert x\rvert = x^2 - 2x - 3$ $2x = x^2 - 2x - 3$ $x^2 - 4x - 3 = 0$ $x = \dfrac{4 \pm \sqrt{28}}{2}$ $x = 2 \pm \sqrt{7}$ however $x = 2 - \sqrt{7}$ is not a solution $\therefore \quad x = 2 + \sqrt{7}$ $-2x = x^2 - 2x - 3$ $x^2 - 3 = 0$ $x = \pm\sqrt{3}$ however $x = \sqrt{3}$ not a solution $\therefore \quad x = -\sqrt{3}$ $\therefore x = -\sqrt{3}$ or $x = 2 + \sqrt{7}$	**3**	**3 marks** • Correctly identifies the two correct answers **2 marks** • Finds one answer after rejecting its conjugate • Finds four answers including the correct two **1 mark** • Identifies two correct cases
8 (c) (i) $f^2(x) = \dfrac{\left(\dfrac{x+\sqrt{3}}{1-x\sqrt{3}}\right) + \sqrt{3}}{1 - \left(\dfrac{x+\sqrt{3}}{1-x\sqrt{3}}\right)\sqrt{3}}$ $= \dfrac{x + \sqrt{3} + \sqrt{3} - 3x}{1 - x\sqrt{3} - x\sqrt{3} - 3}$ $= \dfrac{-2x + 2\sqrt{3}}{-2 - 2x\sqrt{3}}$ $= \dfrac{x - \sqrt{3}}{1 + x\sqrt{3}}$	**1**	**1 marks** • Correct solution
8 (c) (ii) $f^3(x) = \dfrac{\left(\dfrac{x-\sqrt{3}}{1+x\sqrt{3}}\right) + \sqrt{3}}{1 - \sqrt{3}\left(\dfrac{x-\sqrt{3}}{1+x\sqrt{3}}\right)}$ $= \dfrac{x - \sqrt{3} + \sqrt{3} + 3x}{1 + x\sqrt{3} - x\sqrt{3} + 3}$ $= \dfrac{4x}{4}$ $= x$	**1**	**1 marks** • Correct solution
8 (c) (iii) As $f^3(x) = x$ then $f^4(x) = f(x),\ f^5(x) = f^2(x), f^6(x) = f^3(x) = x$, etc $\therefore f^{2016}(x) = f^3(x) = x$	**1**	**1 marks** • Correct solution

Solution	Marks	Comments
QUESTION 8…*continued*		
8 (d) (i)  $d^2 = e^2 + c^2$ $e^2 = a^2 + b^2$ $\therefore d^2 = a^2 + b^2 + c^2$ $d = \sqrt{a^2 + b^2 + c^2}$	**2**	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution
8 (d) (ii) $\cos\alpha = \frac{a}{d}$ Similarly $\cos\beta = \frac{b}{d}$ and $\cos\gamma = \frac{c}{d}$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{a^2}{d^2} + \frac{b^2}{d^2} + \frac{c^2}{d^2}$ $= \frac{a^2 + b^2 + c^2}{d^2}$ $= \frac{d^2}{d^2}$ $= 1$	**2**	**2 marks** • Correct solution **1 mark** • Progress towards a correct solution