Name: ______________________________

Teacher: ____________________________

Class: ______________________________

FORT STREET HIGH SCHOOL

2010

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics Extension I

TIME ALLOWED: 1 HOUR

PLUS 5 MINUTES READING TIME

Outcomes Assessed	Questions	Marks
Demonstrates the ability to manipulate and simplify numeric and algebraic expressions and solves problems involving equations	1	
Solves problems involving inequalities, indices and logs	2	
Uses appropriate techniques to solve problems involving plane and circle geometry	3	

Question	1	2	3	Total	%
Marks	/12	/16	/12	/40	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- <u>Each Question is to be started in a new booklet.</u>

Question 1. *(12 marks)*

a) i) Factorise $y^2 + 4y + 4$ **1**

ii) Factorise $y^3 + 8$ **1**

iii) Hence, simplify $\dfrac{y^3+8}{3y-6} \times \dfrac{y^2-4}{y^2+4y+4}$ **1**

b) Express $1 \cdot 3\dot{8}$ as a fraction in its simplest form. **2**

c) Solve $3x^2 - 5x - 3 = 0$, leaving you answer in exact form **2**

d) Show that $\dfrac{1}{3-\sqrt{2}} + \dfrac{1}{3+\sqrt{2}}$ is a rational number. **2**

f) Solve for *u*, *v* and *w*.

$$\begin{aligned} 3u + v - 4w &= -4 \\ u - 2v + 7w &= -7 \\ 4u + 3v - w &= 9 \end{aligned}$$

3

Question 2. *Start a separate booklet* *(16 marks)*

a) Solve the inequality $\frac{2x+1}{x+4} \geq 1$ **3**

b) i) sketch the graph of $y = |2x-2|$ **2**

ii) hence or otherwise solve $|2x-2| \leq |x-3|$ **2**

c) Simplify $\log_2 64 - \log_2 8$ **2**

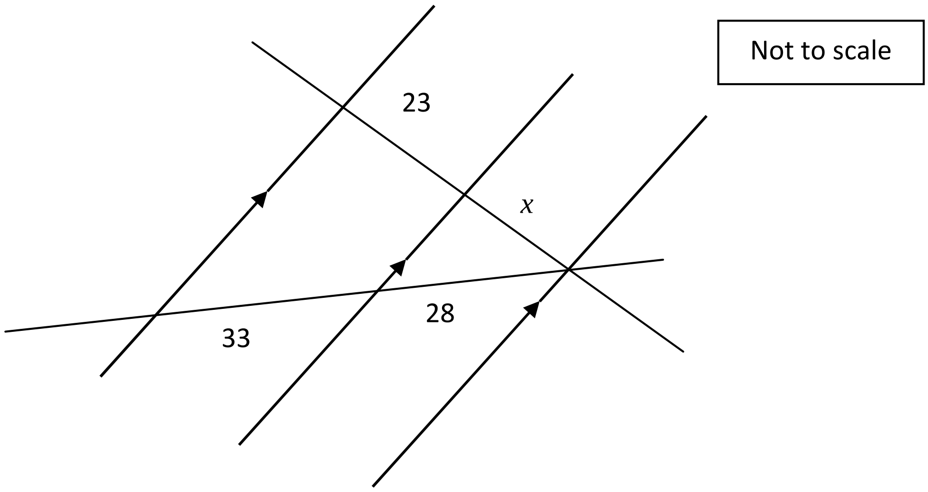
d) If $\log_5 7 = 1\cdot 21$ and $\log_5 2 = 0\cdot 43$, evaluate $\log_5 98$ **2**

e) Solve $3^{x-5} = 238$ correct to three significant figures **2**

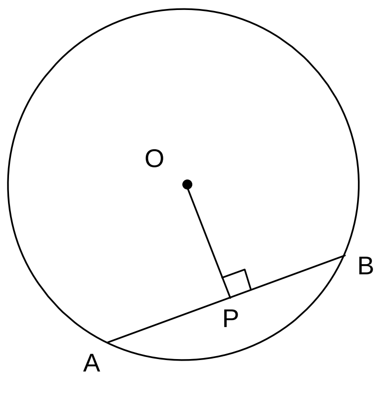
f) Solve $4^x - 9\left(2^x\right) + 8 = 0$ **3**

Question 3. *Start a separate booklet* *(12 marks)*

a) Find the value of x in the following diagram **2**



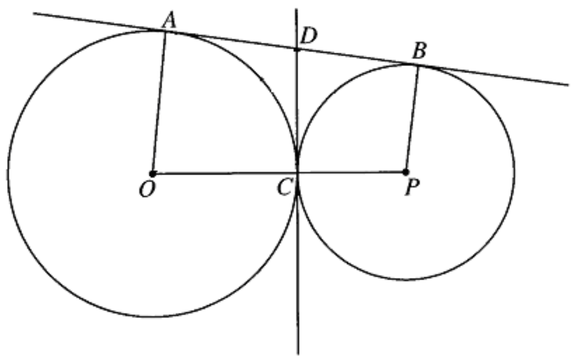
b) Prove the perpendicular from the centre of a circle to a chord bisects the chord. **3**



Question 3 continues on the next page.

c) Two circles, centres *O* and *P*, intersect at point *C* only.

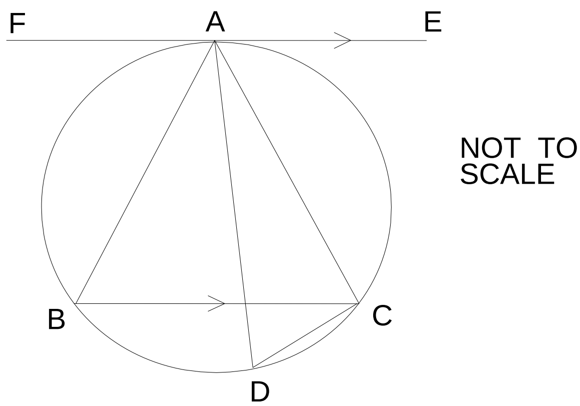
AB is a common tangent to the two circles which meets the tangent through *C* at *D*.



(i) Prove that *DA* = *DB* **2**

(ii) Prove that quadrilateral *AOCD* is cyclic. **2**

d)



In the diagram the points A, B, C and D lie on the circle, FAE is a tangent that touches the circle at A. FE is parallel to BC.

Let $\angle FAB = \alpha$.

(i) Explain why $\angle ACB = \alpha$ **1**

(ii) Hence prove that $\angle ACB = \angle ADC$ **2**

FORT STREET HIGH SCHOOL

2010

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics Extension I

TIME ALLOWED: 1 HOUR

PLUS 5 MINUTES READING TIME

Solutions

Question 1. *(12 marks)*

a) i) Factorise y^2+4y+4

Solution

$$y^2+4y+4=(y+2)^2$$

Marking guideline: **1 mark for correct response**

ii) Factorise y^3+8

Solution

$$y^3+8=(y+2)(y^2-2y+4)$$

Marking guideline: **1 mark for correct response**

iii) Hence, simplify $\frac{y^3+8}{3y-6}\times\frac{y^2-4}{y^2+4y+4}$

Solution

$$\begin{aligned}\frac{y^3+8}{3y-6}\times\frac{y^2-4}{y^2+4y+4}&=\frac{(y+2)(y^2-2y+4)}{3(y-2)}\times\frac{(y+2)(y-2)}{(y+2)^2}\\&=\frac{(y^2-2y+4)}{3}\times\frac{1}{1}\\&=\frac{(y^2-2y+4)}{3}\end{aligned}$$

Marking guideline: **1 mark for correct response**

b) Express $1 \cdot 3\dot{8}$ as a fraction in its simplest form. **2**

Solution

$$x = 1 \cdot 3888...$$
$$10x = 13 \cdot 888...$$
$$100x = 138 \cdot 888...$$

$$100x - 10x = 125$$
$$90x = 125$$
$$x = \frac{125}{90}$$

$$= \frac{25}{18} \quad or \quad 1\frac{7}{8}$$

Marking guideline:	**2 marks for correct response OR**
	1 mark for correct procedure but with arithmetic error

c) Solve $3x^2 - 5x - 3 = 0$, leaving you answer in exact form **2**

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-3)}}{2.3}$$

$$= \frac{5 \pm \sqrt{61}}{6}$$

Marking guideline:	**2 marks for correct response OR**
	1 mark for correct procedure but with arithmetic error OR
	1 mark for not answering in *exact* form.

d) Show that $\dfrac{1}{3-\sqrt{2}}+\dfrac{1}{3+\sqrt{2}}$ is a rational number. **2**

Solution

$$\frac{1}{3-\sqrt{2}}+\frac{1}{3+\sqrt{2}} = \frac{1}{3-\sqrt{2}}\times\frac{3+\sqrt{2}}{3+\sqrt{2}} + \frac{1}{3+\sqrt{2}}\times\frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3+\sqrt{2}}{9-2} + \frac{3-\sqrt{2}}{9-2}$$

$$= \frac{3+\sqrt{2} \ + \ 3+\sqrt{2}}{7}$$

$$= \frac{6}{7}$$

Which is a rational number.

Marking guideline: **2 marks for correct response OR**

1 mark for correct procedure but with arithmetic error

e) Solve for *u*, *v* and *w*.

$$\begin{array}{rcrcrcrl} 3u & + & v & - & 4w & = & -4 & (1) \\ u & - & 2v & + & 7w & = & -7 & (2) \\ 4u & + & 3v & - & w & = & 9 & (3) \end{array}$$

Solution

$(2)\times 3$	$3u$	$-$	$6v$	$+$	$21w$	$=$	-21	(4)
$(2)\times 4$	$4u$	$-$	$8v$	$+$	$28w$	$=$	-28	(5)
$(1)-(4)$			$7v$	$-$	$25w$	$=$	17	(6)
$(3)-(5)$			$11v$	$-$	$29w$	$=$	37	(7)
$(6)\times 11$			$77v$	$-$	$275w$	$=$	187	(8)
$(7)\times 7$			$77v$	$-$	$203w$	$=$	259	(9)
$(9)-(8)$					$72w$	$=$	72	
					w	$=$	1	

Substituting *w = 1* into the original equations

$$\begin{array}{lcccccl} & 3u & + & v & = & 0 & (1A) \\ & u & - & 2v & = & -14 & (2A) \end{array}$$

$$\begin{array}{lcccccl} (2)\times 3 & 3u & - & 6v & = & -42 & (10) \\ (1)-(10) & & & 7v & = & 42 & \\ & & & v & = & 6 & \end{array}$$

Substituting *w = 1* , *v= 6* into (1A)

$$\begin{array}{ccccc} 3u & + & 6 & = & 0 \\ & & 3u & = & -6 \\ & & u & = & -2 \end{array}$$

$$\therefore \quad u = -2, \quad v = 6, \quad w = 1$$

Marking guideline:	**3 marks for correct response OR**
	2 marks for correct procedure but with one arithmetic error OR
	1 mark for correct procedure with 2 or more arithmetic errors

Question 2. *Start a separate booklet* *(16 marks)*

a) Solve the inequality $\dfrac{2x+1}{x+4} \geq 1$

Solution

NOTE: $x \neq -4$

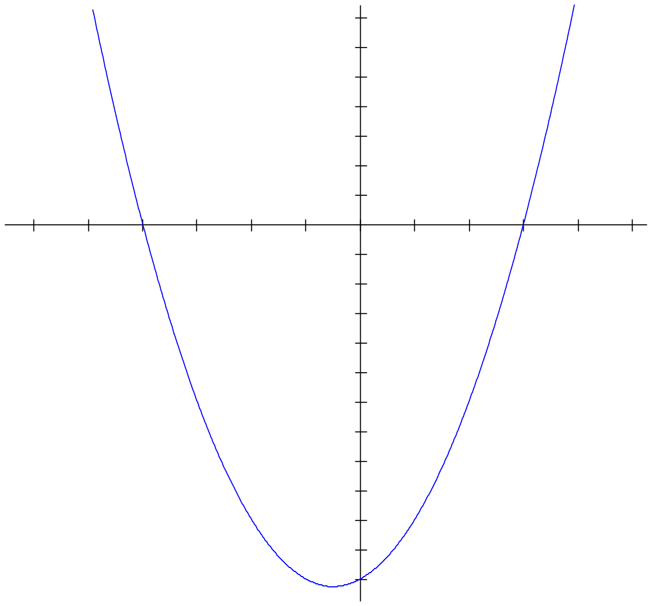
$$\frac{2x+1}{x+4} \geq 1$$

$$\frac{(2x+1)(x+4)^2}{x+4} \geq (x+4)^2$$

$$(2x+1)(x+4)-(x+4)^2 \geq 0$$

$$(x+4)\big((2x+1)-(x+4)\big) \geq 0$$

$$(x+4)(x-3) \geq 0$$



$\therefore \quad x < -4 \quad or \quad x \geq 3$

Marking guideline:

3 marks for correct response OR

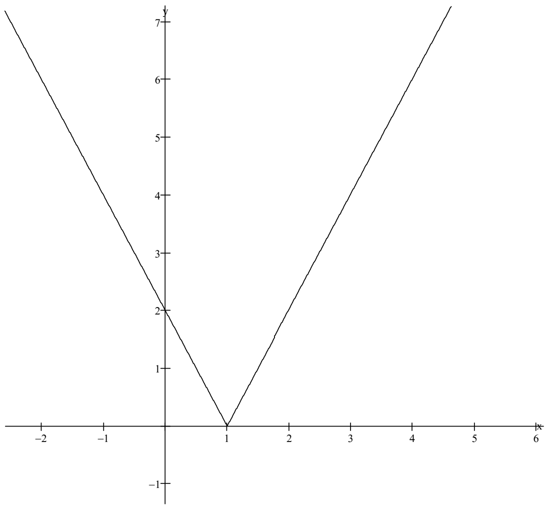
2 marks for correct procedure but with one arithmetic error OR

2 marks for $x \leq -4$ **in the solution and nor the correct** $x < -4$

1 mark for $x > -4 \;\; or \;\; x > 3$ **(i.e. neglects to graph or similar)**

b) i) sketch the graph of $y = |2x - 2|$ **2**

Solution



Marking guideline: **2 marks for correct response OR**

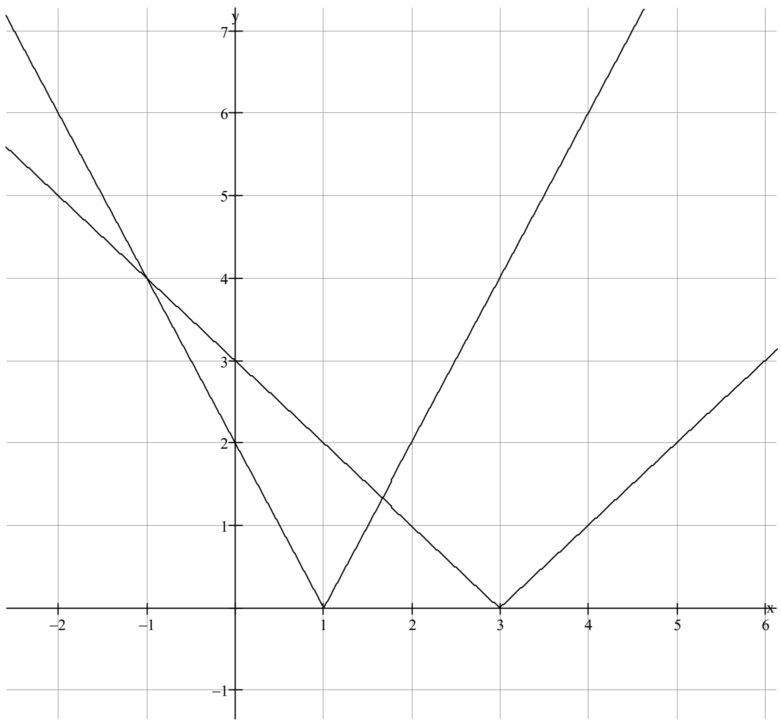
1 mark for correct shape with errors

ii) hence or otherwise solve $|2x - 2| \le |x - 3|$

Solution

Graph

$y = |x - 3|$



From the graph, the intersections occur when both branches of $y = |2x-2|$ cuts the negative branch of $y = |x-3|$.

That is, $y = |2x-2|$ intersects the left branch of $y = |x-3|$ i.e. the line $y = 3-x$

$$(2x-2) \le -(x-3)$$

$$2x-2 \le -x+3$$

$$3x \le 5$$

$$x \le \frac{5}{3}$$

And, the left branch of $y = |2x-2|$ i.e. the line $y = -2x+2$ intersects the left branch of $y = |x-3|$ i.e. the line $y = 3-x$

$$-(2x-2) \le -(x-3)$$

$$-2x+2 \le -x+3$$

$$-x \le 1$$

$$x \ge -1$$

So solution is $-1 \le x \le \frac{5}{3}$

Marking guideline:	**2 marks for correct response OR**
	1 mark for partial solution

c) Simplify $\log_2 64 - \log_2 8$

Solution

$$\log_2 64 - \log_2 8 = \log_2 \frac{64}{8}$$

$$= \log_2 8$$

$$= \log_2 2^3$$

$$= 3\log_2 2$$

$$= 3$$

Marking guideline: **2 marks for correct response OR**

1 mark for evidence of two logs laws

d) If $\log_5 7 = 1\cdot 21$ and $\log_5 2 = 0\cdot 43$, evaluate $\log_5 98$

Solution

$$\log_5 98 = \log_5 (49 \times 2)$$

$$= \log_5 49 + \log_5 2$$

$$= \log_5 7^2 + \log_5 2$$

$$= 2\log_5 7 + \log_5 2$$

$$= 2 \times 1\cdot 21 + 0\cdot 43$$

$$= 2\cdot 85$$

Marking guideline: **2 marks for correct response OR**

1 mark for evidence of two logs laws

e) Solve $3^{x-5} = 238$ correct to three significant figures

Solution

$$3^{x-5} = 238$$

$$\log_{10} 3^{x-5} = \log_{10} 238$$

$$(x-5)\log_{10} 3 = \log_{10} 238$$

$$x - 5 = \frac{\log_{10} 238}{\log_{10} 3}$$

$$x = 5 + \frac{\log_{10} 238}{\log_{10} 3}$$

$$= 9 \cdot 9810754...$$

$$= 9 \cdot 98 \quad \text{Correct to 3 sig. figs.}$$

Marking guideline:	**2 marks for correct response OR**
	1 mark for not correcting to 3 sig. figs.

f) Solve $4^x - 9\left(2^x\right) + 8 = 0$

Solution

Let $m = 2^x$

$$4^x - 9\left(2^x\right) + 8 = 0$$

$$\left(2^x\right)^2 - 9\left(2^x\right) + 8 = 0$$

$$m^2 - 9m + 8 = 0$$

$$(m-8)(m-1) = 0$$

$$m = 8 \;\; or \;\; 1$$

But $m = 2^x$, so

$2^x = 8$ OR $2^x = 1$

$x = 3$ $\quad$ $x = 0$

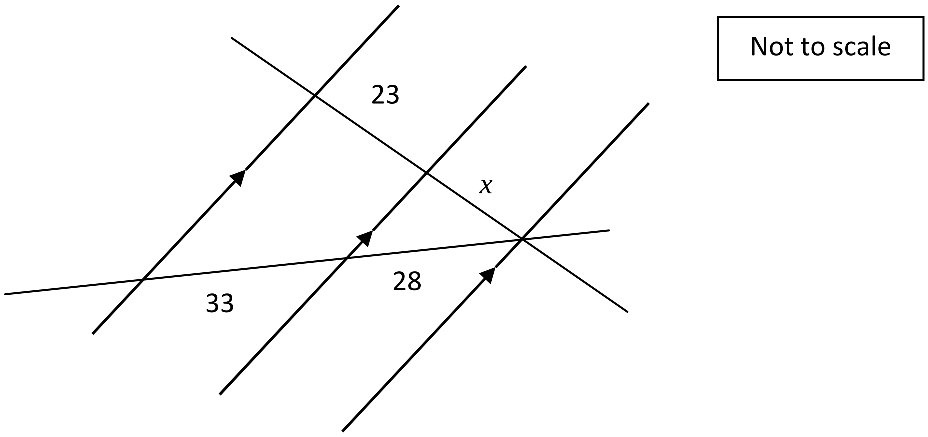
Marking guideline: **3 marks for correct response OR**

2 marks for correct procedure but with one arithmetic error OR

1 mark for finding $m = 8 \ \ or \ \ 1$

Question 3. *Start a separate booklet* *(12 marks)*

a) Find the value of x in the following diagram



Solution

$$\frac{x}{23} = \frac{28}{33}$$ Transversal cuts off intercepts in the same ratio

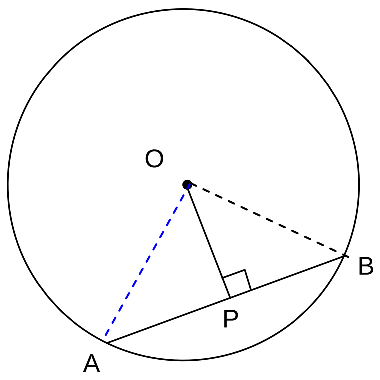
$$x = \frac{644}{33}$$

$$= 19\frac{17}{33}$$

Marking guideline: **2 marks for correct response OR**

1 mark correct procedure with one arithmetic error

b) Prove the perpendicular from the centre of a circle to a chord bisects the chord.



Solution

Construction: Join OA and OB

In ΔAOP and ΔBOP

$\angle APO = \angle BPO$	Given – perpendicular line OP & AB
OP	Common
$OA = OB$	Equal radii
$\therefore \Delta AOP \equiv \Delta BOP$	RHS

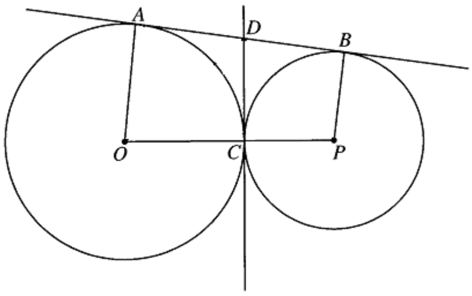
So, $AP = BP$ Corresponding sides of congruent triangles.

$\therefore$ AP bisects AB

Marking guideline:	**3 marks for correct response OR**
	2 marks for correct procedure with one missing reason OR
	2 marks for proving congruent triangles but not stating consequence OR
	1 mark for correct procedure but with two or more missing reasons.

c) Two circles, centres *O* and *P*, intersect at point *C* only.

AB is a common tangent to the two circles which meets the tangent through *C* at *D*.



(i) Prove that *DA* = *DB*

Solution

DA=DC [tangents from an external point are equal]

DB = DC [tangents from an external point are equal]

∴ DA = DB [both intervals are equivalent to DC]

Marking guideline: **2 marks for correct response OR**

1 mark for incorrect or no reasoning

(ii) Prove that quadrilateral *AOCD* is cyclic. **2**

Solution

OAD = 90 [radius meets a tangent at right angles]

OCD = 90 [radius meets a tangent at right angles]

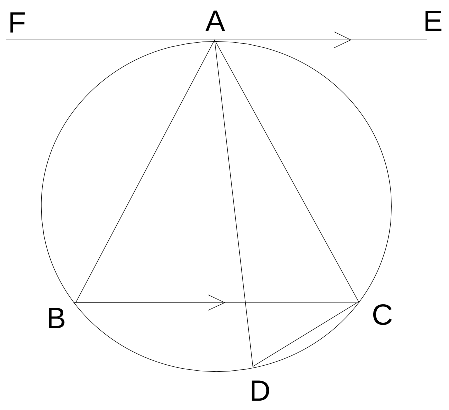
= 180

∴ OACD is a cyclic quadrilateral [opposite angles supplementary]

Marking guideline: **2 marks for correct response OR**

1 mark for lack of reasons

d)



In the diagram the points A, B, C and D lie on the circle, FAE is a tangent that touches the circle at A. FE is parallel to BC.

Let $\angle FAB = \alpha$.

(i) Explain why $\angle ACB = \alpha$

Solution

The angle between a chord and a tangent is equal to the angle subtended by the chord in the alternate segment.

Marking guideline: **1 mark for correct response**

(ii) Hence prove that $\angle ACB = \angle ADC$

Solution

FAB = ABC = **α** [Alternate angles of parallel lines]

ABC = ADC = α [Angles standing on the same arc subtended by the same chord are equal]

And $\angle ACB = \alpha$ [From above]

So $\angle ACB = \angle ADC$

Marking guideline: **2 marks for correct response OR**

1 mark for lack of reasons