Question 1 (15 marks)

a) Express 1.46 as a fraction in its simplest form. 2

b) Express $\frac{1+\sqrt{8}}{1+3\sqrt{2}}$ with a rational denominator in its simplest form. 2

c) Simplify $\frac{x^2-x-6}{2x^3+16} \div \frac{x^2-3x}{2x+4}$ 3

d) Solve for x:

$\frac{x+2}{2} = 4 - \frac{x-1}{3}$ 2

e) Solve the following equations simultaneously: 3

$y = x^3 + x^2$ and $y = x + 1$

f) Solve

$\frac{x}{2x-1} \geq 5$ 3

Question 2 (14 marks)

a) i) Simplify, expressing the answer with positive indices:

$\frac{2^n.4^{n+1}}{8^{n-2}}$ 2

ii) Solve for x:

$\frac{3.5^x-1}{5^x+2} = 2$ 2

iii) Find the value of x correct to 2 decimal places. 1

$2^{-x} = 5$

b) i) Evaluate $\log_4 3$, correct to 4 significant figures. 1

ii) Evaluate $\log_5 100 - \log_5 4$. 1

iii) Show that $3^{\log_3 9} = 9$ 1

c) Solve for x

$\log_2(2x-1) + \log_2(x-4) = 2$ 2

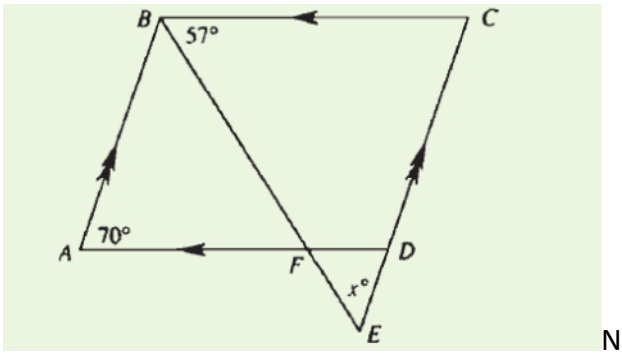
d) i) If $\log_{10} 2 = 0.3010$

Evaluate $\log_{10}(3.2)$ 2

ii) If $\log_a x = 1.8$ and $\log_a y = 2.4$, find $\log_a\left(\frac{a}{xy}\right)$ 2

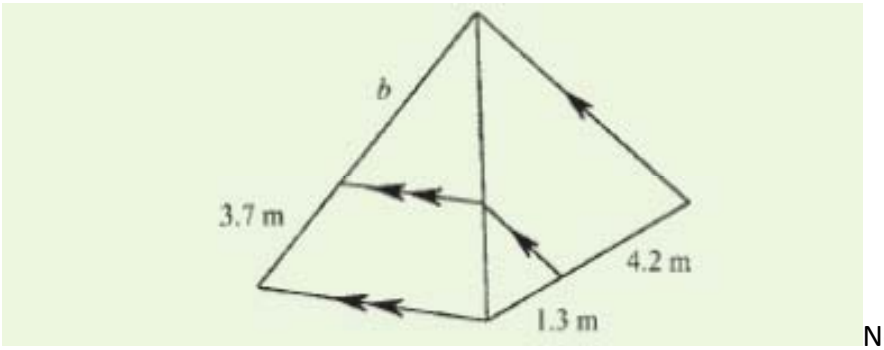
Question 3 (10 marks)

a) Copy the following diagram on to your answer sheet, and
Find the value of x, giving reasons for each step of your working out. 3



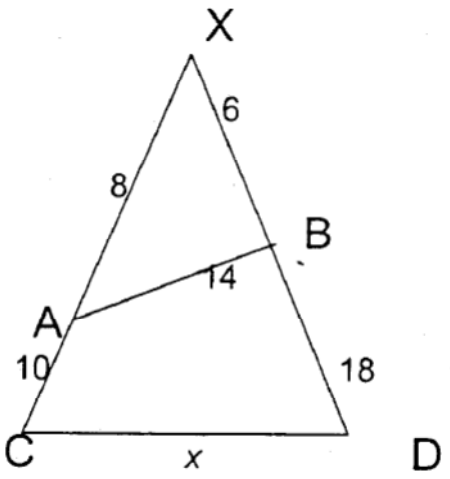
Not to scale

b) Copy the diagram on to your answer sheet and find the value of b.
(correct to 2 decimal places). 3



Not to scale

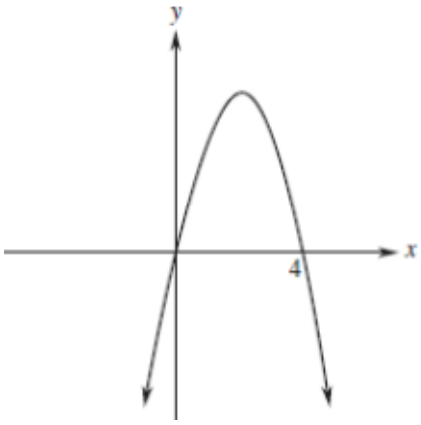
c) Prove that Δ AXB is similar to Δ CXD and hence find the value of x. 4



NOT TO SCALE

Question 4 (11 marks)

a) i) State whether the graph below is odd, even or neither. 1



ii) If $f(x) = \begin{cases} 2x & , x \geq 0 \\ x^2 - 3, & x < 0 \end{cases}$

find $f(5) - f(0) + f(-1)$ 1

iii) State the domain and range of $y = \sqrt{2x - 4}$ 2

iv) Show that $y = x^3 - x$ is an odd function. 1

b) i) On the same set of axes, sketch the graphs of $y = 2|x|$ and $y = |x + 3|$ 1

(note: your graph must be approxamitely 1/3 of a page in size and neatly presented)

ii) Find the point(s) of intersection of $y = 2|x|$ and $y = |x + 3|$ 2

c) Sketch the curve $y = 1 - \frac{4}{x^2 - 4}$ 3

END

Solutions – 2011 Task 1 – Ext 1 Preliminary

Comments

1 a) Let $x = 1.46464646\ldots$ (15)

$100x = 146.46464646\ldots$ ✓

$99x = 145$

$x = \frac{145}{99}$ ✓ $\left(1\frac{46}{99}\right)$

b) $\frac{(1+\sqrt{8})}{(1+3\sqrt{2})} \times \frac{(1-3\sqrt{2})}{(1-3\sqrt{2})}$

$= \frac{1-3\sqrt{2}+\sqrt{8}-3\sqrt{16}}{1-(9\times 2)}$ ✓ ① x by the conjugate and with a rational denominator

$= \frac{1-3\sqrt{2}+2\sqrt{2}-12}{-17}$

$= \frac{-11-\sqrt{2}}{-17}$

$= \frac{11+\sqrt{2}}{17}$ ✓ ① simplified answer

c) $\frac{x^2-x-6}{2x^3+16} \div \frac{x^2-3x}{2x+4}$

$= \frac{(x-3)(x+2)}{2(x^3+8)} \times \frac{2(x+2)}{x(x-3)}$ ✓ ① Partially factorised and simplified.

$= \frac{(x+2)(x+2)}{x(x+2)(x^2-2x+4)}$ ✓ ① factorising sum of two cubes

$= \frac{x+2}{x(x^2-2x+4)}$ ✓ $= \frac{(x+2)}{x(x-2)^2}$ ① Answer

①

d) $\left[\frac{x+2}{2} = 4 - \frac{x-1}{3}\right] \times 6$

$3(x+2) = 24 - 2(x-1)$ ✓

$3x + 6 = 24 - 2x + 2$

$5x = 20$

$x = 4$ ✓

e) $y = x^3 + x^2$ — ①

$y = x + 1$ — ②

② = ①

$x^3 + x^2 = x + 1$

$x^2(x+1) - (x+1) = 0$

$(x+1)(x^2 - 1) = 0$

$(x+1)(x-1)(x+1) = 0$

$x = 1, -1$ ✓

when $x = 1$

$y = 2$ $\therefore (1, 2)$ ✓

When $x = -1$

$y = 0$ $(-1, 0)$ ✓

f) $\frac{x}{2x-1} \geq 5$ (x b.s by $(2x-1)^2$)

$x(2x-1) \geq 5(2x-1)^2$

$5(2x-1)^2 - x(2x-1) \leq 0$

$(2x-1)(5(2x-1) - x) \leq 0$

$(2x-1)(9x-5) \leq 0$ ✓ working

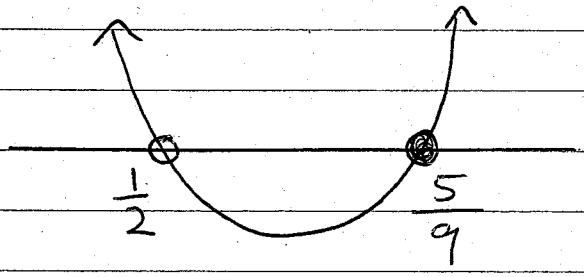
① Working

$(2x-1 \neq 0)$

$x \neq \frac{1}{2}$

① values for x

① correct expression



$\frac{1}{2} < x \leq \frac{5}{9}$ ✓ values

✓ expression

2a) i) $\dfrac{2^n \cdot 4^n \cdot 4}{8^{n-2}}$ (14)

$= \dfrac{2^n \cdot 2^{2n} \cdot 2^2}{2^{3(n-2)}}$ ✓

$= 2^{3n-3n} \cdot 2^{2+6}$

$= 2^8$ ✓ (256)

ii) $\dfrac{3 \cdot 5^x - 1 = 2}{(5^x + 2)}$ ↗

$3 \cdot (5^x) - 1 = 2 \cdot (5^x) + 4$ ✓

$5^x = 5^1$

$x = 1$ ✓

iii) $2^{-x} = 5$

$-x = \dfrac{\ln 5}{\ln 2}$

$x = -2.32$ ✓

b i) $\log_4 3 = \dfrac{\ln 3}{\ln 4}$

$= 0.7925$ ✓

ii) $\log_5 100 - \log_5 4 = \log_5 \left(\dfrac{100}{4}\right)$

$= \log_5 25 = \log_5 5^2$

$= 2$ ✓ (4)

iii) To show that $3^{\log_3 9} = 9$

$= 3^{\log_3 3^2}$

$= 3^{2\log_3 3} = 3^2$ ✓

① Answer with correct working.

c) $\log_2(2x-1) + \log_2(x-4) = 2$

$(2x-1)(x-4) = 2^2$ ✓

$2x^2 - 8x - x + 4 = 4$

$2x^2 - 9x = 0$

$x(2x-9) = 0$

$x \neq 0 \quad \therefore x = \frac{9}{2}$ ✓

d i) $\log_{10} 3.2 = \log_{10}\left(\frac{32}{10}\right)$

$= \log_{10}\left(\frac{2^5}{10}\right)$ ✓

$= 5\log_{10} 2 - \log_{10} 10$

$= 5(0.3010) - 1$

$= 0.505$ ✓

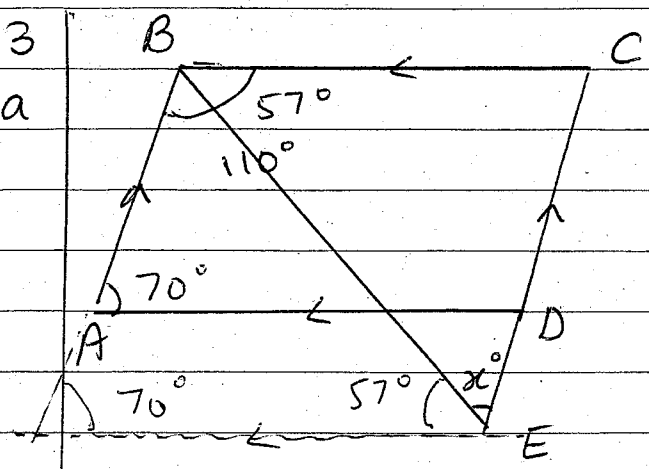
ii) $\log_a\left(\frac{a}{xy}\right) = \log_a a - [\log_a x + \log_a y]$ ✓

$= 1 - (1.8 + 2.4)$

$= -3.2$ ✓

3 (10)

a



$\angle ABC = 110°$ (Cointerior to $\angle BAD$) ✓

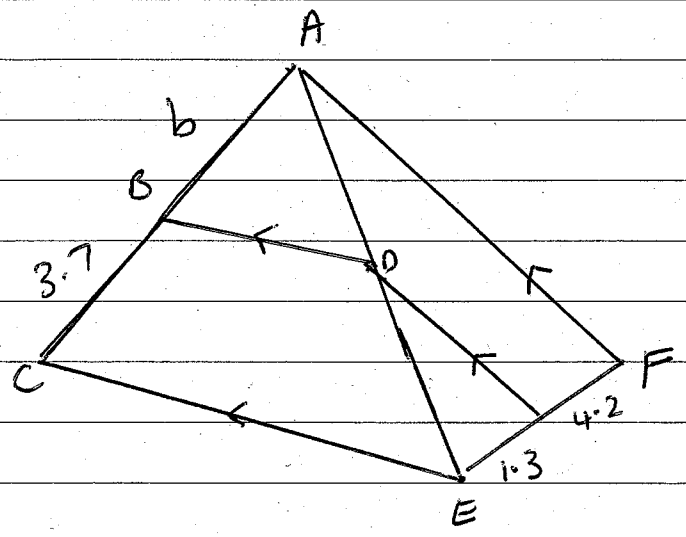
① working
① Answer
① Reason

$\angle ABE = 110° - 57°$

$= 53°$ ✓

$\therefore x° = 53°$ (alternate to $\angle ABE$) ✓

b



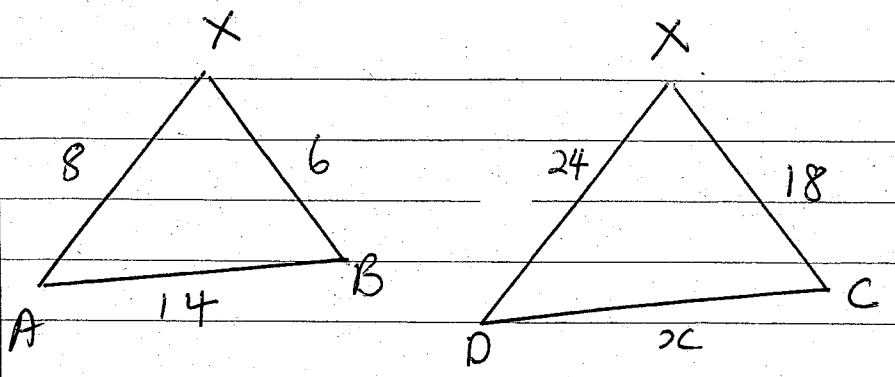
$$\frac{b}{3.7} = \frac{AD}{DE} = \frac{4.2}{1.3}$$ ✓

$$\frac{b}{3.7} = \frac{4.2}{1.3}$$ ✓

$$b = \frac{3.7 \times 4.2}{1.3} = 11.95 \text{ m}$$ ✓

(6)

c)



$\angle X$ is common

$\frac{AX}{DX} = \frac{8}{24} = \frac{1}{3}$ ✓ ① Ratio

$\frac{BX}{CX} = \frac{6}{18} = \frac{1}{3}$ ✓ ① Ratio

✓ ① Reason

$\therefore \Delta AXB \,|||\, \Delta CXD$ (corresponding sides have the same ratio. ① Answer

$\therefore \frac{14}{x} = \frac{1}{3}$

$x = 3 \times 14$

$= 42$ ✓

(11)

4 a

i) Neither ✓

ii) $f(5) - f(0) + f(-1)$

$= 10 - 0 + 1 - 3$

$= 8$ ✓

iii) $y = \sqrt{2x-4}$

$2x - 4 \geq 0$

$x \geq 2$

D: $x \geq 2$ ✓

R: $y \geq 0$ ✓

iv $y = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

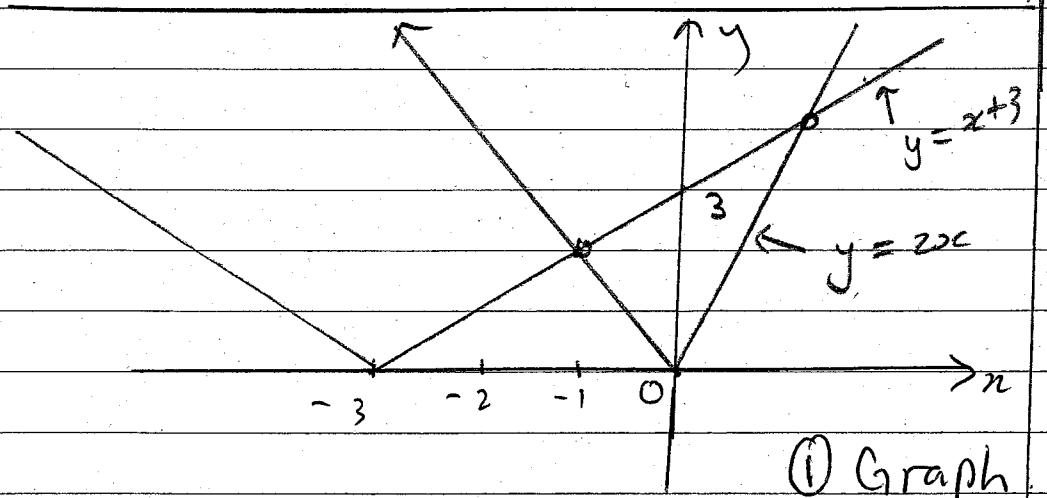
$= -x^3 + x$

$= -(x^3 - x)$ ✓

$= -f(x)$

$\therefore f(-x) = -f(x)$, hence an odd function.

b i)



① Graph

Intersection:

$2x = x + 3$

$x = 3$

$y = 6$

$\therefore (3, 6)$ ✓

$-2x = (x + 3)$

$-3x = 3$

$x = -1$

$y = -1 + 3$

$= 2$

$\therefore (-1, 2)$ ✓

c) $y = 1 - \dfrac{4}{x^2 - 4}$

$y = 1 - \dfrac{4}{(x-2)(x+2)}$

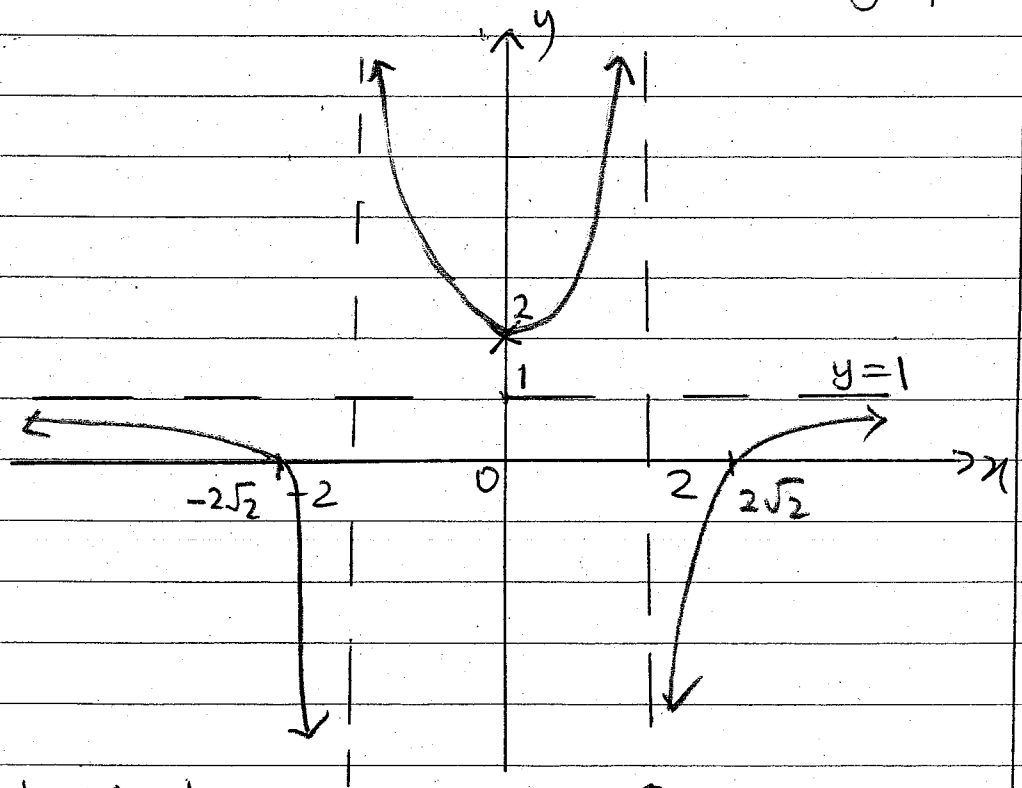
$x \neq 2, -2$

∴ Vertical asymptotes : $x = 2, -2$

$\dfrac{4}{(x-2)(x+2)} = 1 - y$

$4 = (1-y)(x-2)(x+2)$

∴ $y \neq 1$, hence $y = 1$ is the horizontal asymptote



① Shape
① asymptotes
① Intercepts

Intercepts:

$x = 0$

$y = 2$

$y = 0$

$x^2 - 4 = 4$

$x^2 = 8$

$x = \pm 2\sqrt{2}$