Girraween High School **Mathematics Extension**

Year 11 Task 1 (Half Yearly) April, 2003 Time: 80 min

Instructions:

1. Attempt all questions.

2. Write your answers on your own paper.

3. All necessary working must be shown.

4. Marks will be deducted for careless or badly arranged work.

Question 1 (19 marks)

Solve a)

(i) $\frac{x-3}{2} - \frac{x+1}{4} = 5$

(ii) $\frac{3}{x^2} = 2$

4

b)

Solve $\frac{1}{1-x} > 2$

3

Factorise $\frac{x^3-1}{x^2-1}$ c)

3

Solve for $0^0 \le \theta \le 360^0$: d)

(i)

 $\sqrt{3}\sin\theta = \cos\theta$ (ii) $\sin(\theta - 30)^0 = \frac{\sqrt{3}}{2}$ (iii) $2\cos^2\theta - \cos\theta = 1$

9

Question 2 (23 marks)

If $f(x) = 1 - 2x^2$, find: a)

(i) f(2) (ii) f(-x)

(iii) f(x+1) 4

b) What is the domain of each of these functions:

> $y = x^2 + 2$ (i)

(ii) $y = \frac{1}{x}$

(iii) $y = \sqrt{x-1}$

(iv) $y = \sqrt{x^2 - 9}$ (v) $y = \sqrt{4 - x^2}$

(vi) $y = \frac{1}{\sqrt{1-2x}}$

If $f(x) = x^2$ = x+1 x > 0 $x \le 0$ c)

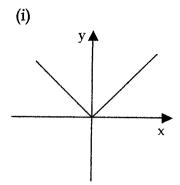
> evaluate f(-2) + f(2)(i)

2

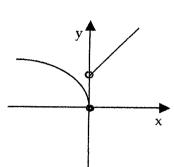
(ii) find the domain and range of the function. 2

d) State whether these curves represent functions:

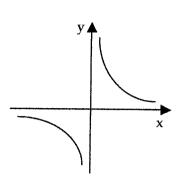
3



(ii)



(iii)



Question 3 (21 marks)

- a) Evaluate:
- (i) 10!
- (ii) $\frac{6!}{3!}$
- (iii) $\frac{100!}{98!}$

4

- b) How many ways can 4 boys and 3 girls be arranged in a line if:
 - (i) there are no restrictions?

1

(ii) the boys are together?

2

(iii) the boys and girls alternate?

- 2
- c) How many ways can 5 different English books, 4 different Science books and 2 different History books be arranged on a shelf if the books in each subject are to be together?
- d) How many ways can the letters of the word SUCCESS be arranged if:
 - (i) there are no restrictions?

2

(ii) the C's must be together?

- 2
- e) There are two small rows of chairs, with 3 in the front row and 4 in the second row. How can 7 people be seated if:
 - (i) 2 particular people must sit in the second row.

2

(ii) Jan will not sit on the same row as Norman and Stan must sit in the first row.

3

Question 4 (16 marks)

- a) A debating team of 4 students is to be selected from 10 students. How many team combinations are possible if:
 - (i) there are no restrictions?

1

(ii) Talitha must be included in the team?

2

c) A standard pack of 52 cards has 4 suites: Hearts, Diamond, Clubs and Spades Each suite has 13 cards: Ace, King, Queen, Jack, 10,9,8,7,6,5,4,3,2

Five cards are selected at random from a standard pack of 52 cards.

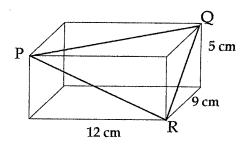
- How many different selections of 5 cards are possible? 1
- (ii) What is the probability that the 5 cards selected are hearts? 2 (iii) What is the probability that 3 kings are in the selected 5 cards? 3
- In a conference room there is a round table surrounded by 10 chairs. The group d) attending the conference consists of 7 women and 3 men.
 - (i) How many seating arrangements are possible if there are no restrictions? 1
 - (ii) How many seating arrangements are possible if the 3 men sit together? 2

Two of the people are elected to attend the national conference. What is the probability that they:

- (iii) sit next to each other?
- (iv) sit opposite each other?

Question 5 (21 marks)

a) For this rectangular prism, find the size of ∠PQR, correct to the nearest degree.



5

3

2

2

- b) A vertical tower AB stands on level ground with A being the top point. Points C and D are at ground level. The angle of elevation of A from C is 25° and the angle of elevation of A from D is 30° and $\angle CBD = 60^{\circ}$. Let the height of the tower AB be h.
 - (i) Draw a sketch showing all this information. 2
 - (ii) Show $CB = h \cot 25^{\circ}$ 2
 - Show $CD^2 = h^2 \cot^2 25^0 + h^2 \cot^2 30^0 2h^2 \cot 25^0 \cot 30^0 \cos 60^0$ (iii) 3
 - Find the height h, correct to 1 decimal place, if the distance CD=50 m (iv)
- Prove: c)

(i)
$$\frac{\cot\theta\cos\theta}{\cot\theta+\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$
 3

 $\frac{\sin^2 \theta}{1 + \cos \theta} + \frac{\sin^2 \theta}{1 - \cos \theta}$ is independent of θ . 3

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-	Solutions Maths Extension	y Yrll Task 1 2003	
	Sugar Land	1 1 2 2	
	Question 1. (19)	d) i) \3 sm0 = w0	
	a) i)	$+ an \Theta = \frac{1}{\sqrt{3}}$	
	$\frac{x-3}{2} = \frac{x+1}{11} = 5$		
-	2 4	0 = 30° or 210° (3)	
	2x-6-(x+1)=20		
	2x-6-x-1=20	\vec{a}) $6-30^{\circ} = 60^{\circ} \text{ or } 120^{\circ}$	
	x-7 = 20	8 = 90° or 150°	
	x = 27 (2)		
		.'. Solution is 90° or 150°	
	$\frac{3}{\sqrt{2}} = 2$	3	
	x2		
	$x^2 = \frac{3}{2}$	$iii) 2\cos^2\theta - \cos\theta = 1$	
		$2\cos^2\theta - \cos\theta - (=0)$	
	$x = \pm \sqrt{\frac{3}{2}} \qquad 2$	(2co+1)(co+1)=0	
ara saar-ar-aan diikan kankan sa mar kee k Anna.			
	b) $\frac{1}{1-x} \geq 2$	1 COD = - 1 or 1	
	1-2 0		
	Discontinuous at x=1	.'. 0 = 120° 240° 0° 360°	
	Solve L = 2	(3)	
	1 = 2-226		
	2x = 1	Question 2. (23)	
		Chestion 2. (23)	
	x = \frac{1}{2}		
	Check points	$a) f(x) = 1 - 2x^2$	
ada al de la composição d	0 1	$\downarrow i) f(2) = -7 \qquad \bigcirc$	
	2		
	$\frac{1}{2}$ 4 \times 4 3	\ddot{u} $f(-x) = 1 - 2(-x)^2$	
		$= 1-2x^2$ (1)	
	e) $x^3 - 1$ $(x - 1)(x^2 + x + 1)$		
	$\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)}$	\tilde{u}) $f(x+1) = 1-2(x+1)^2$	
1	$= x^2 + x + 1$	$= (-2(x^2+2x+i))$	
	$\frac{17171}{x+1}$ 3		
		$\frac{2 - 2x^2 - 4x - 1}{2}$	

	b) i) $y = x^2 + 2$	Question 3 (21)	
_	Domain allreal x 3	a) i) $10! = 3628800$	0_
-		61	
-	ii) $y = \frac{1}{x}$ Domain: all real x, $x \neq 0$	$\frac{11}{3!} = 6 \times 5 \times 4$	~
-	Domain: all real x, x = 0	= 120	0
-	$ \begin{array}{ccc} & \text{iii} & y = \sqrt{x-1} \\ & \text{Domain: } x - 1 > 0 \end{array} $	$\frac{1001}{a21} = \frac{100 \times 99}{}$:
	Domain: X-120	= 9900	(2)
	× > (②	b) i) 7! = 5040	$\overline{\mathbb{O}}$
			<u> </u>
	$iv) y = \sqrt{x^2 - q}$	ii) boys together	
	Domain . x 7,3 , x 6-3	4! x4! = 576	2
	(2)	iii) boys a girls alternate	
	$y = \sqrt{4-x^2}$	вававав	
-	Domain -2 < x < 2	4! x 3! = 144	2
		c) 5E 4S 2H	
	Domain $x < \frac{1}{2}$	(5! x 4! x 2!) x 3! = 34560	(3)
*****	Domain 252	- 34380	ري
	(c) i) f(-2) + f(2)	d) success	
	2.		
	$= (-2+1) + 2^{-1}$ = 3	$\frac{1}{3} + \frac{1}{3} = 420$	2
	ii) Domain all real or		
	Range all really (2)	ii) CC together.	-databa ang talang kanada ya gila at-
		6,	
	d) all curves represent Sunctions	6! = 120	
	functions 3	e) i) (4×3) ×5!	
		= 1440	
		- Itheo	<u>(2)</u>
	(and an angular of the second and an angular and an analysis and an	

		y		
	3	(·44
				LHS
ii) I	d) i) 91 = 362880 ①		(b) A	c) i) coto coso
<u>5</u> N	1) 1) 1,		<i>i</i>)	cote + core
T N S	ü) 7! ×3! = 30240 (2)			COS Q 600 Q
(4×3×2)×4!	10,100	When here yet a series of the	h	= SMG
24 × 41 = 576	iii) P(next+0 8!x2		B (2)	coso coo (x sino)
but Jan and Normatica			600	sno
rows 24576	$= \frac{2}{2} \qquad (2)$			= ces²6
= 1152	3 or, if one person is seated	and the second s	C	COO 8+ COO 85 in 8
C N J Brid	the probability that the person	Managar and discount of the second of the se	ii) In A ABC	cos ² O
Quadinu (10)	is next to her is 3	- Marie de Arigina de La Carte	$ton 25° = \frac{h}{eB}$	= (200 (1+5120)
Question 4 (16)	IS NEXT to her is a	Andreas and the second	F B	= 000
$\frac{a}{2}$ $\frac{10}{10}$ $\frac{10}{10}$ $\frac{10}{10}$ $\frac{10}{10}$ $\frac{10}{10}$ $\frac{10}{10}$	1 iii) Proposite each 1 (2)	haddadanan latera) bet emerika dipatat da artip dakan	$CB = \frac{h}{\tan 25^\circ}$	1+sme = RHS (3)
0 4 = 210	O iii) P(opposite each 2 2)	The second secon	Tunko	
\vec{u}) T+3others = $^{9}C_{3}$			CB = h cot 25° (2)	ii) sm²0 sm²0
= 84		political de l'estables de description de l'estables de l'		1+000 + 1-000
7 24		And the state of t	iii) Similarly BD = h cot 30°	
i) 52 c5 = 259891	Question 5 (21)	To and an analysis of the Contract of the Cont		= 5m20(1-cose) + sun20(1+cose)
11 '		militaria deplication de la companya del companya del companya de la companya de	Using the cosine rule in	(1+000)(1-000)
$ii) f(Hearts) = \frac{18c5}{52c_5}$	$\rho R^2 = 12^2 + 5^2$ $\rho^2 = 12^2 + 9^2$	punti addirent annes, mengenge yip empere at dagah	A CBD:	sur 0 - sur g cos 0 + sur 0 + sur gc
MIL((lear15) - 52c5	$PR^{2} = 169 PQ^{2} = 225$	The special section is a second section of the State of t		- I-co'0
33	$QR^2 = 5^2 + q^2$		$CD^{2} = h^{2} \cot^{2} 25 + h^{2} \cot^{2} 30^{\circ}$	
= 66640	QR = 5 +9 QR = 106	da.mir arvandas pandy	-2 · hcot 25 · h cot 30 cos6	$\frac{2\sin^2 6}{\sin^2 6}$
or ≟ 0.000495	(2)	nagenesis and the same of the		= 2 which is independent
= 0,0004-95		generalise Marjon (in a mass) massivi filish sadis di ti	:. CD = h2 cot25° + h2 cot230°	of 6 (3)
$ \tilde{u} P(3K_{-}) = 4c_3$	In Δ PQR, cosine rule.	pyningung kananalas kananan mengemakan	- 2h2 cot25° cot 30° cos60°	
		pajosilisalimindironiki Hillimindirol	3	
7			(v) $cD = h^2 \left[\omega t^2 + \omega t^2 \right]$	
	qu 2xPQ xQR	and placements of the second s	-2 cot 25° cot 30° cos	41
11	54145 = 225 + 106 - 169 (3) 2×15×106	A A STATE OF THE S	2500 = h2 x [13.8845]	3)
or	Control of the contro	. Springer with a majority and a maj	643.6 = h2	
= 0.001736			h = 25.3688	
	<u> </u>	to similar of the sign of the	= 25.4 3	
		printerna pharter and the factor of the fact		