Girraween High School

Mathematics Extension

Year 11 — April, 2003

Task 1 (Half Yearly) — Time: 80 min

Instructions:
1. Attempt all questions.
2. Write your answers on your own paper.
3. All necessary working must be shown.
4. Marks will be deducted for careless or badly arranged work.

Question 1 (19 marks)

a) Solve (i) $\frac{x-3}{2}-\frac{x+1}{4}=5$ (ii) $\frac{3}{x^2}=2$ **4**

b) Solve $\frac{1}{1-x}>2$ **3**

c) Factorise $\frac{x^3-1}{x^2-1}$ **3**

d) Solve for $0^0 \le \theta \le 360^0$:

(i) $\sqrt{3}\sin\theta=\cos\theta$ (ii) $\sin(\theta-30)^0=\frac{\sqrt{3}}{2}$ (iii) $2\cos^2\theta-\cos\theta=1$ **9**

Question 2 (23 marks)

a) If $f(x)=1-2x^2$, find:
(i) f(2) (ii) f(-x) (iii) f(x+1) **4**

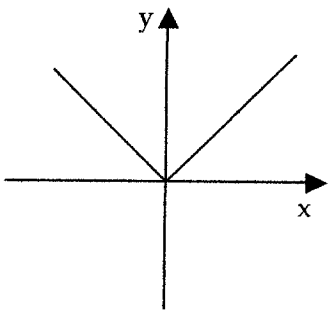
b) What is the domain of each of these functions:
(i) $y=x^2+2$ (ii) $y=\frac{1}{x}$ (iii) $y=\sqrt{x-1}$ **6**
(iv) $y=\sqrt{x^2-9}$ (v) $y=\sqrt{4-x^2}$ (vi) $y=\frac{1}{\sqrt{1-2x}}$ **6**

c) If $f(x)=x^2$ for $x>0$; $f(x)=x+1$ for $x\le 0$

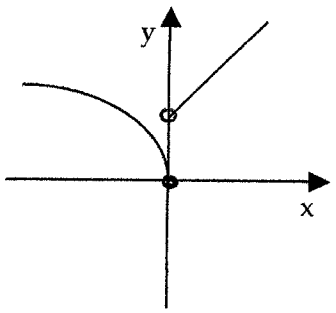
(i) evaluate $f(-2)+f(2)$ **2**
(ii) find the domain and range of the function. **2**

d) State whether these curves represent functions: **3**

(i)



(ii)



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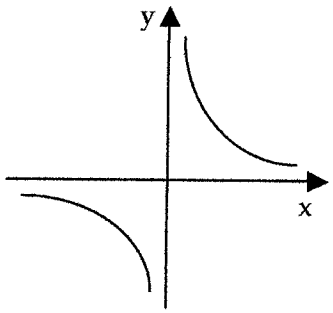


Question 3 (21 marks)

a) Evaluate: (i) $10!$ (ii) $\frac{6!}{3!}$ (iii) $\frac{100!}{98!}$ **4**

b) How many ways can 4 boys and 3 girls be arranged in a line if:
(i) there are no restrictions? **1**
(ii) the boys are together? **2**
(iii) the boys and girls alternate? **2**

c) How many ways can 5 different English books, 4 different Science books and 2 different History books be arranged on a shelf if the books in each subject are to be together? **3**

d) How many ways can the letters of the word SUCCESS be arranged if:
(i) there are no restrictions? **2**
(ii) the C's must be together? **2**

e) There are two small rows of chairs, with 3 in the front row and 4 in the second row. How can 7 people be seated if:
(i) 2 particular people must sit in the second row. **2**
(ii) Jan will not sit on the same row as Norman and Stan must sit in the first row. **3**

Question 4 (16 marks)

a) A debating team of 4 students is to be selected from 10 students. How many team combinations are possible if:
(i) there are no restrictions? **1**
(ii) Talitha must be included in the team? **2**

c) *A standard pack of 52 cards has 4 suites: Hearts, Diamond, Clubs and Spades*
Each suite has 13 cards: Ace, King, Queen, Jack, 10,9,8,7,6,5,4,3,2
Five cards are selected at random from a standard pack of 52 cards.

(i) How many different selections of 5 cards are possible? **1**

(ii) What is the probability that the 5 cards selected are hearts? **2**

(iii) What is the probability that 3 kings are in the selected 5 cards? **3**

d) In a conference room there is a round table surrounded by 10 chairs. The group attending the conference consists of 7 women and 3 men.

(i) How many seating arrangements are possible if there are no restrictions? **1**

(ii) How many seating arrangements are possible if the 3 men sit together? **2**

Two of the people are elected to attend the national conference. What is the probability that they:

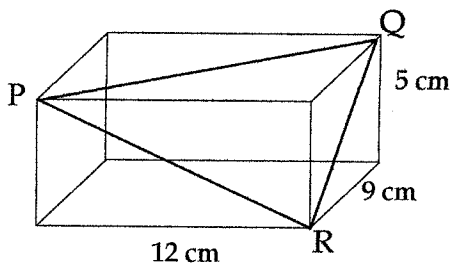
(iii) sit next to each other? **2**

(iv) sit opposite each other? **2**

Question 5 (21 marks)

a) For this rectangular prism, find the size of

$\angle PQR$, correct to the nearest degree.



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b) A vertical tower AB stands on level ground with A being the top point. Points C and D are at ground level. The angle of elevation of A from C is 25^0 and the angle of elevation of A from D is 30^0 and $\angle CBD = 60^0$. Let the height of the tower AB be h.

(i) Draw a sketch showing all this information. **2**

(ii) Show $CB = h\cot 25^o$ **2**

(iii) Show $CD^2 = h^2\cot^2 25^0 + h^2\cot^2 30^0 - 2h^2\cot 25^0\cot 30^0\cos 60^0$ **3**

(iv) Find the height h, correct to 1 decimal place, if the distance CD=50 m **3**

c) Prove:

(i) $\dfrac{\cot\theta\cos\theta}{\cot\theta+\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$ **3**

(ii) $\dfrac{\sin^2\theta}{1+\cos\theta} + \dfrac{\sin^2\theta}{1-\cos\theta}$ is independent of θ. **3**

Solutions Maths Extension Yr11 Task 1 2003

Question 1. (19)

a) i)

$\frac{x-3}{2} - \frac{x+1}{4} = 5$

$2x - 6 - (x+1) = 20$

$2x - 6 - x - 1 = 20$

$x - 7 = 20$

$x = 27$ (2)

ii) $\frac{3}{x^2} = 2$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{3/2}$ (2)

b) $\frac{1}{1-x} \geq 2$

Discontinuous at $x = 1$

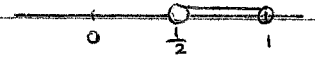
Solve $\frac{1}{1-x} = 2$

$1 = 2 - 2x$

$2x = 1$

$x = \frac{1}{2}$

Check points



$\therefore \frac{1}{2} < x < 1$ (3)

c) $\frac{x^3-1}{x^2-1} = \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$

$= \frac{x^2+x+1}{x+1}$ (3)

d) i) $\sqrt{3}\sin\theta = \cos\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30^\circ$ or 210° (3)

ii) $\theta - 30^\circ = 60^\circ$ or 120°

$\therefore \theta = 90^\circ$ or 150°

$\therefore$ Solution is 90° or 150° (3)

iii) $2\cos^2\theta - \cos\theta = 1$

$2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\therefore \cos\theta = -\frac{1}{2}$ or 1

$\therefore \theta = 120^\circ, 240^\circ, 0^\circ, 360^\circ$ (3)

Question 2. (23)

a) $f(x) = 1 - 2x^2$

i) $f(2) = -7$ (1)

ii) $f(-x) = 1 - 2(-x)^2$

$= 1 - 2x^2$ (1)

iii) $f(x+1) = 1 - 2(x+1)^2$

$= 1 - 2(x^2 + 2x + 1)$

$= -2x^2 - 4x - 1$ (2)

b) i) $y = x^2 + 2$

Domain: all real x (2)

ii) $y = \frac{1}{x}$

Domain: all real x, $x \neq 0$ (2)

iii) $y = \sqrt{x-1}$

Domain: $x - 1 \geq 0$

$x \geq 1$ (2)

iv) $y = \sqrt{x^2 - 9}$

Domain $x \geq 3$, $x \leq -3$ (2)

v) $y = \sqrt{4 - x^2}$

Domain $-2 \leq x \leq 2$ (2)

vi) $y = \frac{1}{\sqrt{1-2x}}$ (2)

Domain $x < \frac{1}{2}$

c) i) $f(-2) + f(2)$

$= (-2+1) + 2^2$

$= 3$ (2)

ii) Domain all real x

Range all real y (2)

d) all curves represent functions (3)

Question 3 (21)

a) i) $10! = 3\,628\,800$ (1)

ii) $\frac{6!}{3!} = 6 \times 5 \times 4$

$= 120$ (1)

iii) $\frac{100!}{98!} = 100 \times 99$

$= 9900$ (2)

b) i) $7! = 5040$ (1)

ii) boys together

$4! \times 4! = 576$ (2)

iii) boys & girls alternate

B G B G B G B

$4! \times 3! = 144$ (2)

c) 5E 4S 2H

$(5! \times 4! \times 3!) \times 3!$

$= 34560$ (3)

d) SUCCESS

i) $\frac{7!}{3! \times 2!} = 420$ (2)

ii) CC together.

$\frac{6!}{3!} = 120$ (2)

e) i) $(4 \times 3) \times 5!$

$= 1440$ (2)

ii) J _ _ _
S _ N
J N S
$(4 \times 3 \times 2) \times 4!$
$24 \times 4! = 576$
but Jan and Normah can swap rows $\therefore 2 \times 576$
$= 1152$ (3)

Question 4 (16)

a)
i) ${}^{10}C_4 = 210$ (1)

ii) T + 3 others $= {}^{9}C_3$
$= 84$ (2)

c) i) ${}^{52}C_5 = 2598960$ (1)

ii) $P(\text{Hearts}) = \dfrac{{}^{13}C_5}{{}^{52}C_5}$
$= \dfrac{33}{66640}$
or
$\doteq 0.000495$ (2)

iii) $P(3K\_) = \dfrac{{}^{4}C_3 \times {}^{48}C_2}{{}^{52}C_5}$
$= \dfrac{94}{54145}$
or
$\doteq 0.001736\ldots$ (3)

d) i) $9! = 362\,880$ (1)

ii) $7! \times 3! = 30\,240$ (2)

iii) $P(\text{next to each other}) = \dfrac{8! \times 2}{9!}$
$= \dfrac{2}{9}$ (2)

or, if one person is seated the probability that the person is next to her is $\frac{2}{9}$

iii) $P(\text{opposite each other}) = \dfrac{1}{9}$ (2)

(same reasoning as above)

Question 5 (21)

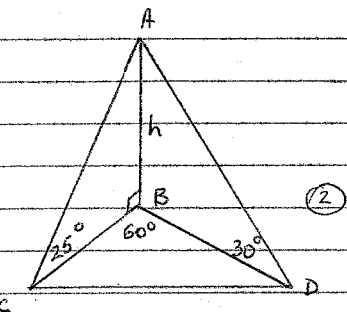
By Pythagoras.

$PR^2 = 12^2 + 5^2$ $\quad P^2 = 12^2 + 9^2$
$\therefore PR^2 = 169$ $\quad PQ^2 = 225$
$QR^2 = 5^2 + 9^2$
$QR^2 = 106$

In ΔPQR, cosine rule.

$\cos \angle PQR = \dfrac{PQ^2 + QR^2 - PR^2}{2 \times PQ \times QR}$
$= \dfrac{225 + 106 - 169}{2 \times 15 \times \sqrt{106}}$
$\cos \angle PQR = 0.52449\ldots$
$\doteq 58^\circ$ (5)

b)
i)



(2)

ii) In ΔABC
$\tan 25^\circ = \dfrac{h}{CB}$
$CB = \dfrac{h}{\tan 25^\circ}$
$CB = h \cot 25^\circ$ (2)

iii) Similarly $BD = h \cot 30^\circ$

Using the cosine rule in ΔCBD:

$CD^2 = h^2 \cot^2 25 + h^2 \cot^2 30^\circ - 2 \cdot h \cot 25^\circ \cdot h \cot 30^\circ \cos 60^\circ$

$\therefore CD^2 = h^2 \cot^2 25^\circ + h^2 \cot^2 30^\circ - 2h^2 \cot 25^\circ \cot 30^\circ \cos 60^\circ$ (3)

iv) $CD^2 = h^2 [\cot^2 25^\circ + \cot^2 30^\circ - 2 \cot 25^\circ \cot 30^\circ \cos 60^\circ]$
$2500 = h^2 \times [3.8845]$
$643.6 = h^2$
$\therefore h = 25.3688$
$= 25.4$ (3)

c) LHS
i) $\dfrac{\cot\theta \cos\theta}{\cot\theta + \cos\theta}$
$= \dfrac{\frac{\cos\theta}{\sin\theta} \cdot \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$ $\quad \left(\times \dfrac{\sin\theta}{\sin\theta}\right)$
$= \dfrac{\cos^2\theta}{\cos\theta + \cos\theta\sin\theta}$
$= \dfrac{\cos^2\theta}{\cos\theta(1 + \sin\theta)}$
$= \dfrac{\cos\theta}{1 + \sin\theta} = RHS$ (3)

ii) $\dfrac{\sin^2\theta}{1 + \cos\theta} + \dfrac{\sin^2\theta}{1 - \cos\theta}$
$= \dfrac{\sin^2\theta(1 - \cos\theta) + \sin^2\theta(1 + \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$
$= \dfrac{\sin^2\theta - \sin^2\theta\cos\theta + \sin^2\theta + \sin^2\theta\cos\theta}{1 - \cos^2\theta}$
$= \dfrac{2\sin^2\theta}{\sin^2\theta}$
$= 2$ which is independent of θ (3)