GIRRAWEEN HIGH SCHOOL

2004

YEAR 11 HALF YEARLY EXAMINATIONS

TASK 1

Mathematics Extension 1

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

Total marks – 108

- Attempt Questions 1 – 5
- Questions are NOT of equal value

Total marks – 113
Attempt Questions 1 – 5
Questions are NOT of equal value
Answer each question on a SEPARATE piece of paper clearly marked Question 1, Question 2, etc. Each piece of paper must show your name.

Marks

Question 1 ***(20 marks)*** Use a *separate* piece of paper

a) If $f(x)=x^2+x$ and $g(x)=x^2+3x+1$, find;

(i) $\dot{f}(1)$ *1*

(ii) $g(-2)$ *1*

(iii) $f(4)-g(3)$ *2*

(iv) $f(x+1)$ *2*

b) Given that $f(x)=\begin{cases} x^2 & ,x>2 \\ \dfrac{1}{x+1} & ,-1<x\le 2 \\ 5 & ,x\le -1 \end{cases}$, find;

(i) $f(3)+f(-1)$ *2*

(ii) the domain of $f(x)$ *1*

(iii) the range of $f(x)$ *1*

c) In your own words describe what is meant by "function" when referring to number plane graphs. *2*

d) Write down the domain and range of;

(i) $y=2x^2-5$ *2*

(ii) $y=\dfrac{4}{x-2}$ *2*

(iii) $y=\sqrt{16-x^2}$ *2*

(iv) $y=\dfrac{5}{x^2+9}$ *2*

Question 2 ***(24 marks)*** Use a *separate* piece of paper

a) Solve the following inequalities;

(i) $\dfrac{2}{x-3}\ge 4$ *3*

(ii) $\dfrac{1}{x}<\dfrac{1}{x+1}$ *3*

Question 2...*continued.* *Marks*

b) Solve for θ, correct to the nearest degree where necessary, where $0^\circ \le \theta \le 360^\circ$

(i) $\cos^2\theta = \cos\theta$ *3*

(ii) $\sqrt{3}\sin\theta - 3\cos\theta = 0$ *3*

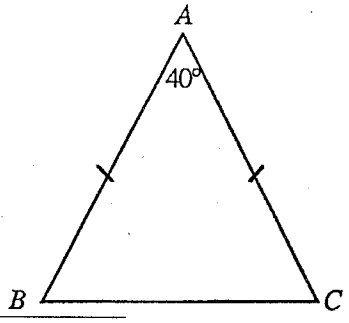
(iii) $2\sin^2\theta - 5\sin\theta + 2 = 0$ *4*

(iv) $\sin 3\theta = \frac{1}{2}$ *4*

(v) $4\tan\theta + 1 - \tan^2\theta = \sec^2\theta$ *4*

Question 3 ***(13 marks)*** Use a *separate* piece of paper

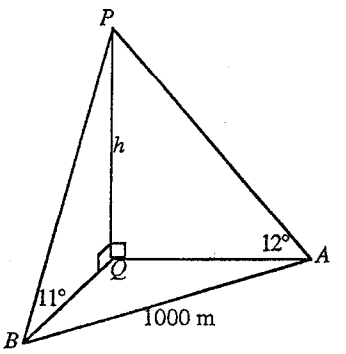
a) The vertical angle of an isosceles triangle is 40° and its area is $40\,\text{cm}^2$.



(i) Show that $AB = \sqrt{80\text{cosec}40^\circ}$ *2*

(ii) Hence, or otherwise, calculate the length of BC, correct to one decimal place *3*

b) The angle of elevation of a tower PQ of height h metres, at a point A due East of it, is 12°. From another point B, the bearing of the tower is 051°T and the angle of elevation is 11°. The points A and B are 1000 metres apart and on the same level as the base Q of the tower.



(i) Show that $\angle AQB = 141^\circ$ *2*

(ii) Consider ΔAPQ, show that $AQ = h\tan 78^\circ$ *2*

(iii) Find a similar expression for BQ *1*

(iv) Hence calculate h, correct to the nearest metre. *3*

Question 4 *(36 marks)* Use a *separate* piece of paper

a) The game of Yahtzee involves rolling five regular six sided dice.

(i) How many ways can the dice land? *2*

(ii) A Yahtzee is when all five dice show the same number. What is the probability of rolling a Yahtzee? *2*

b) An eight person committee is to be chosen from six boys and six girls. In how many way s can this be done if;

(i) there are no restrictions? *2*

(ii) there must be an equal number of boys and girls on the committee? *2*

(iii) the two school captains (one boy and one girl) must be on the committee? *2*

c) How many ways can the letters of the word PARRAMATTA be arranged if;

(i) there are no restrictions? *2*

(ii) an R must be at either end of the word? *2*

(iii) the T's cannot be next to each other? *2*

d) To win Lotto, you choose 6 numbers out of a total of 45.

(i) In how many ways can this be done? *2*

(ii) What is the probability of winning first prize, (that is picking all six numbers correctly), if you have four different entries? *2*

e) Numbers are to be made from the digits 1 to 9, with no digit allowed more than once in the same number.

(i) How many four digit numbers can be made? *2*

(ii) How many of these four digit numbers would be odd? *2*

(iii) What is the probability that a four digit number is divisible by 5? *3*

(iv) How many numbers, (using any number of digits), less than 3000 can be made? *3*

f) Four men and four women are seated around a table

(i) How many ways can they be seated? *2*

(ii) What is the probability that the men and women alternate around the table? *2*

(iii) What is the probability that two particular men do not sit next to each other? *2*

Question 5 ***(15 marks)*** Use a *separate* piece of paper

a) Factorise $\left(a^2-b^2\right)^2-(a-b)^4$ completely *3*

b) If θ is acute and $\sin\theta=\dfrac{1}{\sqrt{3}}$

(i) Show that $\dfrac{\tan\theta}{1-\sec\theta}=-\sqrt{2}-\sqrt{3}$ *3*

(ii) Find the value of this fraction when θ is obtuse. *2*

c) Prove that $\dfrac{1+\cot\theta}{\text{cosec?}}-\dfrac{1+\tan\theta}{\sec\theta}$ is independent of θ *3*

d) Prove that $\dfrac{1+\cos\theta}{1-\cos\theta}=\left(\text{cosec?}+\cot\theta\right)^2$ *4*

Year 11 Extension 1 Half Yearly 2004

Question 1 (20)

a) $f(x) = x^2 + x \quad g(x) = x^2 + 3x + 1$

(i) $f(1) = 1^2 + 1$
$= 2$ (1)

(ii) $g(-2) = (-2)^2 + 3(-2) + 1$
$= -1$ (1)

(iii) $f(4) - g(3)$
$= 4^2 + 4 - [3^2 + 3(3) + 1]$
$= 20 - 19$
$= 1$ (2)

(iv) $f(x+1) = (x+1)^2 + (x+1)$
$= x^2 + 2x + 1 + x + 1$
$= x^2 + 3x + 2$ (2)

b) (i) $f(3) + f(-1) = 3^2 + 5$
$= 14$ (2)

(ii) domain: all real x (1)

(iii) range: $y \geq 0$ (1)

c) A function is a relation such that for every value in the domain there exists no more than one value in the range (2)

d) (i) domain: all real x (1)
range: $y \geq -5$ (1)

(ii) domain: all real x except $x = 2$ (1)
range: all real y except $y = 0$ (1)

(iii) domain: $16 - x^2 \geq 0$
$x^2 \leq 16$
$-4 \leq x \leq 4$ (1)

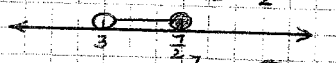
range: $x = 4, \ y = \sqrt{16 - 4^2}$
$= 0$
$x = 0, \ y = \sqrt{16 - 0^2}$
$= 4$
$0 \leq y \leq 4$ (1)

(iv) domain: all real x (1)
range: $x^2 \geq 0$
$-x^2 + 9 \leq 9$
$0 < y \leq \frac{5}{9}$ (1)

Question 2 (24)

a) $\frac{2}{x-3} \geq 4$
$x \neq 3$

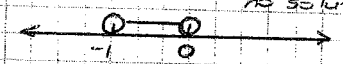
$\frac{2}{x-3} = 4$
$2 = 4x - 12$
$4x = 14$
$x = \frac{7}{2}$



$3 < x \leq \frac{7}{2}$ (3)

(ii) $\frac{1}{x} < \frac{1}{x+1}$
$x \neq 0, \ x \neq -1$

$\frac{1}{x} = \frac{1}{x+1}$
$x = x + 1$
$0 = 1$
no solutions



$-1 < x < 0$ (3)

b) (i) $\cos^2\theta = \cos\theta$
$\cos^2\theta - \cos\theta = 0$
$\cos\theta(\cos\theta - 1) = 0$
$\cos\theta = 0$ or $\cos\theta = 1$
$\theta = 90°, 270°$ $\quad \theta = 0°, 360°$
$\theta = 0°, 90°, 270°, 360°$ (3)

(ii) $\sqrt{3}\sin\theta - 3\cos\theta = 0$
$\sqrt{3}\sin\theta = 3\cos\theta$
$\tan\theta = \sqrt{3}$
Q1, Q3
$\tan\alpha = \sqrt{3}$
$\alpha = 60°$
$\theta = 60°, 240°$ (3)

(iii) $2\sin^2\theta - 5\sin\theta + 2 = 0$
$2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$
$2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$
$(\sin\theta - 2)(2\sin\theta - 1) = 0$
$\sin\theta = 2$ or $\sin\theta = \frac{1}{2}$
no solutions
Q1, Q2
$\sin\alpha = \frac{1}{2}$
$\alpha = 30°$
$\theta = 30°, 150°$
$\theta = 30°, 150°$ (4)

(iv) $\sin 3\theta = \frac{1}{2} \quad 0 \leq 3\theta \leq 1080°$
Q1, Q2
$\sin\alpha = \frac{1}{2}$
$\alpha = 30°$
$3\theta = 30°, 150°, 390°, 510°, 750°, 870°$
$\theta = 10°, 50°, 130°, 170°, 250°, 290°$ (4)

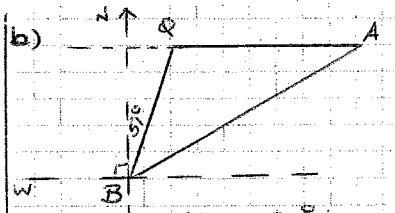
(iv) $4\tan\theta + 1 - \tan^2\theta = \sec^2\theta$
$= 1 + \tan^2\theta$
$2\tan^2\theta - 4\tan\theta = 0$
$2\tan\theta(\tan\theta - 2) = 0$
$\tan\theta = 0 \quad \tan\theta = 2$
$\theta = 0°, 180°, 360°$ $\quad$ Q1, Q3
$\tan\alpha = 2$
$\alpha = 63°$
$\theta = 63°, 143°$
$\theta = 0°, 63°, 143°, 180°, 360°$ (4)

Question 3 (13)

(i) $\frac{1}{2}AB \cdot AC \sin 40° = 40$
$\frac{1}{2}AB^2 \sin 40° = 40$
$AB^2 = \frac{80}{\sin 40°}$
$AB = \sqrt{80}\,\text{cosec}\,40°$ (2)

(ii) $BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cos 40°$
$= 2AB^2 - 2AB^2 \cos 40°$
$= 2AB^2(1 - \cos 40°)$
$= 160\,\text{cosec}\,40°(1 - \cos 40°)$
$BC = 7.6$ cm (3)

b)



$\angle NBQ = 51°$
$\therefore \angle WBQ = 141°$
$\angle AQB = \angle WBQ$ (alternate $\angle$'s, // lines)
$\therefore \angle AQB = 141°$ (2)

(ii) $\frac{AQ}{h} = \tan 78°$
$AQ = h\tan 78°$ (2)

(iii) $\frac{BQ}{h} = \tan 79°$
$BQ = h\tan 79°$ (1)

(iv) $AB^2 = AQ^2 + BQ^2 - 2AQ \cdot BQ \cos Q$
$1000^2 = h^2\tan^2 78° + h^2\tan^2 79° - 2h^2\tan 78° \tan 79° \cos 141°$
$h^2 = \frac{1000^2}{\tan^2 78° + \tan^2 79° - 2\tan 78 \tan 79 \cos 141°}$
$h = \frac{1000}{\sqrt{\tan^2 78° + \tan^2 79° - 2\tan 78° \tan 79° \cos 141°}}$
$h = 108$m (3)

Question 4 (36)

a) (i) Ways $= 6^5$
$= 7776$ (2)

(ii) $P(\text{Yahtzee}) = \frac{6}{7776}$
$= \frac{1}{1296}$ (2)

b) (i) Ways $= {}^{12}C_8$
$= 495$ (2)

(ii) Ways $= {}^6C_4 \times {}^6C_4$
$= 225$ (2)

(iii) Ways $= 1 \times {}^{10}C_6$
$= 210$ (2)

c) (i) Ways $= \frac{10!}{4!2!2!}$
$= 37800$ (2)

(ii) Ways $= 1 \times \frac{8!}{4!2!}$
$= 840$ (2)

(iii) Ways Ts together $= \frac{9!}{4!2!}$
$= 7560$

$\therefore$ Ways Ts not together
$= 37800 - 7560$
$= 30240$ (2)

d) (i) Ways $= {}^{45}C_6$
$= 8\,145\,060$ (2)

(ii) $P(\text{win}) = \frac{4}{{}^{45}C_6}$
$= \frac{1}{2\,036\,265}$ (2)

e) (i) 4 digit #'s $= {}^9P_4$
$= 3024$ (2)

(ii) odd 4 digit #'s $= {}^5P_1 \times {}^8P_3$
$= 1680$ (2)

(iii) $\div 5$ means # ends in 5
4 digit $\div 5 = 1 \times {}^8P_3$
$= 336$
$P(\div 5) = \frac{336}{3024}$
$= \frac{1}{9}$ (3)

(iv) 1 digit = 9

2 digit = ${}^9P_2$
= 72

3 digit = ${}^9P_3$
= 504

4 digit = $2 \times {}^8P_3$
= 672

#'s < 3000 = 9 + 72 + 504 + 672
= 1257 (3)

(i) (ii) Ways = $7!$ (2)
= 5040

(iii) P(alternate) = $\frac{3!\,4!}{7!}$
= $\frac{1}{35}$ (2)

(iii) P(not together) = 1 − P(together)
= $1 - \frac{2!\,6!}{7!}$
= $1 - \frac{2}{7}$
= $\frac{5}{7}$ (2)

c) $\frac{1+\cot\theta}{\operatorname{cosec}\theta} - \frac{1}{\sec\theta}$

$= \frac{1+\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{1+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \sin\theta + \cos\theta - (\cos\theta + \sin\theta)$

$= 0$ (3)

d) $\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{1 + 2\cos\theta + \cos^2\theta}{1 - \cos^2\theta}$

$= \frac{1 + 2\cos\theta + \cos^2\theta}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta + 2\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta} + \cot^2\theta$

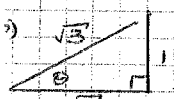
$= \operatorname{cosec}^2\theta + 2\cot\theta\operatorname{cosec}\theta + \cot^2\theta$

$= (\operatorname{cosec}\theta + \cot\theta)^2$ (4)

Question 5 (15)

a) $(a^2-b^2)^2 - (a-b)^4$
$= (a+b)^2(a-b)^2 - (a-b)^4$
$= (a-b)^2[(a+b)^2 - (a-b)^2]$
$= (a-b)^2(4ab)$
$= 4ab(a-b)^2$ (3)

b)



$\frac{\tan\theta}{1-\sec\theta} = \frac{\frac{1}{\sqrt{2}}}{1-\frac{\sqrt{3}}{\sqrt{2}}}$

$= \frac{1}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}$

$= \frac{\sqrt{2}+\sqrt{3}}{-1}$

$= -\sqrt{2}-\sqrt{3}$ (3)

(ii) if θ obtuse;

$\frac{\tan\theta}{1-\sec\theta} = \frac{-\frac{1}{\sqrt{2}}}{1+\frac{\sqrt{3}}{\sqrt{2}}}$

$= \frac{-1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$= \frac{-\sqrt{2}+\sqrt{3}}{-1}$

$= \sqrt{2}-\sqrt{3}$ (2)