2005 HY Yr11 Ext 1

Question One

a) Solve $6x^2 - 5x + 1 = 0$ **2**

b) Find the value of y if $a^{4y-3} = \dfrac{1}{\sqrt{a}}$ **2**

c) If $\cos A = \dfrac{2}{5}$ and $\tan A < 0$ find the exact value of sin A and cot A. **3**

d) i) P divides the interval from A(-4,-6) to B(6,-1) in the ratio k:2. Write down the coordinates of P. **2**

ii) If P lies on the line $4x - 3y = 17$ find the exact coordinates of P. **2**

e) i) Neatly sketch $y = |3x - 6|$ **2**

ii) Hence or otherwise solve $|3x - 6| = 2x - 5$ **2**

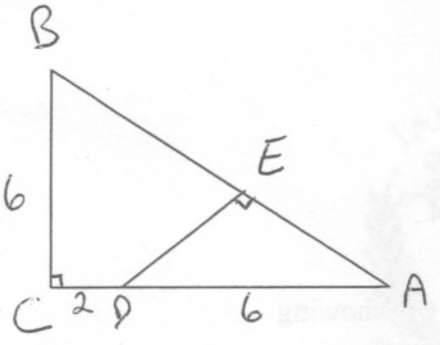
Question Two (Start a new page)

a) Given that $f(x) = x^2 + x$, simplify $\dfrac{f(x-1)}{x-1}$ **2**

b) If the perpendicular distance between (-1, 3) and the line $ax + 3y - 15 = 0$ is 2 units, find the exact value/s of a. **2**

c) i) Is the function $y = \dfrac{2}{x^2}$ odd or even? Justify your answer. **2**

ii) Neatly sketch $y = \dfrac{2}{x^2}$, clearly showing all important features. **2**

d) 

In ΔABC, AD = 6 units, DC = 2 units and BC = 6 units.

i) Prove $\Delta ABC \,|||\, \Delta ADE$. **3**

ii) Find DE, giving reasons. **2**

e) Simplify $\frac{1}{2}[\sin(A + B) - \sin(A - B)]$ **2**

<u>Question Three</u> (Start a new page)

a) Differentiate the following with respect to x:

i) $3x^3 - x^5$ **1**

ii) $x\sqrt{x}$ **2**

iii) $(x^2 + 1)^2$ **2**

iv) $\dfrac{\ln x}{x}$ **2**

b) When is $f(x) = 2x^3 - \dfrac{5}{2}x^2 + x + 6$ concave up? **2**

c) Find the exact gradient of the tangent to $y = \sin x.\tan x$ when $x = \dfrac{\pi}{6}$ **3**

d) Find $\lim\limits_{x \to 2} \dfrac{x-2}{x+3}$ **1**

e) If the roots of $kx^2 - 2x + (1 - 2k) = 0$ are reciprocals, find the exact value of their sum. **2**

<u>Question Four</u> (Start a new page)

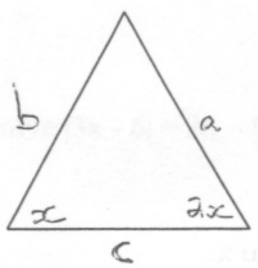
a) Write down the domain and range of $y = \sqrt{x^2 - 4}$. **2**

b) Neatly sketch using calculus $y = 2x^3 - \frac{5}{2}x^2 + x + 6$ showing all important features. **4**

c) Find the locus of a point P(x,y) which moves so that its distance from A(a,2a) is equal to its distance from B(2a, a). **2**

d) Find the values of k for which $y = e^{-kx}(x - k)$ is stationary at $x = 2.5$ **3**

e)



i) Prove that $\cos x = \frac{b}{2a}$ (Hint use the sine rule). **2**

ii) Prove that $c = \frac{b^2 - a^2}{a}$ or c = a **2**

END OF PAPER

Yr 11 Ext. 1 Solutions 2005

Question One

a) $6x^2 - 5x + 1 = 0$ $\times 6, -5$

$6x^2 - 3x - 2x + 1 = 0$

$3x(2x-1) - (2x-1) = 0$

$(3x-1)(2x-1) = 0$ (2)

$\therefore x = \frac{1}{3}$ or $x = \frac{1}{2}$

b) $a^{4y-3} = \frac{1}{\sqrt{a}}$

$a^{4y-3} = a^{-\frac{1}{2}}$ (1)

$\therefore 4y - 3 = -\frac{1}{2}$

$4y = 2\frac{1}{2}$

$\therefore y = \frac{5}{8}$ (1) (or $y = 0.625$)

c) $\cos A = \frac{2}{5}$ (½)

$\frac{S \mid A}{T \mid C}$ (½)

$\sqrt{21}$, 5, 2, A

$\therefore \sin A = \frac{-\sqrt{21}}{5}$ (1) $\cot A = -\frac{2}{\sqrt{21}}$ (1)

d) i) $(-4,-6)$ $(6,-1)$

$k:2$

$\therefore P = \left(\frac{-8+6k}{k+2}, \frac{-12-k}{k+2}\right)$ (1) (1)

ii) $4\frac{(-8+6k)}{k+2} - 3\frac{(-12-k)}{k+2} = 17$

$-32 + 24k + 36 + 3k = 17k + 34$

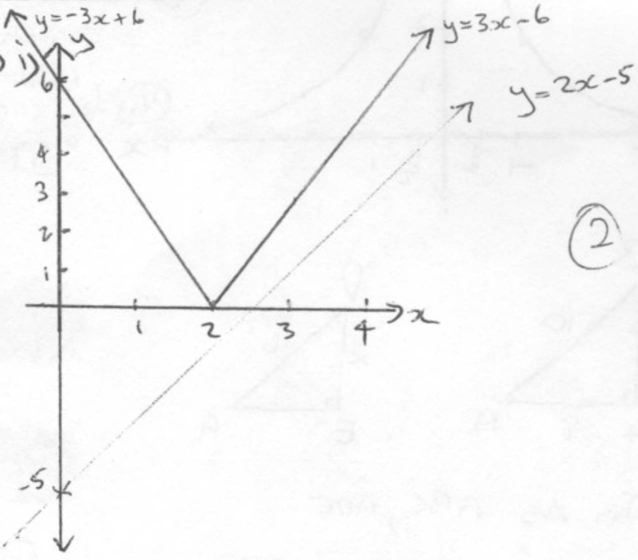
$27k - 17k = 34 - 4$

$10k = 30$

$\therefore k = 3$ (1)

$\therefore P$ is $(2,-3)$ (1)

e) i)



(2)

ii) $|3x-6| = 2x-5$

no real solution (1)

(+ 1 mark for graph or 1 mark for solving algebraically and 1 mark for explaining that there's no real solution).

Question Two

a) $\frac{f(x-1)}{x-1} = \frac{(x-1)^2 + (x-1)}{x-1}$ (1)

$= x - 1 + 1$

$= x$ (1)

b) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$2 = \frac{|-a+9-15|}{\sqrt{a^2+9}}$ (1)

$4 = \frac{(-a-6)^2}{a^2+9}$

$4a^2 + 36 = a^2 + 12a + 36$

$3a^2 - 12a = 0$

$3a(a-4) = 0$ (1)

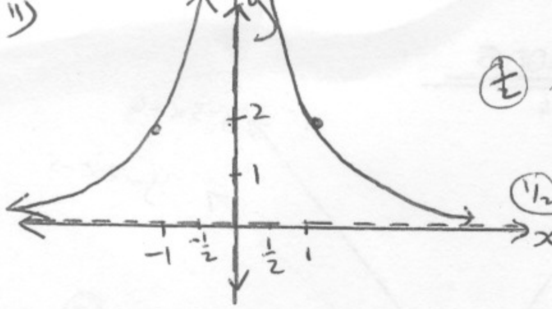
$\therefore a = 0$ or 4

c) i) $f(x) = \frac{2}{x^2}$

$f(-x) = \frac{2}{(-x)^2} = \frac{2}{x^2} = f(x)$ (1)

$\therefore$ even function (1)

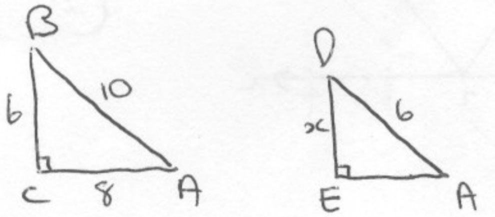
c) ii)



(½) for each branch (total = 2)

(½) for each asymptote

d) i)



In $\triangle$s ABC, ADE (1)

$\hat{A}$ is common (1)

$B\hat{C}A = D\hat{E}A$ (both $90°$)

$\therefore \triangle ABC \parallel\!| \triangle ADE$ (equiangular) (1)

ii) $\frac{x}{6} = \frac{6}{10}$ (1) (corresponding sides in similar triangles are in the same ratio

$x = 3\frac{3}{5}$ units (1)

e) $\frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$= \frac{1}{2}\left[\sin A\cos B + \cos A\sin B - \sin A\cos B + \cos A\sin B\right]$ (1)

$= \frac{1}{2} \times 2\cos A\sin B$

$= \cos A\sin B$ (1)

Question Three

a) i) $9x^2 - 5x^4$ (1)

ii) $y = x^{3/2}$ (1)

$\therefore y' = \frac{3}{2}x^{1/2}$

$= \frac{3}{2}\sqrt{x}$ (1)

iii) $y = x^4 + 2x^2 + 1$ (1)

$\frac{dy}{dx} = 4x^3 + 4x$ (1)

iv) $y = \frac{\ln x}{x}$

$y' = \frac{x \cdot \frac{1}{x} - \ln x}{x^2}$ (1) $= \frac{1 - \ln x}{x^2}$ (1)

b) $f(x) = 2x^3 - \frac{5}{2}x^2 + x + 6$

$f'(x) = 6x^2 - 5x + 1$ (½)

$f'(x) = 12x - 5$

concave up when $f''(x) > 0$ (½)

$\therefore 12x - 5 > 0$

(1) $12x > 5$

$\therefore x > \frac{5}{12}$

c) $y = \sin x . \tan x$

$\frac{dy}{dx} = \tan x . \cos x + \sin x . \sec^2 x$ (1)

$= \sin x + \tan x . \sec x$

when $x = \pi/6$,

$M = \sin\pi/6 + \tan\frac{\pi}{6} \cdot \frac{1}{\cos\pi/6}$ (1)

$= \frac{1}{2} + \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}}$

$= 1\frac{1}{6}$ (1) (must be exact!!)

d) $\lim_{x \to 2} \frac{x-2}{x+3}$

$= \frac{0}{5}$ (1)

$= 0$

e) let roots be α and $\frac{1}{\alpha}$

$\therefore \alpha + \frac{1}{\alpha} = \frac{2}{K}$ - - - - (1)

$1 = \frac{1-2K}{K}$ - - - (2)

from (2) $K = 1 - 2K$

$3K = 1$

$K = 1/3$ (1)

sub into (1)

$\therefore \alpha + \frac{1}{\alpha} = 2/_{1/3}$

$= 6$ (1)

Question Four

a) $y=\sqrt{x^2-4}$

D: $x\leq -2$ or $x\geq 2$ (1)

R: $y\geq 0$ (1)

b) $y=2x^3-\frac{5}{2}x^2+x+6$

$\frac{dy}{dx}=6x^2-5x+1$

stationary points when $\frac{dy}{dx}=0$

ie at $x=1/3$ or $\frac{1}{2}$ [from 1 a)] (1)

$\frac{d^2y}{dx^2}=12x-5$

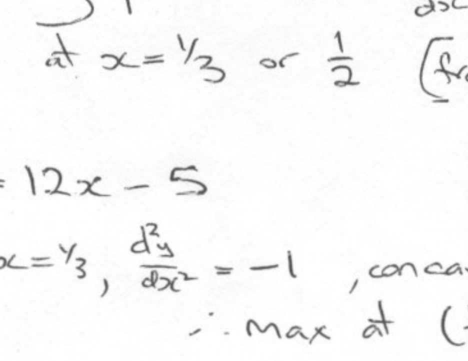
when $x=1/3$, $\frac{d^2y}{dx^2}=-1$, concave down

$\therefore$ Max at $(\frac{1}{3}, 6\frac{7}{54})$ (1)

when $x=\frac{1}{2}$, $\frac{d^2y}{dx^2}=1$, concave up

$\therefore$ min. at $(\frac{1}{2}, 6\frac{1}{8})$ (1)

intercept $(0,6)$



(1)

c) $PA=2PB$

$(x-a)^2+(y-2a)^2=(x-2a)^2+(y-a)^2$ (1)

$x^2-2ax+a^2+y^2-4ay+4a^2=x^2-4ax+4a^2+y^2-2ay+a^2$

$2ax-2ay=0$ (1/2)

$2a(x-y)=0$

$\therefore$ The locus is the line $y=x$. (1/2)

d) $y=e^{-kx}(x-k)$

$\frac{dy}{dx}=e^{-kx}+-k(x-k)e^{-kx}$

$=e^{-kx}\left[1-k(x-k)\right]$

$=e^{-kx}(1-kx+k^2)$

Stat. points when $\frac{dy}{dx}=0$ (1)

ie when $e^{-kx}(1-kx+k^2)=0$

but $e^{-kx}\neq 0$ $\therefore$ when $1-kx+k^2=0$

now $x=2.5$

$\therefore 1-k(2.5)+k^2=0$

$k^2-2.5k+1=0$

$k=\frac{2.5\pm\sqrt{2.5^2-4}}{2}$

$=\frac{2.5\pm 1.5}{2}$

$\therefore k=2$ or $1/2$ (1)

e) i) $\frac{b}{\sin 2x}=\frac{a}{\sin x}$ (1)

$\frac{\sin x}{2\sin x\cos x}=\frac{a}{b}$ (1/2)

$\frac{1}{\cos x}=\frac{2a}{b}$

$\therefore \cos x=\frac{b}{2a}$ (1/2)

(11)

PTO

an alternative solution for

(e) ii) using the cos rule

$$a^2 = b^2 + c^2 - 2bc \cos x$$

$$a^2 = b^2 + c^2 - 2bc \cdot \frac{b}{2a} \qquad \text{from (i)}$$

$$a^2 = b^2 + c^2 - \frac{b^2c}{a}$$

$$a^3 = ab^2 + ac^2 - b^2c$$

$$\therefore ac^2 - b^2c + ab^2 - a^3 = 0$$

$$\therefore c = \frac{b^2 \pm \sqrt{b^4 - 4a(ab^2 - a^3)}}{2a}$$

$$= \frac{b^2 \pm \sqrt{b^4 - 4a^2b^2 + 4a^4}}{2a}$$

$$= \frac{b^2 \pm \sqrt{(b^2 - 2a^2)^2}}{2a}$$

$$= \frac{b^2 \pm (b^2 - 2a^2)}{2a}$$

$$= \frac{2b^2 - 2a^2}{2a} \quad \text{or} \quad \frac{2a^2}{2a}$$

$$= \frac{b^2 - a^2}{a} \quad \text{or} \quad a$$