Question 1. **Marks**

(a) Factorise fully: $a^2c - b^2c - abc^2 + ab$. **2**

(b) Evaluate: $\lim\limits_{x \to 0} \dfrac{x \cos x}{\sin 3x}$. **2**

(c) Find the equation of the line that passes through the point of intersection of the lines $7x - 3y + 6 = 0$, $4x + 11y - 5 = 0$ and the point (1, 1). **2**

(d) Given $G(x) = 3^{2x}$, find $G'(0)$. **2**

(e) The point P divides the interval AB externally in the ratio $1:2$. Find the coordinates of P when $A = (-2, 1)$ and $B = (1, -5)$. **2**

(f) Shade the region, on the Cartesian number plane, that satisfies both **3**

$$y \le \sqrt{9 - x^2} \text{ and } y \ge x^3 \text{ simultaneously.}$$

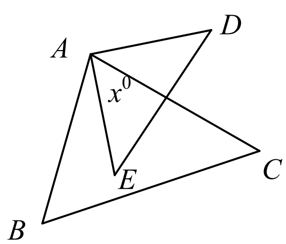
(g) Solve for x: $3\tan 2x = \sqrt{3}$, for $0 \le x \le \pi$. **2**

Question 2. **[START A NEW PAGE]**

(a) Simplify: $\sin(A+B)\sin(A-B) + \cos(A+B)\cos(A-B)$. **2**

(b) Given the function: $H(x) = x\left(\dfrac{2^x - 1}{2^x + 1}\right)$, show that $H(x)$ is an even function. **2**

(c) Solve for x: $|x - 3| < 1 + |x|$. **2**

(d) 

In the diagram, triangles ABC and ADE are equilateral, and $\angle CAE = x^0$. **3**

Not to scale

Copy the diagram onto your writing paper and Prove that: $\Delta BAE \equiv \Delta CAD$.

(e) Find $\dfrac{dy}{dx}$ for: (i) $y = ex$. **1**

(ii) $y = \sec 3x$. **1**

(f) Given $y = xe^{-x} + 3$, $y' = (1 - x)e^{-x}$ and $y'' = (x - 2)e^{-x}$,

(i) Show that the curve $y = xe^{-x} + 3$ has a point of inflection. **2**

(ii) Find the equation of the inflectional tangent. **2**

Question 3. **[START A NEW PAGE]** **Marks**

(a) (i) Find the derivative of $\sqrt{1+6x}$. **1**

(ii) Hence, or otherwise, differentiate $x^3\sqrt{1+6x}$. **1**

(b) Find $\frac{dy}{dx}$ when: $y = \ln\left[\frac{x^4}{x-1}\right]$. **2**

(c) For what values of x is the curve: $y = \frac{5}{2}x^2 - \frac{1}{3}x^3 + 1$ increasing? **2**

(d) Solve for x: $\frac{4}{x^2-4} - \frac{1}{x-2} = 1$. **2**

(e) Show, by sketching, that: $\pi \sin x \geq 2x$ in the interval $0 \leq x \leq \frac{\pi}{2}$. **2**

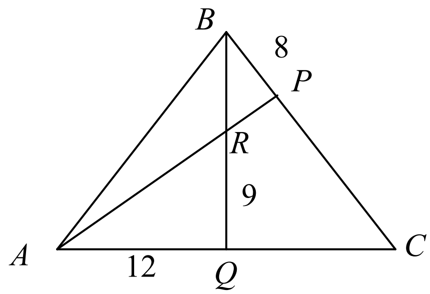
(f) Solve for x, given that $a > 1$: $a^{2x} + a = (1+a)a^x$. **2**

(g) Find the simplified result of: $\frac{d}{dx}\left[\frac{x\sin x}{1+\cos x}\right]$. **3**

Question 4. **[START A NEW PAGE]** **Marks**

(a) Show that $\log_{a^n} 3 = \frac{1}{n}\log_a 3$, for $a > 0$ and $n \neq 0$. **1**

(b) In ΔABC, $AP \perp BC$ at P and $BQ \perp AC$ at Q, as shown in the diagram.



Not to scale

Copy the diagram onto your writing paper and

(i) Prove that $\Delta AQR \; ||| \; \Delta BPR$. **3**

(ii) If $AQ = 12$, $QR = 9$ and $BP = 8$, find the area of ΔABR. **2**

(c) Given the function: $f(x) = \dfrac{27(x-1)^2}{(x+1)^3}$.

(i) Show that: $f'(x) = \dfrac{27(x-1)(5-x)}{(x+1)^4}$. **2**

(ii) Find the stationary points of the function $y = f(x)$. **2**

(iii) Determine the nature of these stationary points. **2**

(iv) Sketch the curve $y = f(x)$, showing any asymptotes, intercepts and turning points. **3**

THE END

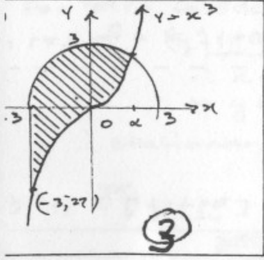
☹ 😐 ☺ 👍

YII HALF YEARLY M. EXT I, 2006

MATHEMATICS Extension 1 : Question 1

Suggested Solutions	Marks	Marker's Comments
(a) $a^2c - b^2c - abc^2 + ab$ $= a^2c - abc^2 + ab - b^2c$ $= ac(a - bc) + b(a - bc)$ $= (a - bc)(ac + b)$ (2)	1 1	
(b) $\lim_{x\to 0} \frac{x\cos x}{\sin 3x} = \lim_{x\to 0} \frac{3x}{3\sin 3x}\cdot\cos x$ METHOD 1 $= \frac{1}{3}\lim_{x\to 0}\frac{3x}{\sin 3x}\cdot\cos x$ $= \frac{1}{3}\times 1\times\cos 0$ $= \frac{1}{3}$ (2)	1 1	other methods · subst. $\theta = 3x$ · small ∠ approx.
(c) Let the line be: $7x - 3y + 6 + k(4x + 11y - 5) = 0$ (2) satisfies $\therefore 7 - 3 + 6 + k(4 + 11 - 5) = 0$ $10 + 10k = 0$ $\therefore k = -1$ so $7x - 3y + 6 - 1(4x + 11y - 5) = 0$ $\Rightarrow 3x - 14y + 11 = 0$	1 1	Point of intersection $\left(-\frac{51}{89}, \frac{59}{89}\right)$ $m = \frac{3}{14}$ $y - 1 = \frac{3}{14}(x - 1)$
(d) $G(x) = 3^{2x} = 9^x$ (2) $G'(x) = 3^{2x}\cdot\ln 9$ $G'(0) = 3^0\cdot\ln 9 = \ln 9 = 3\ln 2$	1 1	
(e) $A(-2, 1)$ $-1:2$ $B(1, -5)$ (2) $P = \left(\frac{-1\times 1 + 2\times -2}{-1+2}, \frac{-1\times(-5) + 2\times 1}{-1+2}\right)$ $P = (-5, 7)$	1, 1	

(f)



(3)

$\frac{1}{2}$ ea curve

1 region

(g) $3\tan 2x = \sqrt{3}$

$\tan 2x = \frac{1}{\sqrt{3}}$

$0 \le x \le \pi$

$0 \le 2x \le 2\pi$

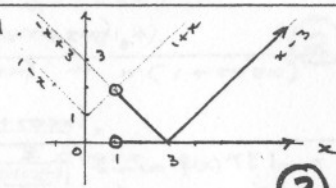
$\therefore 2x = \frac{\pi}{6}$ or $\pi + \frac{\pi}{6}$

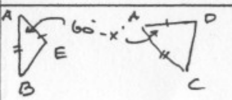
$\therefore x = \frac{\pi}{12}$ or $\frac{7\pi}{12}$ — 1, 1

(2)

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
(a) $\sin(A+B)\sin(A-B) + \cos(A+B)\cos(A-B)$ $= \cos[(A+B) - (A-B)]$ $= \cos 2B$ (2)	1 1	Method 2: Expansion of LHS and $\sin^2\theta + \cos^2\theta = 1$
(b) $H(-x) = (-x)\left(\frac{2^{-x} - 1}{2^{-x} + 1}\right) = -x\cdot\frac{(1 - 2^{+x})}{(1 + 2^x)}$ $= -x\cdot -1\frac{(2^x - 1)}{2^x + 1} = x\cdot\left(\frac{2^x - 1}{2^x + 1}\right)$ i.e. $H(-x) = H(x)$ (2) $\therefore H(x)$ is an EVEN FUNCTION	1 1	
(c) $\lvert x - 3\rvert < 1 + \lvert x\rvert$ Graphically $-x + 3 = 1 + x$ $2 = 2x$ $\therefore x = 1$ $\therefore x > 1$ (2)	1 1	Algebraic: $\lvert x-3\rvert - \lvert x\rvert < 1$ $x < 0$: × ; $0 \le x < 3$: $1 < x < 3$; $x \ge 3$: $x \ge 3$ $\Rightarrow x > 1$
(d) In Δs BAE and CAD 1. AE = AD (given data) 2. ∠BAE = ∠CAD = $(60 - x)^\circ$ (All angles in equilateral Δ are 60° each) 3. AB = AC (given) ∴ ΔBAE ≡ ΔCAD (SAS) (3)		1 For #1 & 3 1. For explaining #2 1 For SAS.
(e) (i) $y = ex$ $\therefore \frac{dy}{dx} = e$ — 1 (ii) $y = \sec 3x$ $\therefore \frac{dy}{dx} = 3\sec 3x\tan 3x$ — 1	1 each	
(f) (i) For possible points of inflection $y'' = 0$ $\therefore (x-2)e^{-x} = 0$ $\therefore x = 2$ as $e^{-x} \ne 0$ $y = 2e^{-2} + 3$ (2)	1	





TEST:

x	1.5	2	2.5
y''	(−)(+) ⊖ −0.11	0	(+)(+) ⊕ 0.04

since y is cont + diff'ble over $1.5^- \le x \le 2.5^+$

and y'' changes sign (− 0 +)

$\therefore$ change in concavity

$\therefore$ a PoI at $x = 2$ — 1

(ii) Gradient of Tangent: $m_T = -1e^{-2}$ — 1

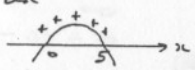
$\therefore$ Equation

$y - (2e^{-2} + 3) = -1e^{-2}(x - 2)$ — 1

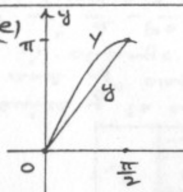
$y = -e^{-2}x + 3 + 4e^{-2}$

(2)

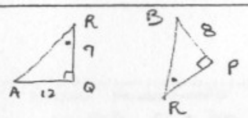
MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
(i) $\frac{d}{dx}[\sqrt{1+6x}] = \frac{d}{dx}[(1+6x)^{\frac{1}{2}}]$ $= \frac{1}{2}(1+6x)^{-\frac{1}{2}} \times 6$ $= \frac{3}{\sqrt{1+6x}}$ (1)	1	
Let $y = x^3\sqrt{1+6x}$ $\frac{dy}{dx} = 3x^2\sqrt{1+6x} + x^3 \times \frac{3}{\sqrt{1+6x}}$ $= \frac{3x^2(1+7x)}{\sqrt{1+6x}}$ (1)	1	
$y = \ln[\frac{x^4}{x-1}]$ (2) $= 4\ln x - \ln(x-1)$ $\frac{dy}{dx} = \frac{4}{x} - \frac{1}{x-1} = \frac{3x-4}{x(x-1)}$	1 1	
$y = \frac{5}{2}x^2 - \frac{1}{3}x^3 + 1.$ $\frac{dy}{dx} = 5x - x^2$ For y to be increasing: $\frac{dy}{dx} > 0.$ $\therefore 5x - x^2 > 0$ $x(5-x) > 0$  $\therefore 0 < x < 5$ to be increasing (2)		1 For $5x - x^2 > 0$ 1 For $0 < x < 5$
$\frac{4}{x^2-4} - \frac{1}{x-2} = 1.$ i.e. $\frac{4}{(x-2)(x+2)} - \frac{1}{x-2} = 1$ so $x \neq \pm 2$ $\Rightarrow 4 - (x+2) = x^2 - 4$ $2 - x = x^2 - 4$ so $x^2 + x - 6 = 0$ $(x+3)(x-2) = 0$ $\therefore x = -3$ or 2 but $x \neq 2$ $\therefore x = -3$ only (2)	$\frac{1}{2}$ 1 $\frac{1}{2}$	Note: $\frac{2-x}{x^2-4} = 1$ so $\frac{-1}{x+2} = 1$, $x \neq 2$ processing.

MATHEMATICS Extension 1 : Question 3 CONTINUED.

Suggested Solutions	Marks	Marker's Comments
(e)  $Y = \pi \sin x$ $y = 2x$ $0 \leq x \leq \frac{\pi}{2}$ $\therefore Y \leq y$ (2)	1 1	sketching each correctly.
(f) $a^{2x} + a = (1+a)a^x$ $a^{2x} - (1+a)a^x + a = 0$ $(a^x - 1)(a^x - a) = 0$ $a^x = 1$ or $a^x = a$ $\therefore x = 0$ or $x = 1$ (2)		• Let $u = a^x$
(g) $\frac{d}{dx}[\frac{x\sin x}{1+\cos x}]$ $= \frac{(1.\sin x + x\cos x)(1+\cos x) - x\sin x \times (-\sin x)}{(1+\cos x)^2}$ i.e. $= \frac{(\sin x + x\cos x)(1+\cos x) + x\sin^2 x}{(1+\cos x)^2}$ $= \frac{\sin x + \sin x\cos x + x\cos x + x\cos^2 x + x\sin^2 x}{(1+\cos x)^2}$ $= \frac{\sin x(1+\cos x) + x\cos x + x \times 1}{(1+\cos x)^2}$ $= \frac{\sin x(1+\cos x) + x(1+\cos x)}{(1+\cos x)^2}$ $= \frac{x + \sin x}{1+\cos x}$ (3)	2 1	

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
$\log_{a^n} 3 = \frac{\log_a 3}{\log_a a^n} = \frac{\log_a 3}{n\log_a a} = \frac{\log_a 3}{n}$ (1)	1	
(i) In Δs AQR and BPR 1. $\angle AQR = \angle BPR = 90^\circ$ (data) 2. $\angle ARQ = \angle BRP$ (vertically opposite angles are equal) 3. $\therefore \Delta AQR \parallel\!\!\mid \Delta BPR$ (equiangular) (2)	$\frac{1}{2}$ 1 $\frac{1}{2}$	
$\frac{9}{PR} = \frac{12}{8}$ (corresponding sides in similar triangles are in proportion) $\therefore PR = 6$	1	
$AR = 15$ (Pyth. Triad $<3,4,5>$) $BR = 10$ (" " $<3,4,5>$)	1	
Area $ABR = \frac{1}{2}\times 12\times(9+10) - \frac{1}{2}\times 12\times 9$ $= 60$ square units (3)	1	
$f(x) = \frac{27(x-1)^2}{(x+1)^3}$ $f'(x) = \frac{27[2(x-1)(x+1)^3 - (x-1)^2 \times 3(x+1)^2]}{(x+1)^6}$	1	
$= \frac{27(x-1)\cancel{(x+1)^2}[2(x+1) - 3(x-1)]}{\cancel{(x+1)^6}\,4}$	1	
$= \frac{27(x-1)[2x+2-3x+3]}{(x+1)^4}$ $= \frac{27(x-1)(5-x)}{(x+1)^4}$ q.e.d. (2)		
For S.P.s to occur $f'(x) = 0$ $\therefore (x-1)(5-x) = 0$ $\therefore x = 1$ or 5 $\therefore y = 0$ or 2	1	
$\therefore$ Stationary points are $(1,0)$ and $(5,2)$ (2)	1	

MATHEMATICS Extension 1 : Question 4 cont.

Suggested Solutions	Marks	Marker's Comments
(iii) TEST NATURE. at $x = 1$		

x	0	1	2
$f'(x)$	$\frac{27(-)(+)}{(+)}$ $\ominus$ ..... -135	0	$\frac{27(+)(+)}{(+)}$ $\oplus$ 1
	\	—	/

Since $f(x)$ is cont + diff'ble over $0 \le x \le 2$ and $f'(x)$ changes sign $(- \; 0 \; +)$
$\therefore$ a RELATIVE MIN T.P at $(1,0)$ (2) — 1

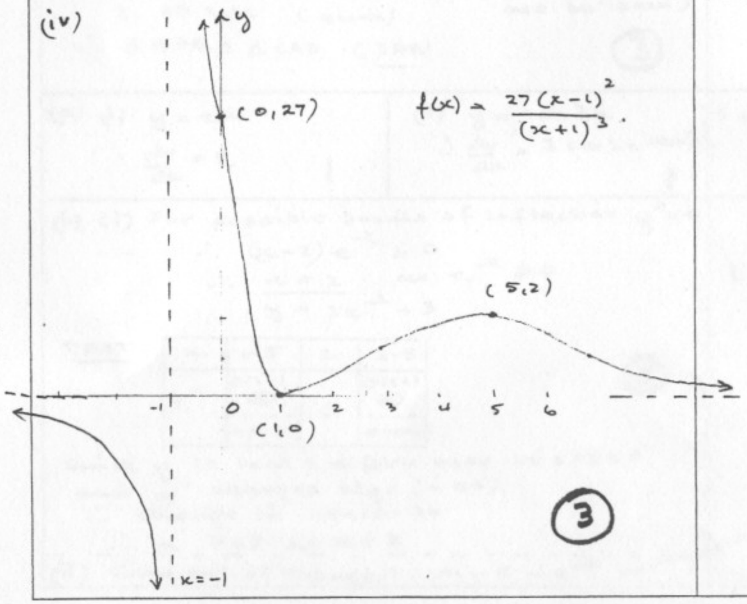
at $x = 5$ — 1

x	4	5	6
$f'(x)$	$\frac{27(+)(+)}{(+)}$ $\oplus$ ..... $0\cdot12..$	0	$\frac{27(+)(-)}{(+)}$ $\ominus$ ..... $0\cdot056..$
	/	—	\

Since $f(x)$ is cont + diff'ble over $4 \le x \le 6$ and $f'(x)$ changes sign $(+ \; 0 \; -)$

$\therefore$ RELATIVE MAX. T.P. at $(5,2)$

(iv)



$f(x) = \frac{27(x-1)^2}{(x+1)^3}$.

(3)

Marker's Comments:
1 For indicating VA $x = -1$, HA $y = 0$
1 For Min $(1,0)$, Max $(5,2)$
1 For correct shape and $(0,27)$

* CHECK CFPR // CFE.