Question 1. **Marks**

(a) Rationalise the denominator of: $\dfrac{4\sqrt{2}-3}{\sqrt{2}+5}$. **2**

(b) Simplify: $\dfrac{2}{x^2-4}-\dfrac{3}{x+2}+\dfrac{5}{x-2}$. **2**

(c) Evaluate: $\lim\limits_{x \to 0} \dfrac{\sin\left(\frac{x}{5}\right)}{2x}$. **2**

(d) Solve for t: $(\pi - t)(t-3) < 0$. **2**

(e) Differentiate the following with respect to x.

(i) ex^2. **1**

(ii) 7^x. **1**

(iii) $\dfrac{1}{\sqrt{1+x^2}}$. **2**

(iv) $\ln\left(\dfrac{x+1}{\sqrt{\sin 2x - 1}}\right)$. **3**

Question 2. **[Start a New page]** **Marks**

(a) If α and β are the roots of the equation $x^2 - 2x - 4 = 0$,

(i) Show that $\alpha^2 + \beta^2 = 12$. **2**

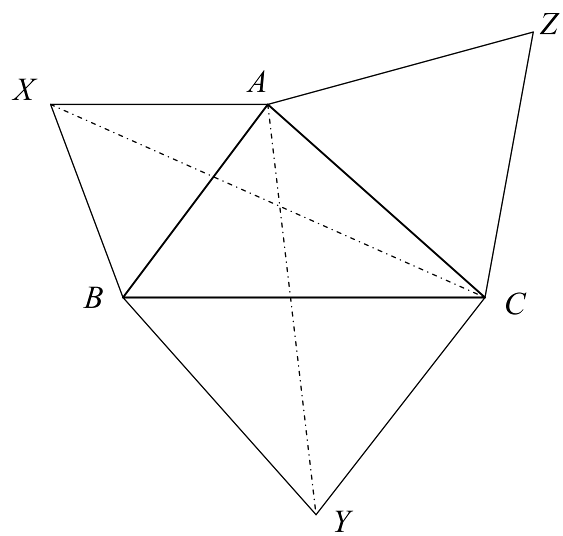
(ii) Hence, or otherwise find $\alpha^3 + \beta^3$. **1**

(b) Solve for x: $|x - 2| < 3 - |x|$. **3**

(c) Given $f(x) = 3x \ln^2 x$, find $f'(x)$. **3**

(d) Shade the region on the number plane where: $y > \sqrt{x}$. **2**

(e) Equilateral triangles *ABX, CBY* and *CAZ* are constructed externally on the sides *AB, BC* and *CA* of ΔABC respectively, as shown below.



Not to scale

(i) Copy the diagram onto your writing paper and Prove that: $\Delta BXC \equiv \Delta BAY$. **2**

(ii) Hence, or otherwise, explain why $BZ = AY = CX$. **2**

Question 3. **[Start a New Page]** **Marks**

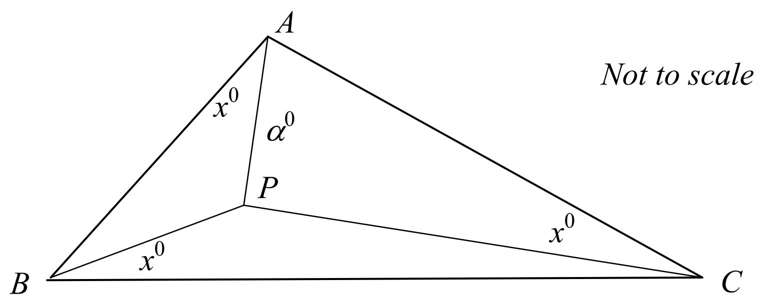
(a) Solve for x: $\sqrt{3}\tan(x-\frac{\pi}{3})=1$, for $0 \le x \le 2\pi$. **2**

(b) (i) Sketch the graph of $y=1-\cos\pi x$ over the domain $0 \le x \le 2$. **2**

(ii) Find the equation of the tangent to $y=1-\cos\pi x$ at $x=\frac{1}{3}$. **4**

(c) Prove that: $\dfrac{\sin\theta}{\cos\theta+\sin\theta}+\dfrac{\sin\theta}{\cos\theta-\sin\theta} \equiv \tan 2\theta$. **2**

(d) In $\Delta\, ABC$, $\angle CAB=\alpha^0$, $\angle ABC=\beta^0$ and $\angle BCA=\gamma^0$.
The point P is chosen internally in $\Delta\, ABC$ so that:
$\angle PAB=\angle PBC=\angle PCA=x^0$ as shown in the diagram.



(i) Show that: $\sin(\beta-x)^0=\dfrac{PA}{PB}.\sin x^0$. **1**

(ii) Hence show that: $\sin(\alpha-x)^0\sin(\beta-x)^0\sin(\gamma-x)^0=\sin^3 x^0$. **2**

(iii) Using the identity: $\cot p-\cot q=\dfrac{\sin(q-p)}{\sin p\sin q}$, deduce that: **2**

$(\cot x^0-\cot\alpha^0)(\cot x^0-\cot\beta^0)(\cot x^0-\cot\gamma^0)=\operatorname{cosec}\alpha^0\operatorname{cosec}\beta^0\operatorname{cosec}\gamma^0$.

Question 4. **[Start a New Page]** **Marks**

(a) Consider the statement: **2**

' For a point of inflection to exist at $x = c$ on a continuos curve $y = f(x)$, then $f'(c) = f''(c) = 0$ only '.
Discuss the truthfulness of this statement, giving reasons.

(b) Consider the curve: $y = (x + \frac{1}{x})^2 - 5(x + \frac{1}{x}) + 6\frac{1}{4}$.

(i) Determine the x-intercepts. **2**

(ii) Show that: $\frac{dy}{dx} = \left(1 - \frac{1}{x^2}\right)\left(2x - 5 + \frac{2}{x}\right)$. **2**

(iii) Hence determine the stationary points of this curve. **3**

(iv) Determine the nature of these stationary points. **3**

(v) Hence sketch the curve $y = (x + \frac{1}{x})^2 - 5(x + \frac{1}{x}) + 6\frac{1}{4}$. **3**
[show all essential detail].

THE END

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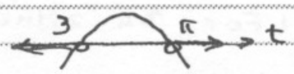
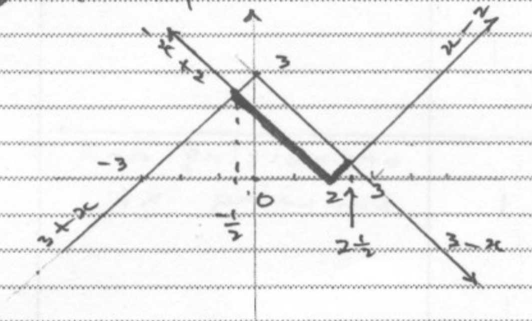
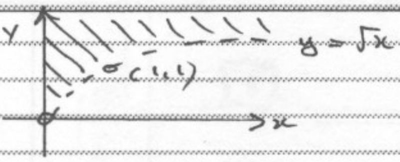
Y11 HALF YEARLY TERM 2, 2007.

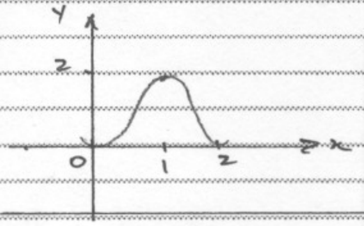
MATHEMATICS Extension 1 : Question…1….

Suggested Solutions	**Marks**	**Marker's Comments**
Q1(a) $\dfrac{4\sqrt{2}-3}{\sqrt{2}+5} \times \dfrac{(5-\sqrt{2})}{(5-\sqrt{2})} = \dfrac{20\sqrt{2}-8-15+3\sqrt{2}}{25-2}$ $\boxed{2}$	1	$\times \dfrac{\sqrt{2}-5}{\sqrt{2}-5}$
$= \dfrac{23\sqrt{2}-23}{23} = \underline{\sqrt{2}-1}$	1	
(b) $\dfrac{2}{x^2-4} - \dfrac{3}{x+2} + \dfrac{5}{x-2} = \dfrac{2}{(x+2)(x-2)} - \dfrac{3}{x+2} + \dfrac{5}{x-2}$		
$= \dfrac{2-3(x-2)+5(x+2)}{(x+2)(x-2)}$	1	
$= \dfrac{2-3x+6+5x+10}{(x+2)(x-2)} = \dfrac{2x+18}{(x+2)(x-2)}$ $\boxed{2}$	1	
(c) METHOD 1 $\lim\limits_{x\to 0} \dfrac{\sin\frac{x}{5}}{2x} = \lim\limits_{x\to 0} \dfrac{\sin\frac{x}{5}}{2\left(\frac{x}{5}\right)5}$	1	organising and showing ×1
$= \dfrac{1}{10}\times 1 = \dfrac{1}{10}$ $\boxed{2}$	1	
(d) $(\pi-t)(t-3)<0$ $\quad \pi>3$ $\boxed{2}$  $\therefore\ \underline{t<3 \text{ or } t>\pi}$	1, 1	1 for '$t<3$' 1 for "or $t>\pi$' Al 1 for $3<t<\pi$
(e) (i) Let $y=ex^2$ $y' = \underline{2ex}$ ✓ $\boxed{1}$		
(ii) $y=7^x = e^{x\ln 7}$ $y' = \underline{\ln 7 . 7^x}$ ✓ $\boxed{1}$		
(iii) $y = \dfrac{1}{\sqrt{1+x^2}} = (1+x^2)^{-\frac{1}{2}}$		
$y' = -\frac{1}{2}(1+x^2)^{-\frac{3}{2}} \times 2x$ $\boxed{2}$	1	
$= -x(1+x^2)^{-3/2}$	1	
(iv) $y = \ln \dfrac{x+1}{\sqrt{\sin 2x-1}} = \ln(x+1) - \frac{1}{2}\ln(\sin 2x-1)$	1	
$y' = \dfrac{1}{x+1} - \dfrac{\cancel{2}\cos 2x}{\cancel{2}(\sin 2x-1)}$	1, 1	
$= \dfrac{1}{x+1} - \dfrac{\cos 2x}{\sin 2x-1}$ $\boxed{3}$		

MATHEMATICS Extension 1 : Question 2

Suggested Solutions	Marks	Marker's Comments
Q 2(a) $x^2 - 2x - 4 = 0$ $\boxed{2}$		
(i) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$	1	
$= 2^2 - 2\times(-4)$	1	
$= 4 + 8 = \underline{12}$ q.e.d.		
(ii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)$ $\boxed{1}$		$\frac{1}{2}$
$= 2(12 - (-4))$		
$= \underline{32}$	1	$\frac{1}{2}$
(b) $\lvert x-2 \rvert < 3 - \lvert x \rvert$		
		
$-x+2 = 3+x$		
$-1 = 2x$		
$\therefore x = -\frac{1}{2}$	1	
$3-x = x+2$		
$5 = 2x$		
$x = 2\frac{1}{2}$	1	
$\therefore \underline{-\frac{1}{2} < x < 2\frac{1}{2}}$ $\boxed{3}$	1	
(c) $f(x) = 3x\ln^2 x$		1 For $3\ln^2 x +$
$f'(x) = 3\ln^2 x + 3x \times 2\ln x \times \frac{1}{x}$	1, 1	1 For $3x \cdot 2 \cdot \ln x \cdot \frac{1}{x}$
$= 3\ln^2 x + 6\ln x$ $\boxed{3}$	1	
(d) $y > \sqrt{x}$		
 $\boxed{2}$		
(e) (i) In Δs BXC and BAY		1 For BX = AB or BC = BY
S 1. BX = AB (given data)		
A 2. $\angle XBC = \angle ABY = 60° + \angle ABC$ ✓	1	
(all angles in equilateral Δ are $60°$)		
S 3. BC = BY (given data) $\boxed{2}$		
$\therefore \Delta BXC \equiv \Delta BAY$ (SAS)		
(ii) CX = AY (corresponding sides in congruent triangles are equal) 1		
By similar argument $\Delta ABY \equiv \Delta ZCB$	1	1 For a sound argument.
Hence BZ = AY = CX $\boxed{2}$		

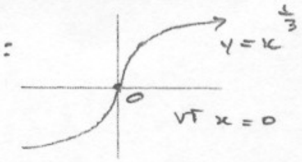
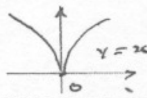
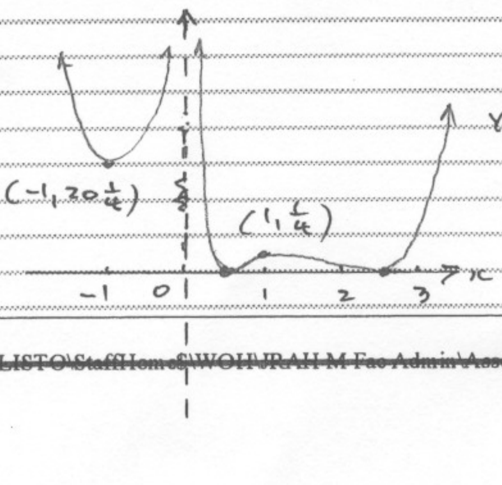
MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
Q3 **(a)** $\sqrt{3}\tan\left(x-\frac{\pi}{3}\right) = 1$ $\tan\left(x-\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$ $\boxed{2}$ $x-\frac{\pi}{3} = \frac{\pi}{6}$ or $\pi+\frac{\pi}{6}$ $x = \frac{3\pi}{6}$ or $\frac{9\pi}{6}$ $\quad \frac{\pi}{2}, \frac{3\pi}{2}$	1, 1	
(b) (i)  $y = 1-\cos\pi x, \ 0 \le x \le 2$ $\boxed{2}$		
(ii) $y = 1-\cos\pi x$ at $x=\frac{1}{3}$: $y = 1-\cos\frac{\pi}{3} = \frac{1}{2}$ ✓ $\left(\frac{\pi}{3}, \frac{1}{2}\right)$ $\boxed{4}$ $y' = \pi\sin\pi x$ ✓ Gradient of tangent at $x=\frac{1}{3}$: $m_T = \pi\times\frac{\sqrt{3}}{2}$ ✓ Equation of tangent at $x=\frac{1}{3}$ $y-\frac{1}{2} = \frac{\pi\sqrt{3}}{2}\left(x-\frac{1}{3}\right)$ ✓ $y = \frac{\pi\sqrt{3}}{2}x + \frac{1}{2} - \frac{\pi\sqrt{3}}{6}$	1 1 1 1	$3\pi\sqrt{3}x - 6y + 3 - \pi\sqrt{3} = 0$
(iii)	1 1	
(c) $\frac{\sin\theta}{\cos\theta+\sin\theta} + \frac{\sin\theta}{\cos\theta-\sin\theta} =$ $\boxed{2}$ $= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta\cos\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$ $= \frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$ ✓ $= \frac{\sin 2\theta}{\cos 2\theta}$ $= \tan 2\theta$ q.e.d.	1 1	

MATHEMATICS Extension 1 : Question 3

Suggested Solutions	Marks	Marker's Comments
3(d) (i) In ΔAPB, using the sine rule		
$\dfrac{PA}{\sin(\beta - x)^\circ} = \dfrac{PB}{\sin x^\circ}$ or $\boxed{1}$		
$\dfrac{\sin(\beta - x)^\circ}{PA} = \dfrac{\sin x^\circ}{PB}$	1	
$\therefore \sin(\beta - x)^\circ = \dfrac{PA}{PB} . \sin x^\circ$		
(ii) From (i) $\sin(\alpha - x)^\circ = \dfrac{PC}{PA} \sin x^\circ$	1	
and $\sin(\gamma - x)^\circ = \dfrac{PB}{PC} \sin x^\circ$		
$\therefore \sin(\alpha - x)^\circ \sin(\beta - x)^\circ \sin(\gamma - x)^\circ$ $\boxed{2}$		
$= \dfrac{PC}{PA} \sin x^\circ \times \dfrac{PA}{PB} \sin x^\circ \times \dfrac{PB}{PC} \sin x^\circ$	1	
$= \sin^3 x^\circ$ qed		
(iii) $(\cot x^\circ - \cot \alpha^\circ)(\cot x^\circ - \cot \beta^\circ)(\cot x^\circ - \cot \gamma^\circ)$		
$= \dfrac{\sin(\alpha - x)^\circ}{\sin \alpha^\circ \sin x^\circ} \times \dfrac{\sin(\beta - x)^\circ}{\sin \beta^\circ \sin x^\circ} \times \dfrac{\sin(\gamma - x)^\circ}{\sin \gamma^\circ \sin x^\circ}$	1	
$= \dfrac{\cancel{\sin^3 x^\circ}}{\cancel{\sin^3 x^\circ} \sin \alpha \sin \beta \sin \gamma}$ using (ii) $\boxed{2}$	1	
$= \dfrac{1}{\sin \alpha^\circ \sin \beta^\circ \sin \gamma^\circ}$		
$= \operatorname{cosec} \alpha^\circ \operatorname{cosec} \beta^\circ \operatorname{cosec} \gamma^\circ$ qed.		

MATHEMATICS Extension 1 : Question 4

Suggested Solutions	Marks	Marker's Comments
4(a) This is not 100% True. A POI is a point where the curve changes its concavity. Eg: $y = x^{1/3}$ at $x = c = 0$ $f(0) = 0$ but $f'(0)$ and $f''(0) \neq 0$ $\therefore$ V.P.O.I at $c = 0$ $\boxed{2}$	1 1	Eg:   $y = \frac{a}{b + ce^{-x}}$
(b)(i) Let $u = x + \frac{1}{x}$ $\therefore u^2 - 5u + 6\frac{1}{4} = 0$ $u = \frac{5}{2}$ $\boxed{2}$ $x + \frac{1}{x} = \frac{5}{2}$ $2x^2 - 5x + 2 = 0$ } $x = \frac{1}{2}$ or 2	1, 1	x-intercepts $x = \frac{1}{2}$ and 2
(ii) $y = (x + \frac{1}{x})^2 - 5(x + \frac{1}{x}) + 6\frac{1}{4}$ $\boxed{2}$ $\frac{dy}{dx} = 2(x + \frac{1}{x})(1 - \frac{1}{x^2}) - 5(1 - \frac{1}{x^2})$ $= (1 - \frac{1}{x^2})(2(x + \frac{1}{x}) - 5)$	1 1	
(iii) For SPs to occur $\frac{dy}{dx} = 0$ $\therefore (1 - \frac{1}{x^2})(2x - 5 + \frac{2}{x}) = 0$ $\boxed{3}$ $\therefore x = \pm 1, \frac{1}{2}$ or 2 $\therefore$ SPs are $(-1, 20\frac{1}{4})$, $(1, \frac{1}{4})$, $(\frac{1}{2}, 0)$ and $(2, 0)$	• 1, 1 1	
(iv) • From (i) and (iii) $(\frac{1}{2}, 0)$, $(2, 0)$ are min TPs. At $(-1, 20\frac{1}{4})$ is a min TP $\boxed{3}$ At $(1, \frac{1}{4})$ is a max TP		1 1 For testing at $x = -1$ 1 For testing at $x = 1$ [1st or 2nd deriv]
(v)  $\boxed{3}$		$\frac{1}{2}$ For VA at $x = 0$ $\frac{1}{2}$ For min TPs $\frac{1}{2}$ For max TP $\frac{1}{2}$ For shape

logistic curves
pH curved

Note: $y = (x + \frac{1}{x})^2 + 5(x + \frac{1}{x}) + 6\frac{1}{4}$
$= x^2 + 5x + x^{-2} + 5x^{-1} + 8.25$