Question 1 (15 Marks) **Marks**

(a) Solve for x: $5^x.25^{2x+1} = 125^x$. **2**

(b) Solve for m: $|m + 2| > 1 - |m|$ **2**

(c) Evaluate $\sqrt{2}\tan 60^\circ - \dfrac{\cos 30^\circ}{\sin 45^\circ}$ **2**

(d) If $x^2 + 3xy - 4y^2 = 0$, then the ratio $\dfrac{x}{y}$ has one value of unity. **1**

Find the other value.

(e) If $\tan A = -\dfrac{12}{5}$ and $\sin A > 0$, evaluate $\sec A$. **1**

(f) Differentiate

(i) $3\sqrt{\pi} - 2x^5$ **1**

(ii) $\sin x \cos x$ **2**

(iii) $\sqrt{(\ln 2x)^7}$ **2**

(iv) $\dfrac{2x}{4x+1}$ **2**

Question 2 (15 Marks) [Start a New Page]

(a) If $\log_{10} y = \dfrac{2}{3}\log_{10} x - 2$, express y in terms of x with no logarithmic expression. **3**

(b) Find the gradient of the tangent to the curve $y = e^{x^2}$ when $x = 1$. **2**

(c) $ABCDE$ is a regular pentagon. Diagonals BD and CE intersect at P.

(i) With the aid of a diagram, show that $\Delta CDE \equiv \Delta BCD$. **3**

(ii) Show that $PD = PC$. **2**

(iii) Hence, or otherwise, show that $ABPE$ is a kite. **2**

(d) Differentiate, by first principles the function, $f(x) = \sqrt{x+2}$. **3**

Question 3 (15 Marks) [Start a New Page] **Marks**

(a) Write the domain and range of $y = \ln(1 - x^2)$. **2**

(b) Find the values of x for which the curve $y = x^3 - 12x$, is monotonically increasing. **2**

(c) Find $\lim\limits_{x \to 0} \dfrac{2\sin x}{\tan\left(\dfrac{x}{2}\right)}$, giving reasons. **2**

(d) Show that $y = \dfrac{x}{x^3 - x}$ an even function & state the restriction on x. **2**

(e) If $\cos x = \dfrac{9 - a^2}{9 + a^2}$, find $\sin x$ in terms of a. **2**

(f) Draw a neat sketch of $y = \dfrac{(x+1)^2}{(x-1)(x+2)}$, clearly showing all important features (calculus is not required). **3**

(g) Find the point P that divides the interval AB internally in the ratio 5:3, if $A = (3, -7)$ and $B = (8, 2)$. **2**

Question 4 (15 Marks) [Start a New Page] **Marks**

(a) Find the first 2 positive values of x such that $\sin 2x = 0 \cdot 1$. . (Answer to 1 decimal place). **2**

(b) (i) Show that the locus of a point P such that $AP \perp BP$, where A and B are (1, 2) and (3, 5), respectively, is $(x - 2)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{13}{4}$ **2**

(ii) Show that the line $3x + y = 2$ does not touch nor cross the equation given in part b (i). **3**

(c) Find any points of inflexion on the curve $y = x^4 - x^2 + 1$ **3**

(d) Consider the function $y = \sqrt[3]{x^4} - 1$.

(i) Show that it has only one stationary point. **2**

(ii) Test the nature of this stationary point. **2**

(iii) Draw a neat sketch of this function. **1**

THE END

year 11 Ext 1 2008 T1 exam

Q1

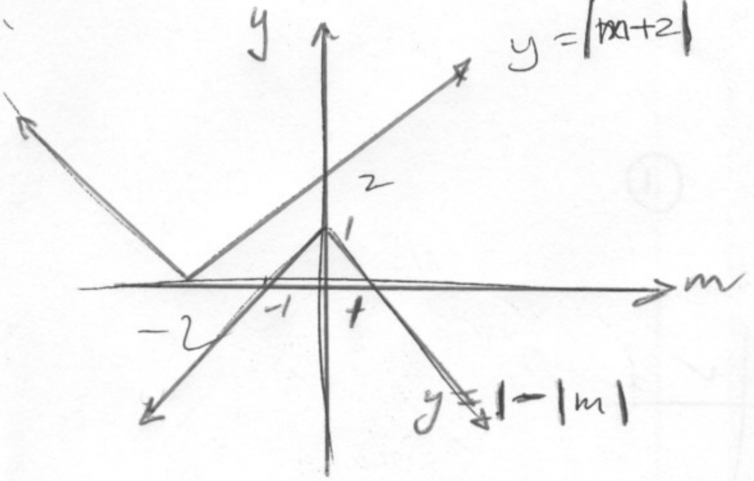
(a) $5^{x} \times 25^{2x+1} = 125^{x}$

$\therefore x + 4x + 2 = 3x$ 1

$2x = -2$

$x = -1$ 1

(b)



all real m.

(c) $\sqrt{2}\tan 60^\circ - \dfrac{\cos 30^\circ}{\sin 45^\circ} = \sqrt{2} \times \dfrac{\sqrt{3}}{2} - \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$ (1)

$= \dfrac{\sqrt{3}}{\sqrt{2}} - \dfrac{\sqrt{3}}{\cancel{2}\sqrt{2}} \times \cancel{\sqrt{2}}$ (1)

$= 0.$

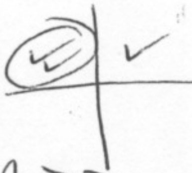
(d) $x^2 + 3xy - 4y^2 = 0$

$(x + 4y)(x - y) = 0$

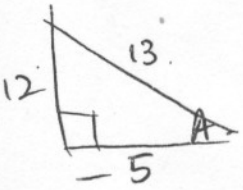
$x = -4y$ or $x = y$

$\dfrac{x}{y} = -4$ $\quad \therefore \dfrac{x}{y} = 1.$ (1)

e) $\tan A = -\dfrac{12}{5}.$



second Q as $\sin A > 0.$



$\sec A = \dfrac{1}{\cos A}$

$= \dfrac{-13}{5}.$ (1)

(f) (i) $\dfrac{d}{dx}\left(3\sqrt{\pi} - 2x^5\right) = -10x^4$ (1)

(ii) $\dfrac{d}{dx} \underset{u}{\sin x}\ \underset{v}{\cos x} =$

$= \sin x(-\sin x) + \cos x(\cos x$

$= \cos^2 x - \sin^2 x$ (1)

or $1 - 2\sin^2 x$

or $2\cos^2 x - 1$

(f)(iii) $\frac{d}{dx}[\ln 2x]^{\frac{7}{2}} = \frac{7}{2}[\ln 2x]^{5/2} \times \frac{1}{x}$ 3

$= \frac{7}{2x}[\ln 2x]^{5/2}$ 2

(iv) $\frac{d}{dx}\left(\frac{2x}{4x+1}\right) = \frac{(4x+1)(2)-(2x)(4)}{(4x+1)^2}$ 1

$= \frac{2}{(4x+1)^2}$ 1

Q2.

(a) $\log_{10} y = \frac{2}{3}\log_{10} x - 2$

$= \log_{10} x^{2/3} - 2.$

$= \log_{10} x^{2/3} - \log_{10} 100$ 1

$= \log_{10}\left(\frac{x^{2/3}}{100}\right)$ 1

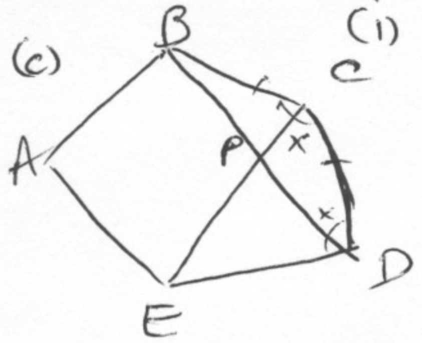
$\therefore y = \frac{x^{2/3}}{100}.$ 1

(b) $y = e^{x^2}$

$y' = 2xe^{x^2}$ 1

$\therefore$ when $x=1$, $y' = 2e.$ 1

(c)



(i) In $\triangle CDE$ & $\triangle BCD$.

$BC = CD$ (ABCDE is a regular pentagon)

$\angle BCD = \angle CDE$ (equal angles of a regular pentagon).

$CD = ED$ (equal sides of regular pentagon)

$\therefore \triangle CDE \equiv \triangle BCD$ (SAS) 3.

(ii) $\angle ECD = \angle CDB$ (corresponding angles of congruent triangles equal).

$\therefore PC = PD$ (equal sides opposite equal angles in $\triangle CDP$)

2

(c) (iii) BD = CE (corresponding sides of congruent triangles equal) $\frac{1}{2}$ (4)

PD = PC (from ii)

$\therefore$ BD − PD = CE − CP

$\therefore$ BP = PE $\frac{1}{2}$

since AB = AE. (sides of regular pentagon) $\frac{1}{2}$

$\therefore$ ABPE is a kite (two pairs of adjacent sides equal) $\frac{1}{2}$

(d) $f(x) = \sqrt{x+2}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \quad (1)$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \times \frac{\sqrt{x+h-2} + \sqrt{x+2}}{\sqrt{x+h-2} + \sqrt{x+2}}$$

$$= \lim_{h \to 0} \frac{x+h+2 - (x+2)}{h(\sqrt{x+h-2} + \sqrt{x+2})}$$

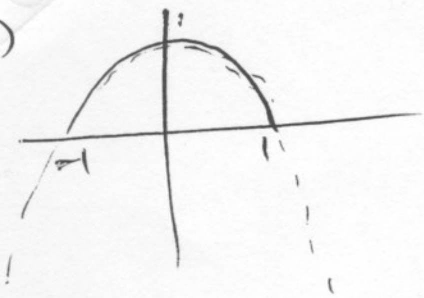
$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h-2} + \sqrt{x+2}} \quad (1)$$

$$= \frac{1}{2\sqrt{x+h}}. \quad (1)$$

5

Q3.

(a)



Domain : $\{x : -1 < x < 1\}$ 1

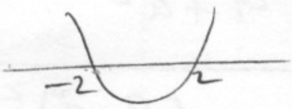
Range : $\{y : y \le 0\}$. 1

(b) $y = x^3 - 12x$

$y' = 3x^2 - 12$ 1

$y' > 0$ when $3x^2 > 12$

$x^2 > 4$.



$\therefore \{x : x < -2 \cup x > 2\}$. 1

(c) $\lim\limits_{x \to 0} \dfrac{2\sin x}{\tan(\frac{x}{2})} = \lim\limits_{x \to 0} \dfrac{4 \sin(\frac{x}{2})\cos(\frac{x}{2})}{\frac{\sin(\frac{x}{2})}{\cos(\frac{x}{2})}}$ 1

$= \lim\limits_{x \to 0} 4\cos^2(\frac{x}{2})$

$= 4.$ 1

Alternatively: $\lim\limits_{x \to 0} 2\sin x \times \dfrac{1}{\tan(\frac{x}{2})}$

$= \lim\limits_{x \to 0} 2 \dfrac{\sin x}{x} \times \dfrac{x}{\cancel{1}} \times \dfrac{1}{\frac{x}{2}} \times \dfrac{\frac{x}{2}}{\tan\frac{x}{2}}$

$= 2 \times 1 \times x \times \dfrac{2}{x} \times 1$

$= 4$

$x^3 - x$, $x \neq 0$.

$$f(-x) = \frac{-x}{(-x)^3 - (-x)}$$

$$= \frac{-x}{-x^3 + x}$$

$$= \frac{x}{x^3 - x}$$

$$= f(x) \quad \therefore \text{ even.} \quad (2)$$

e) $$\cos x = \frac{9 - a^2}{9 + a^2}$$

$$\sin x = \pm \sqrt{1 - \frac{(9-a^2)^2}{(9+a^2)^2}} \quad \left(\text{using } \sin^2 x + \cos^2 x = 1\right)$$

(1)

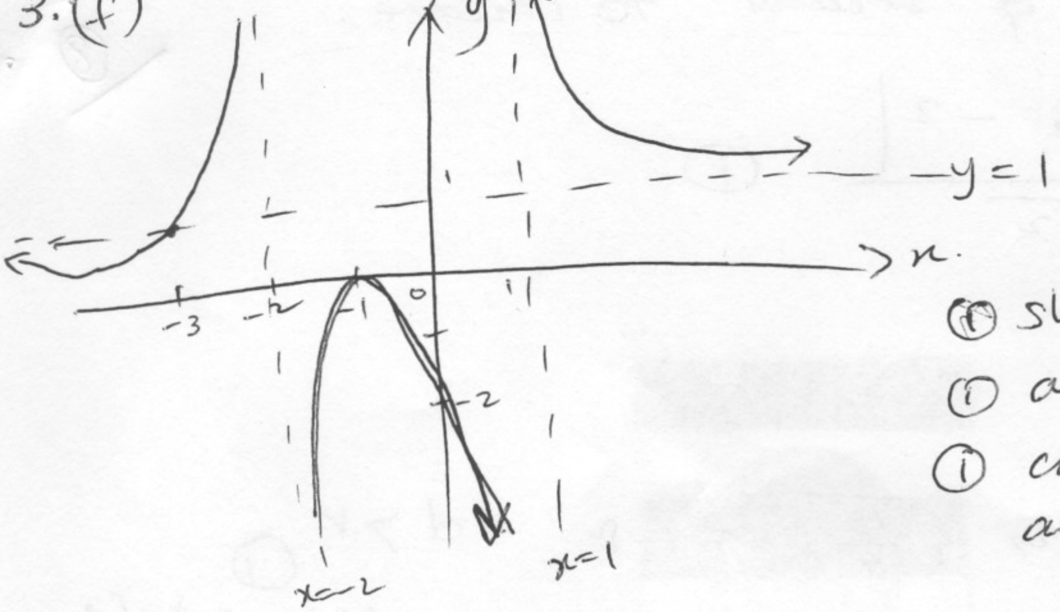
$$= \pm \frac{\sqrt{(9+a^2)^2 - (9-a^2)^2}}{9+a^2}$$

$$= \pm \frac{\sqrt{81 + 18a^2 + a^4 - 81 + 18a^2 - a^4}}{9 + a^2}$$

$$= \pm \frac{\sqrt{36a^2}}{9+a^2}$$

$$= \pm \frac{6a}{9+a^2} \quad (1)$$

3.(f)



① shape
① asymptotes
① critical pt $x=-3$ and intercepts.

⑦

(g) $A(3,-7)$ $B(8,2)$

$5:3$

$$P = \left[\frac{9+40}{8}, \frac{-21+10}{8}\right] = \left(6\tfrac{1}{8}, -1\tfrac{3}{8}\right)$$ ①

① correct formula.

Q4.

(a) $2x = 0.1$ or 3.04.

$x = 0.05$ or 1.52

$x = 0.1$ and 1.5 (1 dp). ②

(b) (i) $\dfrac{y-2}{x-1} \times \dfrac{y-5}{x-3} = -1$ ①

$\therefore y^2 - 7y + 10 = -(x^2 - 4x + 3)$

$x^2 + 4x + 4 + y^2 - 7y + \frac{49}{4} = -13 + 4 + \frac{49}{4}$ ①

$\therefore (x+2)^2 + (y - \frac{7}{2})^2 = \frac{13}{4}$.

4(b) (ii) Distance of secant to centre.

8

$$d = \frac{\left| 3\times 2 + \frac{7}{2} - 2 \right|}{\sqrt{3^2+1^2}} \quad (1)$$

$$= \frac{15}{2\sqrt{10}} \quad (1)$$

$$\approx 2.37$$

Now radius is $\sqrt{\frac{13}{4}} \approx 1.8$ $\therefore d > r$ (1)

$\therefore$ line doesn't touch nor cross the circle.

(c) $y = x^4 - x^2 + 1$

$y' = 4x^3 - 2x$) (1)

$y'' = 12x^2 - 2$

For possible points of inflexion $y'' = 0$

$\therefore x^2 = \frac{1}{6}$ $\therefore x = \pm\frac{1}{\sqrt{6}}$

test. $x = \frac{1}{\sqrt{6}}$ $x = -\frac{1}{\sqrt{6}}$

x	0	$\frac{1}{\sqrt{6}}$	0.5
y''	-2	0	1
	$-$	0	$+$

$\therefore$ change in concavity

x	$-\frac{1}{2}$	$-\frac{1}{\sqrt{6}}$	0
y''	1	0	-2
	$+$	0	$-$

(1)

$\therefore$ change in concavity.

$\therefore$ points of inflexion at

$\left(\frac{1}{\sqrt{6}}, \frac{31}{36}\right)$ and $\left(-\frac{1}{\sqrt{6}}, \frac{31}{36}\right)$ (1)

(d) $y = x^{4/3} - 1$

(i) $y' = \frac{4}{3}x^{\frac{1}{3}}$ 9

For stat. pt $y' = 0$ $\therefore x = 0$. only

$\therefore$ stat pt at $(0, -1)$. 1

(ii) now $y'' = \frac{4}{9}x^{-2/3}$

at $x = 0$, $y'' = 0$ $\therefore$ use $\frac{dy}{dx}$ to test.

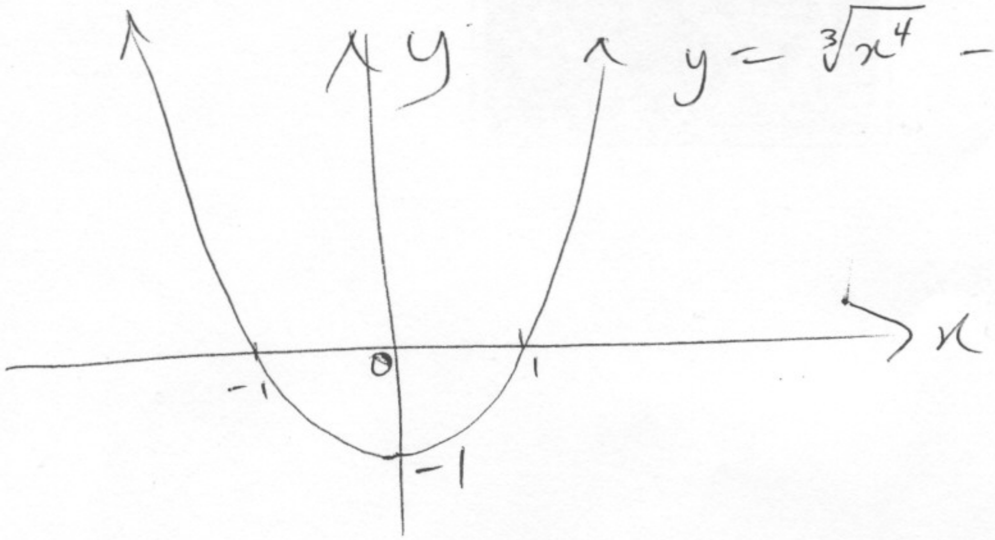
x	-0.1	0	0.1
y'	-0.62	0	0.62

1

$-$ 0 $+$

$\therefore$ minimum at $(0, -1)$. 1

(iii) $y = \sqrt[3]{x^4} - 1$



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