KNOX GRAMMAR SCHOOL

YEAR 11 HALF YEARLY EXAMINATION

1997

MATHEMATICS

3/4 UNIT COMMON PAPER

Time allowed: Two hours
(includes reading time)

INSTRUCTIONS

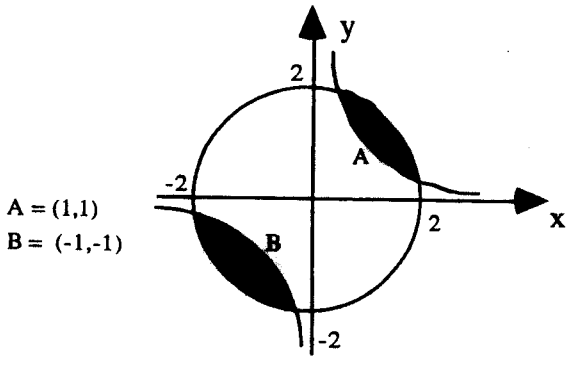
ALL questions should be attempted
ALL questions are of equal value
ALL necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved calculators may be used.
Each question should begin on a new page

QUESTION 1 16 marks

(a) Solve $|3-x| \leq 2$

(b) Find the domain and range of $y = \sqrt{4-x^2}$

(c) Write a set of inequalities that would describe this shaded area.



(d) (i) Find the perpendicular distance from the point (-3,1) to the line $3x-4y-2=0$

(ii) Hence find the equation of a circle with centre at (-3,1) which has the line $3x-4y-2=0$ as a tangent.

(e) Determine with adequate reasoning whether the function $y = 2^x + 2^{-x}$ is ODD or EVEN or NEITHER.

(f) $P(x) = 5-2x+3x^2-3x^3$ has roots α, β, γ. Find the values of

(i) $\alpha+\beta+\gamma$ (ii) $\alpha\beta+\alpha\gamma+\beta\gamma$ (iii) $\alpha^2+\beta^2+\gamma^2$

(g) For $kx^2-(k+1)x+5=0$ find the value of k if its roots are the reciprocals of each other.

QUESTION 2 (16 marks) **Start a new page**

(a) Find $\frac{dy}{dx}$ if (i) $x = 2t$ and $y = \frac{1}{t}$

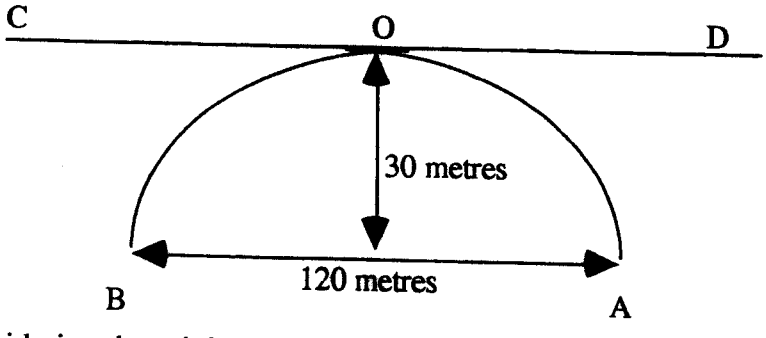
(ii) $y = \frac{x^2+1}{(x-9)^2}$, and give answer in simplest form.

(b) Evaluate $\lim_{x\to\infty} \frac{3x^2-5x+4}{7x^2+2}$.

(c) (i) Differentiate $y = x^2 + 2x - 1$ using the first principles method,

(ii) Hence find the slope of the tangent at the point where $x = 1$.

(d) The bridge CD drawn below is supported by an inverted parabolic arch which spans a river between its banks A and B, 120 metres wide. The top of the bridge is 30 metres above the water.



Considering the origin to be at point O,

(i) write down the coordinates of A

(ii) find the cartesian equation describing the parabolic bridge support.

(e) (i) Using a neat, fully labelled diagram sketch the graph of $y = (x+1)^2(x-3)^3$. Include both x and y intercepts.

(ii) Using the graph in part (i) find all values of x such that $(x+1)^2(x-3)^3 \geq 0$

QUESTION 3. 16 marks **Start a new page**

(a) Find the values of a, b, and c if $2x^2 - x - 9 \equiv a(x-1)^2 + b(x-1) + c$

(b) Find any value of k so that $x^2 - 2x + (3k-1) = 0$ will have rational roots. Justify your answers.

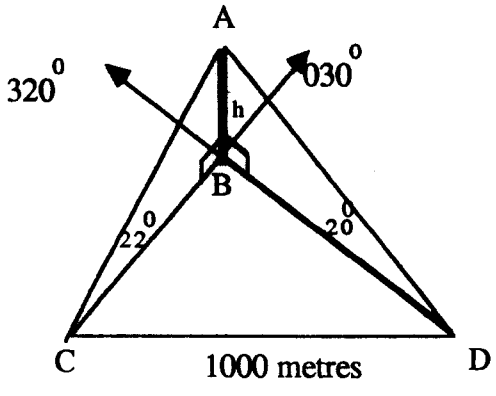
(c) Solve $\dfrac{2}{3-2x} < 5$

(d) The tower AB is vertical, and stands on horizontal ground. From a point C on ground level the bearing of the tower is 030^0 T and the angle of elevation of the top of the tower AB is 22^0.
From a point D on ground level the bearing of the tower AB is 320^0T and the angle of elevation of the top of the tower is 20^0.

(i) Find the size of $\angle CBD$.

(ii) Find expressions for CB, and BD in terms of h

(iii) If CD is 1000 metres use the Cosine Rule to find the height of the tower h to the nearest metre.



(f) When $P(x) = ax^3 - 6bx^2 - 2x + 3$ is divided by $x - 2$ the remainder is 3, and $x - 1$ is a factor of $P(x)$. Find the values of a and b.

QUESTION 4 16 marks. **Start a new page**

(a) Given ΔABC in which $\angle A = 40^0$, BC = 20 cm, and AC = 30 cm.

(i) Find two values for $\angle$ B (to the nearest degree) and draw a triangle ΔABC for each separate value of $\angle$ B.

(ii) For the obtuse angle value of $\angle$ B, find the length of AB to 1 decimal place.

(b) Prove $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2\tan\theta$

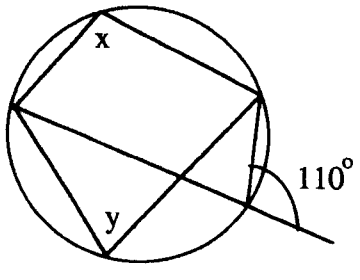
(c) Find all values of θ for which $3\tan^2 2\theta - 1 = 0$ for $0 \le \theta \le 360^0$

(d) Find the coordinates of the points of intersection of the curve $y = x^3 + x + 1$ and the line $y = 2x + 1$.

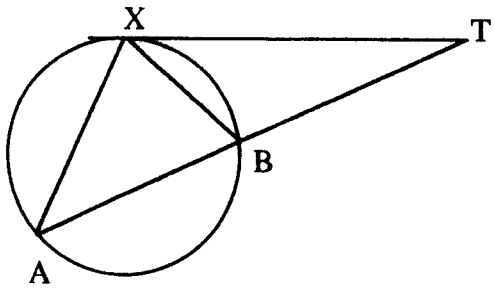
(e) Use Newton's method once to find another approximation to a root near $x = 1$ for $f(x) = x^3 + 3x - 2$.

QUESTION 5 16 marks. **Start a new page**

(a) Find the value of the pronumerals giving reasons.



(b) In the diagram below XT is a tangent to a circle at X. A, B are intercepts made on the circle by the secant drawn from the external point T.



(i) Prove ΔAXT is similar to ΔBXT

(ii) Show that $XT^2 = TA.TB$

(iii) Given XT = 6cm, AB = 7cm, find the length of TB in surd form.

(Hint: Let TB = x cm).

(c) (i) Factorize $p^3 - q^3$

(ii) Write down the coordinates of the point which divides the interval $A(x_1, y_1)$ $B(x_2, y_2)$ in the ratio of m : n

(d) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$ which form a focal chord. The chord PQ is divided <u>externally</u> in the ratio of q: p to produce point R (x,y).

(i) Given that for any focal chord $pq = -1$, show that the coordinates of the point R(x,y) are given by $x = 2a(q + p)$, $y = a(q^2 + pq + p^2)$.

(ii) Show that the locus of the point R is given by the parabola $x^2 = 4a(y - a)$

End of examination.