QUESTION 1 *(12 marks)*

(a) Factorise

(i) $16ab - 2a^4b$,

(ii) $a^2 - (b-1)^2$.

(b) Solve for x:

(i) $|x+3| = 2x$,

(ii) $\dfrac{x+4}{x-2} \geq 3$.

(c) Prove the following is a rational number:

$$\frac{2}{3-\sqrt{3}} + \frac{11}{6+\sqrt{3}}.$$

QUESTION 2 *(12 marks)* **Start this question on a new page.**

(a) Solve $x^2 + 2x - 4 + \dfrac{3}{x^2+2x} = 0$, by making a suitable substitution.

(b) (i) On the same set of axes, sketch the graphs of $y = 2|x|$ and $y = |x+3|$.

(ii) On your diagram, shade in the region where $y \leq 2|x|$ and $y \geq |x+3|$ hold simultaneously.

(c) A is the point (-2, -1). B is the point (1, 5). Find the co-ordinates of the point Q, which divides AB externally in the ratio 5:3

QUESTION 3 *(12 marks)* **Start this question on a new page.**

(a) (i) Show that $y = \dfrac{1}{1+x^2}$, is an even function.

(ii) What value(s) does y approach as x approaches large values.

(iii) Find the range of the function.

(iv) Hence, sketch the curve, $y = \dfrac{1}{1+x^2}$.

(b) (i) State the conditions that the quadratic expression $ax^2 + bx + c$ is negative definite.

(ii) Hence or otherwise show that the expression $(k^2 - k)x^2 - (2k-6)x + 2$ can never be negative definite.

(iii) Find the range of values of k for which the expression is positive definite.

(c) A man wishes to form a rectangular enclosure using an existing fence as one side. If he has 20 metres of fencing available to form the other three sides, find the area of the largest enclosure he can form and its dimensions.

Q4 (cont).../page 2

QUESTION 4 *(12 marks)* **Start this question on a new page.**

(a) For the function $y = \frac{x-4}{x-2}$, $x \neq 2$, sketch the graph of the function, showing clearly the co-ordinates of any points of intersection with the x-axis and the y-axis, and also the equations of any asymptotes. Hence, or otherwise, give the domain and range.

(b) (i) Show that there is only one possible solution to the pair of simultaneous equations below:

$$(x-4)^2 + y^2 = 4$$

$$\frac{x}{\sqrt{3}} - y = 0$$

(ii) Describe what this result means in terms of the graphs of these two equations.

(c) Express $4x^2 - 15x + 16$ in the form $A(x-1)^2 + B(x-1) + C$.

QUESTION 5 *(12 marks)* **Start this question on a new page.**

(a) If the roots of $4kx^2 - (3k-4)x + 1 = 0$ are reciprocals, find the value of k.

(b) (i) Find the centre and the radius of the circle C whose equation is

$$x^2 + y^2 - 4x + 6y - 12 = 0.$$

(ii) Find, in terms of the constant k, the length of the perpendicular from the centre of C to the line L whose equation is $3x + 4y = k$.

(iii) Hence find the values of k for which L is a tangent to C.

(c) The equation $(x - 3y + 5) + k\,(x + 2y) = 0$ represents a family of straight lines passing through a fixed point P.

(i) For what value of k does one of the lines in the family pass through the the point (0,5).

(ii) For what value of k is one of the lines in the family parallel to the straight line $x + y = 2$?

(iii) Find the co-ordinates of P.

END OF TEST

Newington College Extension 1 Common Test 2 1999

Question 1

a) (i) $16ab - 2a^4b = 2ab(8 - a^3)$

$= 2ab(2-a)(4+2a+a^2)$

(ii) $a^2 - (b-1)^2 = [a-(b-1)][a+(b-1)]$

$= (a-b+1)(a+b-1)$

b) (i) $|x+3| = 2x$

$x+3 = 2x$ or $x+3 = -2x$

$-x = -3$ $\quad$ $3x = -3$

$x = 3$ $\quad$ $x = -1$

CHECK

LHS $= |x+3|$ $\quad$ LHS $= |x+3|$

$= |3+3|$ $\quad$ $= |-1+3|$

$= 6$ $\quad$ $= 2$

RHS $= 2x$ $\quad$ RHS $= 2x$

$= 2(3)$ $\quad$ $= 2(-1)$

$= 6$ $\quad$ $= -1$

LHS = RHS $\quad$ LHS $\neq$ RHS

$\therefore x = 3$ $\quad$ $\therefore x \neq -1$

$\therefore x = 3$ is the only solution

(ii) $\dfrac{x+4}{x-2} \geq 3$ $\quad$ $\boxed{x \neq 2}$

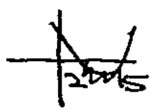
$(x-2)(x+4) \geq 3(x-2)^2$

$3(x-2)^2 - (x-2)(x+4) \leq 0$

$(x-2)[3(x-2)-(x+4)] \leq 0$

$(x-2)(2x-10) \leq 0$

$2(x-2)(x-5) \leq 0$



$2 < x \leq 5$

c) $\dfrac{2}{3-\sqrt{3}} = \dfrac{2}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \dfrac{6+2\sqrt{3}}{9-3}$

$= \dfrac{3+\sqrt{3}}{3}$

$\dfrac{11}{6+\sqrt{3}} = \dfrac{11}{6+\sqrt{3}} \times \dfrac{6-\sqrt{3}}{6-\sqrt{3}}$

$= \dfrac{66-11\sqrt{3}}{36-3}$

$= \dfrac{6-\sqrt{3}}{3}$

$\therefore \dfrac{2}{3-\sqrt{3}} + \dfrac{11}{6+\sqrt{3}} = \dfrac{3+\sqrt{3}+6-\sqrt{3}}{3}$

$= \dfrac{9}{3}$

$= 3$

which is rational

Question 2

a) $x^2+2x-4+\dfrac{3}{x^2+2x} = 0$

Let $y = x^2+2x$

$y - 4 + \dfrac{3}{y} = 0$

$y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

$y = 3, 1$

when $y = 3$

$x^2+2x = 3$

$x^2+2x-3 = 0$

$(x-1)(x+3) = 0$

$x = 1, -3$

when $y = 1$

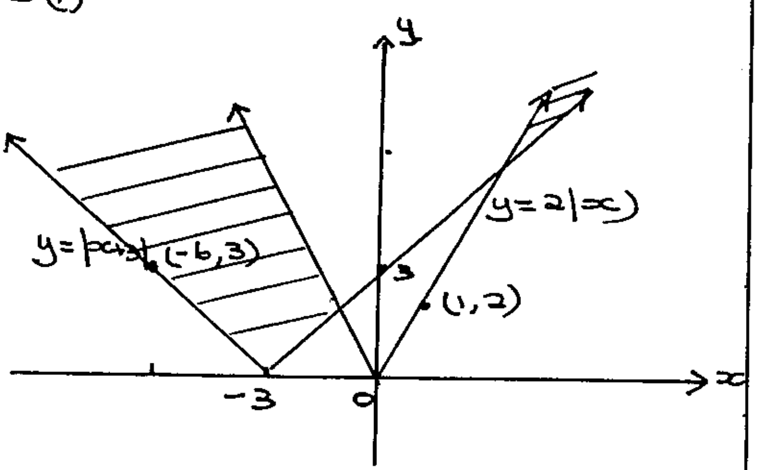
$x^2+2x = 1$

$x^2+2x-1 = 0$

$x = \dfrac{-2 \pm \sqrt{4+4}}{2}$

$= -1 \pm \sqrt{2}$

b (i)



c) $m:n = 5:-3$ $\quad A(-2,-1)\ B(1,5)$

$$x = \frac{mx_2+nx_1}{m+n} \qquad y = \frac{my_2+ny_1}{m+n}$$

$$= \frac{5(1)-3(-2)}{5-3} \qquad = \frac{5(5)-3(-1)}{5-3}$$

$$= \frac{11}{2} \qquad = \frac{28}{2}$$

$$= 14$$

$\therefore$ Q is $\left(\frac{11}{2}, 14\right)$

Question 3

a) Let $f(x) = \frac{1}{1+x^2}$

$$f(-x) = \frac{1}{1+(-x)^2}$$

$$= \frac{1}{1+x^2}$$

$$\therefore f(x) = f(-x)$$

$\therefore$ $y = f(x) = \frac{1}{1+x^2}$ is even

ii) $\lim\limits_{x\to\infty} \frac{1}{1+x^2}$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{x^2}{x^2}}$$

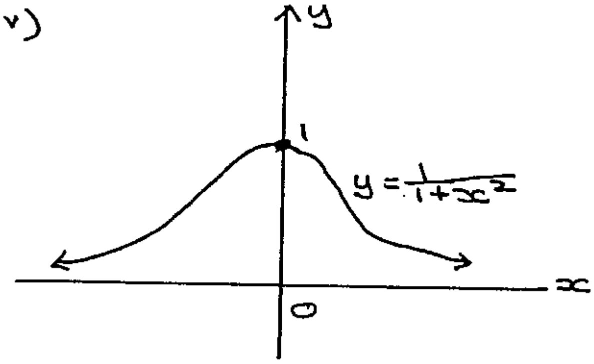
$$= 0$$

iii) when $x=0$ $\quad y = \frac{1}{1+x^2}$

$$= 1$$

$\therefore$ range $0 < y \leq 1$

iv)



b) If $y = ax^2+bx+c = 0$

negative definite if

$a < 0$

$\Delta = b^2-4ac < 0$

ii) $(k^2-k)x^2-(2k-6)x+2=y$

at $x=0$, $y=2$ (y-intercept)

$\therefore$ not negative definite

(iii) For positive definite

(i) $a>0$ $\quad \therefore (k^2-k)>0$

$k(k-1)>0$

$k<0 \quad k>1$

AND

(ii) $\Delta < 0$

$$\Delta = b^2-4ac$$
$$= (6-2k)^2-4(k^2-k)2$$
$$= 36-24k+4k^2-8k^2+8k$$
$$= -4k^2-16k+36$$
$$= -4(k^2+4k-9)$$

i.e. $k^2+4k-9>0$

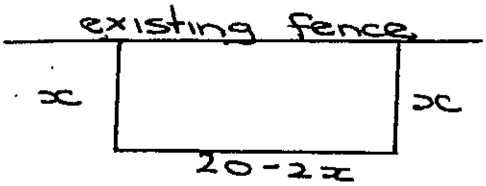
Roots at $k = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$= \frac{-4\pm\sqrt{16+36}}{2}$$

$$= -2\pm\sqrt{13}$$

$\therefore k<-2-\sqrt{13}$, $\quad k>-2+\sqrt{13}$

a)



Let the width be x

length be $20-2x$

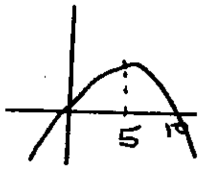
$A = lb$

$= x(20-2x)$

$= 2x(10-x)$

Maximum area at

$x = 5$



$\therefore$ length $= 20-2(5)$

$= 10$

$\therefore$ Area $= 5\times 10$

$= 50m^2$

Question 4

(i) vertical asymptote $x=2$

(ii) horizontal asymptote

$$\lim_{x\to\infty} \frac{x-4}{x-2}$$

$$\lim_{x\to\infty} \frac{\frac{x}{x}-\frac{4}{x}}{\frac{x}{x}-\frac{2}{x}}$$

$= 1$

iii) y-intercept $\Rightarrow x=0$

$y = \frac{x-4}{x-2}$

$= 2$

(iv) x-intercept $\Rightarrow y=0$

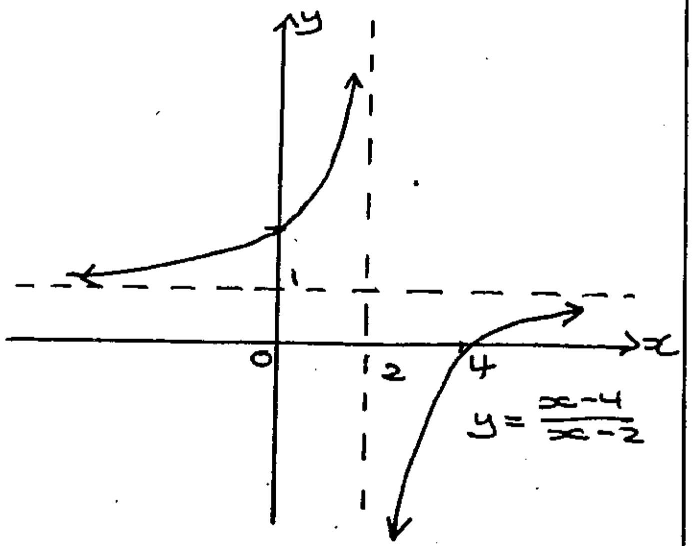
$y = \frac{x-4}{x-2}$

$0 = x-4$

$x=4$

Domain $x \neq 2$ $x\in\mathbb{R}$

Range $y\neq 1$ $y\in\mathbb{R}$



b) $(x-4)^2+y^2=4$ —— ①

$\frac{x}{\sqrt{3}} - y = 0$ —— ②

$y = \frac{x}{\sqrt{3}}$ —— ③

Sub ③ in ①

$(x-4)^2+y^2=4$

$(x-4)^2 + \frac{x^2}{3} = 4$

$3x^2-24x+48+x^2=12$

$4x^2-24x+36=0$

$4(x^2-6x+9)=0$

$4(x-3)^2=0$

$(x-3)^2=0$

$x=3$

$\therefore$ $x=3$, $y=\frac{\sqrt{3}}{3}$ (NB +ve gradient)

$\therefore$ The line is a tangent to the circle

(c) $4x^2-15x+16=A(x-1)^2+B(x-1)+C$

$=Ax^2-2Ax+A+Bx-B+C$

$=Ax^2+(B-2A)x+(A-B+C)$

$A=4$ $B-2A=-15$ $A-B+C=16$

$B-2(4)=-15$ $4+7+C=16$

$B=-7$ $C=5$

$\therefore 4x^2-15x+16=4(x-1)^2-7(x-1)+5$

Question 5

a) $4kx^2-(3k-4)x+1=0$

Let the roots be $\alpha, \frac{1}{\alpha}$

$\alpha \times \frac{1}{\alpha} = \frac{c}{a}$

$1 = \frac{1}{4k}$

$4k=1$

$k=\frac{1}{4}$

b) $x^2+y^2-4x+6y-12=0$

$x^2-4x+y^2+6y = 12$

$(x-2)^2+(y+3)^2 = 12+4+9$

$(x-2)^2+(y+3)^2 = 5^2$

centre $(2,-3)$ radius 5

(ii) $3x+4y=k$

$3x+4y-k=0$ $(2,-3)$

$d=\frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$=\frac{|3(2)+4(-3)-k|}{\sqrt{3^2+4^2}}$

$=\frac{|6-12-k|}{5}$

$=\frac{|-6-k|}{5}$

$=\frac{|6+k|}{5}$

(iii) If the line is a tangent

d = radius

$\frac{|6+k|}{5}=5$

$|6+k|=25$

$6+k=25$ $6+k=-25$

$k=19$ $k=-31$

c) $(x-3y+5)+k(x+2y)=0$

(i) point $(0,5)$

$(0-3(5)+5)+k(0+2(5))=0$

$-10+10k=0$

$10k=10$

$k=1$

(ii) $x-3y+5+k(x+2y)=0$

$x+kx-3y+2ky+5=0$

$(2k-3)y=-(1+k)x-5$

$y=\frac{-(1+k)}{(2k-3)}x-\frac{5}{2k-3}$

$m_1 = m_2$

But $\frac{-(1+k)}{2k-3}=-1$

$-2k+3=-1-k$

$-k=-4$

$k=4$

NB $x+y=2$

$y=-x+2$

$\therefore$ gradient -1

(iii) when $k=1$

$(x-3y+5)+1(x+2y)=0$

$2x-y+5=0$ — (1)

when $k=4$

$(x-3y+5+4(x+2y)=0$

$5x+5y+5=0$

$x+y+1=0$ — (2)

$2x-y+5=0$ — (1)

(2)+(1) $3x+6=0$

$x=-2$

Sub (2) $x+y+1=0$

$-2+y+1=0$

$y=1$

$\therefore$ P is $(-2,1)$