NEWINGTON COLLEGE

Preliminary Common Assessment 2001

MATHEMATICS

Extension 1

Time allowed - 2 hours

DIRECTIONS TO CANDIDATES:

All questions are of equal value.
All questions may be attempted.
In every question, show all necessary working.
Marks may not be awarded for careless or badly arranged work.
Approved silent calculators may be used.
The answers to the seven questions in this paper are to be returned in separate bundles clearly marked Question 1, Question 2 etc.
Each bundle must show the candidate's computer number.
Start each question on a new page.
The questions are not necessarily arranged in order of difficulty. Candidates are advised to read the whole paper carefully at the start of the examination.
Unless otherwise stated candidates should leave their answers in simplest exact form.

OUTCOMES TO BE ASSESSED:

P2 Provides reasoning to support conclusions which are appropriate in the context;

P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities;

P4 Chooses and applies appropriate arithmetic, graphical, trigonometric and geometric techniques;

P5 Understands the concept of a function and the relationship between a function and its graph;

PE2 Uses multi-step deductive reasoning in a variety of contexts;

PE3 Solves problems involving permutations and combinations, **inequalities**, polynomials, circle geometry and parametric representations.

Question 1 **(12 marks)** 7 min

(a) Write in simplest form: $2\sqrt{27}-\frac{\sqrt{12}}{2}+3\sqrt{48}$.

(b) Solve $|x-3|=11$.

(c) Factorise fully: $16x^3-54y^3$.

(d) Express as a rational number in simplest terms: $2.03\dot{9}$.

(e) Give the natural domain for each of the following:

(i) $f(x)=3x^2-2x+1$.

(ii) $g(x)=\sqrt{x^2-9}$.

(iii) $h(x)=\frac{1}{3x-1}$.

Question 2 **Start this question on a new page** **(12 marks)**

6 min

(a) Solve for θ, to the nearest minute,

$\sin\theta=-0.3$, for $0°\le\theta\le360°$.

(b) Draw a solution for the following inequation on a number line

$x^2\le16$.

(c) Express $1+\frac{2}{\sqrt{3}-1}$, in the form $a+b\sqrt{3}$, where a and b are rational.

(d) (i) Sketch the curve $y=\frac{1}{2x+1}$, showing all features.

(ii) If $f(x)=\frac{1}{2x+1}$ is moved horizontally 1 unit to the left, find the equation of the curve, **without** redrawing it. Simplify your answer.

Question 3 **Start this question on a new page** **(12 marks)**

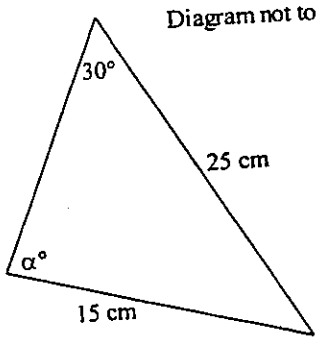
(a) Solve for x, $\tan^2x+2\sec x+1=0$, for $0°\le x\le360°$.

(b) The sides of a triangle are in the ratio 3: 5 : 6. Find the size of the smallest angle (to the nearest minute).

Q3 cont..../page 3

Question 3 (cont.)

(c) Find the value of α, (to the nearest minute).



(d) Prove $\sin^2\theta\cos\theta+\cos^3\theta=\sin(90°-\theta)$.

Question 4 Start this question on a new page **(12 marks)**

(a) If $f(x)=x^2+2$ and $F(x)=2x+3$, then find $f(F(3))$.

(b) Simplify

(i) $\dfrac{1}{x^2-xy}-\dfrac{1}{x^2-2xy+y^2}$

(ii) $\tan\theta+\dfrac{\cos\theta}{1+\sin\theta}$.

(c) Find $\sin\theta$, if $\cos\theta=-\dfrac{12}{13}$ and $90°\le\theta\le180°$.

(d) Solve for x:

(i) $|2x-11|=3x-4$

(ii) $\dfrac{2}{x+1}=\dfrac{x+4}{x+11}$

Question 5 Start this question on a new page **(12 marks)**

(a) Solve for x:

$$1-\sin^2 2x=\frac{1}{2},\qquad -90°\le x\le 90°.$$

(b) A rectangular property next to a river has an area of 20 000 m^2. If a farmer uses the river as one boundary and has only 400 m of fence for the other three sides, what are the dimensions of the field?

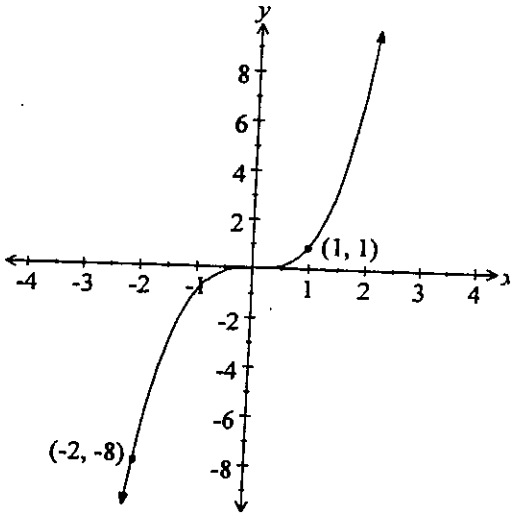
Q5...Cont./ page 4

Question 5 (cont.)

(c) (i) Give the equation of the curve below:

(ii) Is this function an even or odd function or is it neither? Give reasons.

(iii) Find the equation for an inverse function.



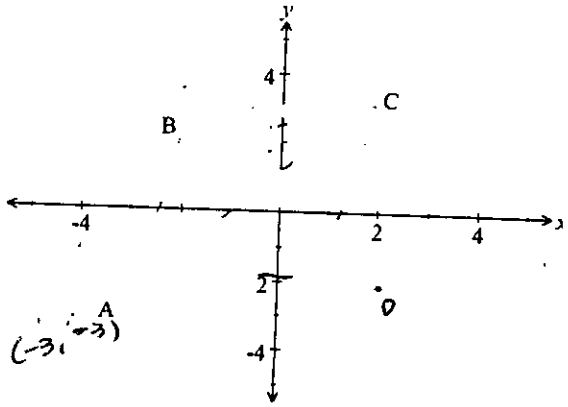
Question 6 Start this question on a new page (12 marks)

(a) Find the co-ordinates of the point P that divide the interval AB in the ratio 2:3, where A is (–3, 5) and B is (–6, –10)

(i) internally.

(ii) externally.

(b) In the diagram below, A (–3, –3), B (–2, 2) and C (2, 3) are three points of a parallelogram ABCD. Find the fourth point D.



Q6...cont./page 5

Question 6 (cont.)

(c) Two ships, A and B, sail from a port simultaneously with A sailing twice as fast as B. Ship A is on a course of 170°T whilst Ship B's course is 290°T. After 1 hour, the two ships are found to be 70 nautical miles apart.

(i) Draw a diagram to describe the information given above.

(ii) Find the speed of each ship, in knots, correct to two decimal places and determine also the bearing of B from A at this time, expressed as a true bearing, correct to the nearest degree. (NOTE: a knot is defined as a speed of one nautical mile per hour.).

Question 7 Start this question on a new page (12 marks)

(a) By completing the square, find the centre and radius of the following circle:

$$x^2 - 3x + y^2 + y - 12 = 0$$

and hence, graph the curve, showing the centre and all intercepts.

(b) Sketch the region defined by the following inequations:

$$y \le \sqrt{9 - x^2} \text{ and } y \ge x^2 - 9$$

(c) Solve for x:

(i) $|3x - 4| + 2|x + 3| \le 5$

(ii) $\dfrac{x-1}{x+1} \le \dfrac{2x}{2x-1}$

END OF PAPER

PRELIM. EXT 1 | COMMON ASSESS 2001.

QUESTION 1

a) $2 \times 3\sqrt{3} - \frac{2\sqrt{3}}{2} + 3 \times 4\sqrt{3}$

$= 6\sqrt{3} - \sqrt{3} + 12\sqrt{3}$.

$= 17\sqrt{3}$

b) $|x-3| = 11$

$x = 14$ or -8

c) $2(8x^3 - 27y^3)$

$= 2(2x-3y)(4x^2+6xy+9y^2)$

d) $2.039\dot{9} \times 1000 = 2.039\dot{9}$

$2.039\dot{9} \times 100 = 203.\dot{9}$

$2.039\dot{9} \times 900 = 1836$

$\therefore 2.039\dot{9} = \frac{1836}{900}$

$= 2\frac{1}{25}$

$\left(= -\frac{51}{25}\right)$

e) i/ $x \in R$

ii/ $-3 \leq x \leq 3$??

iii/ all $x \in R$, $x \neq \frac{1}{3}$

QUESTION 2

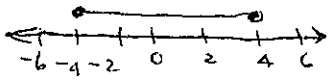
a) $\sin\theta = -0.3$

$\frac{1}{x1}$ $\quad \text{rel } \theta = 17°27'$

$\therefore \theta = 197°27'$ or $342°33'$

b) $x^2 - 16 \leq 0$.

$-4 \leq x \leq 4$.



c) $1 + \frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= 1 + \frac{2\sqrt{3}+2}{3-1}$

$= 1 + \sqrt{3} + 1$

$=$

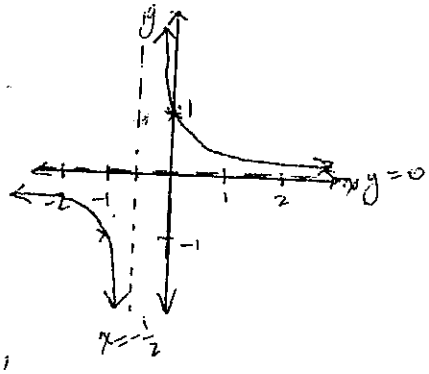
d) i/ $D: x \neq -\frac{1}{2}$, $R: y \neq 0$.

$y\text{-int} = 1$

$x\text{-int} = \text{none}$

check $x = -1$

$y = -1$



ii/ $f(x) = \frac{1}{2(x+1)+1}$

$= \frac{1}{2x+2+1}$

$f(x) = \frac{1}{2x+3}$

QUESTION 3

a) $\tan^2 x + 2\sec x + 1 = 0$

$\sec^2 x - 1 + 2\sec x + 1 = 0$

$\sec^2 x + 2\sec x = 0$.

Let $u = \sec x$.

$u^2 + 2u = 0$.

$u(u+2) = 0$.

$\therefore u = 0$, $u = -2$.

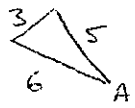
$\therefore \sec x = 0$ or $\sec x = -2$.

No soln $\quad \cos x = -\frac{1}{2}$

$\frac{x}{x}$ $\quad \text{rel } x = 60°$

$x = 120°, 240°$.

b)



$\cos A = \frac{5^2+6^2-3^2}{2 \times 5 \times 6}$.

$A = 29°56'$

c) $\frac{25}{\sin\alpha} = \frac{15}{\sin 30°}$.

$15\sin\alpha = 25\sin 30°$.

$\sin\alpha = \frac{25 \times \frac{1}{2}}{15}$

$\therefore \alpha = 56°27'$

Bd)

$$\text{LHS} = \sin^2\theta\cos\theta + \cos^3\theta$$
$$= \cos^3\theta\left(\frac{\sin^2\theta}{\cos^2\theta} + 1\right)$$
$$= \cos^3\theta(\tan^2\theta + 1)$$
$$= \cos^3\theta \cdot \sec^2\theta$$
$$= \cos^3\theta \cdot \frac{1}{\cos^2\theta}$$
$$= \cos\theta$$
$$= \sin(90-\theta)$$
$$= \text{RHS}.$$

$\therefore \sin^2\theta\cos\theta + \cos^3\theta = \sin(90^\circ - \theta)$

QUESTION 4

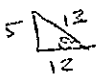
a) $F(3) = 2\times 3 + 3$
$= 9$

$f(F(3)) = f(9)$
$= 9^2 + 2$
$= 83$

b) i) $\frac{1}{x(x-y)} - \frac{1}{(x-y)^2}$
$= \frac{x - y - x}{x(x-y)^2}$
$= \frac{-y}{x(x-y)^2}$

ii) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta - \sin\theta\cos\theta}{1-\sin^2\theta}$
$= \frac{\sin\theta\cos\theta}{\cos^2\theta} + \frac{\cos\theta - \sin\theta\cos\theta}{\cos^2\theta}$
$= \frac{\cos\theta}{\cos^2\theta}$
$= \frac{1}{\cos\theta}$
$= \sec\theta.$

c) $\cos\theta = -\frac{12}{13}$



$\sqrt{13^2 - 12^2} = 5$ (x)

$\therefore \sin\theta = \pm\frac{5}{13}$??

d) i) $|2x-11| = 3x-4$

$\therefore 2x - 11 = 3x - 4$ or $-(2x-11) = 3x-4$

$-7 = x$ or $15 = 5x$

$\therefore x = 3$ or $x = -7$?? $x = 3$.

ii) $2(x+11) = (x+1)(x+4)$

$2x + 22 = x^2 + 5x + 4.$

$x^2 + 3x - 18 = 0.$

$(x+6)(x-3) = 0$

$\therefore x = -6$ or 3

QUESTION 5

a) $1 - \sin^2 2x = \frac{1}{2}.$

$-\sin^2 2x = -\frac{1}{2}.$

$\sin 2x = \pm\frac{1}{\sqrt{2}}$ $\quad -90^\circ \le x \le 90^\circ.$

Let $u = 2x.$ $\quad -180^\circ \le u \le 180^\circ$

$\sin u = \pm\frac{1}{\sqrt{2}}.$

$\therefore u = 45^\circ, 135^\circ,$
$-45^\circ, -135^\circ.$

$\therefore x = \pm 22.5^\circ, \pm 67.5^\circ$

b)

$xy = 20000$

$2x + y = 400$

$\therefore y = 400 - 2x.$

$\therefore x(400-2x) = 20000.$

$2x^2 - 400x + 20000 = 0.$

$x^2 - 200x + 10000 = 0.$

$(x-100)^2 = 0.$

$\therefore x = 100_m \quad y = 200_m$

c) i) $y = x^3$

ii) odd $\quad f(x) = x^3 \quad f(-x) = (-x)^3$
$= -x^3$
$= -f(x)$

iii) $x = y^3$

$y = \sqrt[3]{x}$

$\therefore f^{-1}(x) = \sqrt[3]{x}$

QUESTION 6

a) $2:3 \quad (-3,5) \quad (-6,-10)$

i) $x = \frac{3\times -3 + 2\times -6}{3+2}$

$= -\frac{21}{5}$

$y = \frac{3\times 5 + 2\times -10}{3+2}$

$= -1$

$\left(-\frac{21}{5}, -1\right)$

ii) $2:-3 \quad (-3,5) \quad (-6,-10)$

$x = \frac{-3\times -3 + 2\times -6}{-3+2}$

$= 3$

$y = \frac{-3\times 5 + 2\times -10}{-3+2}$

$= 35$

$(3, 35)$

i) $m_{BC} = \frac{1}{2+2}$

$= \frac{1}{4}$

$\therefore m_{AD} = \frac{1}{4}$.

Eqn of AD

$y+3 = \frac{1}{4}(x+3)$ $\quad 4y+12 = x+3$

$x - 4y - 9 = 0$.

$AB = \frac{2+3}{-2+3}$

$= 5$

$\therefore m_{CD} = 5$.

Eqn of CD

$y-3 = 5(x-2)$

$5x - y - 7 = 0$.

Pt of int of AD + CD

$5x - y - 7 = 0$ (1)

$x - 4y - 9 = 0$ (2)

(2) × 5 $\quad 5x - 20y - 45 = 0$ (2A)

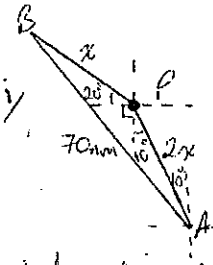
(1) − (2A) $\quad 19y + 35 = 0$

$y = -2$

$\therefore x = 1$

$\therefore D(1, -2)$

c) i)



ii) Let x = speed of B

$70^2 = x^2 + (2x)^2 - 2x^2\cos 120°$

$4900 = 3x^2 - 2x^2\left(-\frac{1}{2}\right)$

$4900 = 3x^2 + x^2$

$4x^2 = 4900$

$x^2 = 1225$

$x = 35$ knots

$\therefore$ Ship B → 35 knots

Ship A → 70 knots.

$\frac{\sin A}{35} = \frac{\sin 120°}{70}$.

$\sin A = \frac{35\sin 120°}{70}$

$\therefore A = 26°$

$26 + 10 = 36°$

$360 - 36 = 324°T$

The bearing of B from A is $324°T$.

QUESTION 7

a) $x^2 - 3x + y^2 + y = 12$.

$x^2 - 3x + \left(\frac{3}{2}\right)^2 + y^2 + y + \left(\frac{1}{2}\right)^2 = 12 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\therefore \left(x - \frac{3}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{29}{2}$.

$\therefore$ centre $\left(\frac{3}{2}, -\frac{1}{2}\right)$

radius $\sqrt{\frac{29}{2}}$ or $\frac{\sqrt{58}}{2}$.

x-int → $x^2 - 3x - 12 = 0$

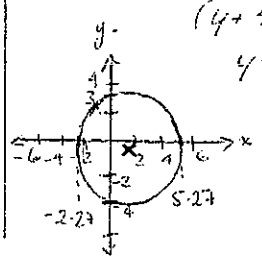
$x = \frac{3 \pm \sqrt{57}}{2}$

$= -2.27$ or 5.27.

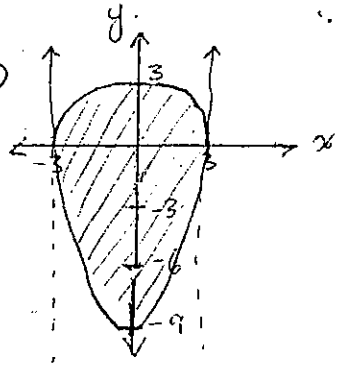
y-int → $y^2 + y - 12 = 0$

$(y+4)(y-3) = 0$

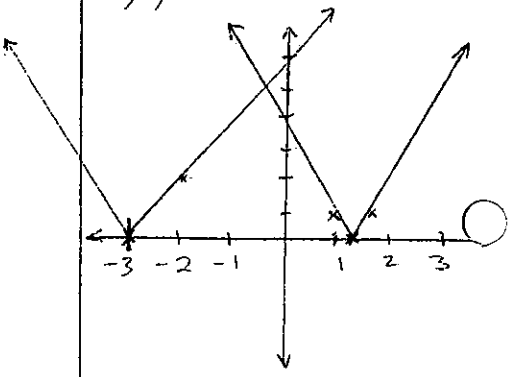
$y = -4, 3$.



b)



c) i) $|3x-4| + 2|x+3| \leq 5$



Case ① $x \leq -3$

$-(3x-4) + 2(x+3) \leq 5$

$-3x + 4 - 2x - 6 \leq 5$

$-5x - 1 \leq 5$.

$-5x \leq 4$.

$x \geq -\frac{4}{5}$

Check $x = -\frac{3}{5}$ ✗

Case ② $-3 \leq x \leq \frac{4}{3}$.

$3x - 4 - 2(x+3) \leq 5$.

$x \leq 15$

check $x = 14$ ✗

Case ③ $x \geq \frac{4}{3}$

$3x - 4 + 2(x+3) \leq 5$

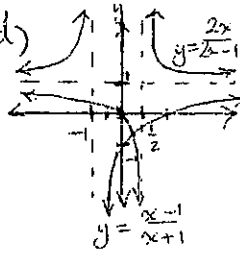
$5x + 2 \leq 5$.

$x \leq \frac{3}{5}$

check $x = \frac{3}{5}$ ✗

$\therefore$ No solutions.

d)



Pt of int $\left(\frac{1}{5}, -\frac{2}{3}\right)$

$\therefore -1 \leq x \leq \frac{1}{5}$.