Question 1 *(12 marks)* ***Marks***

(a) Factorize $2m^3 - 128$ *2*

(b) Simplify $\dfrac{1}{y-x} + \dfrac{x}{(x-y)^2}$ *3*

(c) Solve $\dfrac{x^2-4}{x} \geq 0$ *3*

(d) Solve $x - y = 6$ and $x^2 + y^2 = 18$.
Explain the geometric significance of your answer. *4*

Question 2 *(12 marks)* **Start this Question on a new page.**

(a) If $x - y + \sqrt{x+y} = \sqrt{6}$ find x and y. *4*

(b) If $x = \dfrac{3-\sqrt{3}}{3+\sqrt{3}}$ find $x - \dfrac{1}{x}$ and hence $x^2 + \dfrac{1}{x^2}$. *4*

(c) (i) If $\phi(x) = (x-1)^2$ show that $\phi(P+1) = \phi(1-P)$ *4*

(ii) If $f(x) = x^2 + 2$ and $F(x) = 2x + 3$, find $f(F(x))$ and $F(f(x))$.

Question 3 *(12 marks)* **Start this Question on a new page.**

(a) Sketch $y = |x+2|$ and $y = |x-4|$ on the same axes and then use your graph to solve $|x+2| \leq |x-4|$ *3*

(b) Shade the intersection of the regions represented by $x^2 + y^2 > 1$ and $x^2 + y^2 \leq 9$ *3*

(c) Shade the region $y < \sqrt{9-x^2}$ keeping in mind any restrictions on the domain. *3*

(d) If $\cot\theta = \dfrac{3}{4}$ and $180^\circ < \theta < 360^\circ$ use exact values to show that
$\dfrac{9\sec\theta\tan\theta}{1+\sin\theta} = -100$ *3*

Marks

Question 4 *(12 marks)* **Start this Question on a new page.**

(a) Prove $$\frac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$$ 3

(b) Eliminate θ from the pair of equations $x=2+3\cos^2\theta$, $y=2\sin\theta-5$ 2

(c) Solve for $0^\circ\le x\le 360^\circ$ (i) $\sin\frac{x}{2}=\frac{1}{2}$ 3

(ii) $2\cos^2\theta-3\sin\theta-3=0$ 4

Question 5 *(12 marks)* **Start this Question on a new page.**

(a) (i) Find the size of the largest angle of a triangle with sides 3cm, 4cm and 6cm. 3

(ii) Hence find the area of the triangle. 2

(b) From a point A, on the deck of a ship, the angle of elevation of an aircraft (P) is 34° and from an observation point B, 25m vertically above A, the angle of elevation is $32^\circ 30'$. Calculate, to the nearest 10m, the height of the aircraft above the deck. 7

Question 4 *(12 marks)* **Start this Question on a new page.** *Marks*

(a) Prove $\dfrac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$ *3*

(b) Eliminate θ from the pair of equations $x=2+3\cos^2\theta$, $y=2\sin\theta-5$ *2*

(c) Solve for $0^\circ \le x \le 360^\circ$ (i) $\sin\dfrac{x}{2}=\dfrac{1}{2}$ *3*

(ii) $2\cos^2\theta-3\sin\theta-3=0$ *4*

Question 5 *(12 marks)* **Start this Question on a new page.**

(a) (i) Find the size of the largest angle of a triangle with sides 3cm, 4cm and 6cm. *3*

(ii) Hence find the area of the triangle. *2*

(b) From a point A, on the deck of a ship, the angle of elevation of an aircraft (P) is 34° and from an observation point B, 25m vertically above A, the angle of elevation is $32^\circ 30'$. Calculate, to the nearest 10m, the height of the aircraft above the deck. *7*

Question 6 *(12 marks)* **Start this Question on a new page.**

(a) Noting any restrictions on the domain of the function sketch **2**

$$y = \frac{x^2 - 9}{x + 3}$$

(b) Examine $y = \dfrac{1 + 2x^2}{1 - x^2}$ noting

- any restrictions on the domain
- symmetry
- intercepts
- asymptotes both horizontal and vertical

and then draw the sketch. **4**

(c) For the curve $y = f(x) = \dfrac{1}{\sqrt{x-1}}$

(i) By considering the natural domain, any intercepts and the y values as $x \to 1^+$ and $x \to \infty$ sketch the curve. **3**

(ii) Find the inverse function $f^{-1}(x)$ and sketch this on the same axes. **3**

Solutions Math Ext Assess 2003

Question 1

(a) $2m^3 - 128 = 2(m^3 - 64)$

$= 2(m-4)(m^2+4m+16)$

(b) $\dfrac{1}{y-x} + \dfrac{x}{(x-y)^2}$

$= \dfrac{-1(x-y)}{(x-y)(x-y)} + \dfrac{x}{(x-y)^2}$

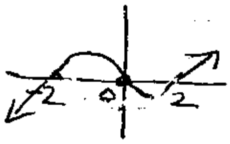
$= \dfrac{-x+y+x}{(x-y)^2}$

$= \dfrac{y}{(x-y)^2}$

c) $x^2 \times \dfrac{x^2-4}{x} \geq 0 \times x^2, \quad x \neq 0$

$x(\quad -4) \geq 0$

$x-2)(x+2) \geq 0$



$-2 \leq x \leq 0$ and $x \geq 2$

) $x - y = 6$ — ①

$x^2+y^2=18$ — ②

earrange ①]

$x = y+6$

$(y+6)^2+y^2=18$

$y^2+12y+36+y^2-18=0$

$y^2+12y+18=0$

$y^2 \quad y+9=0$

$(y \quad 3)^2 = 0$

$\therefore y = -3$

ub in y] in ①

$x - -3 = 6$

$x = 3$

eck in ② $3^2+(-3)^2 = 18$ ✓

$\therefore$ Solution $x=3$; $y=-3$

eometric significance.

There is one point of intersection between a line and a circle. The line is a tangent to the circle.

Question 2

b) $x-y+\sqrt{x+y} = \sqrt{6}$

$(x-y)+\sqrt{x+y} = 0+\sqrt{6}$

$\therefore x-y=0$ — ①

$x+y=6$ — ②

Solve simultaneously.

[①+②] $2x=6$

$\therefore x=3$

[sub x in ①] $3-y=0$

$y=3$

[check in ②] $3+3=6$ ✓

$\therefore x=3$ and $y=3$

c)(i) $\phi(x) = (x-1)^2$

$\phi(p+1) = (p+1-1)^2$

$= p^2$

$\phi(1-p) = (1-p-1)^2$

$= (-p)^2$

$= p^2$

$\therefore \phi(p+1) = \phi(1-p)$

(ii) $f(x) = x^2+2$

$F(x) = 2x+3$

find $f(F(x)) = (2x+3)^2+2$

$= 4x^2+6x+9+2$

$= 4x^2+6x+11$

$F(f(x)) = 2(x^2+2)+3$

$= 2x^2+4+3$

$= 2x^2+7$

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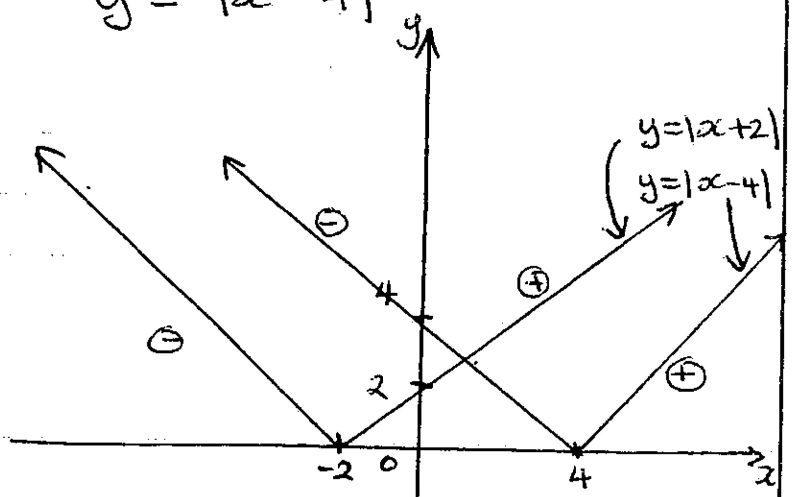
Question 3

(a) $y = |x+2|$

$y = |x-4|$



point of intersection

$+(x+2) = -(x-4)$

$x+2 = -x+4$

$2x = 2$

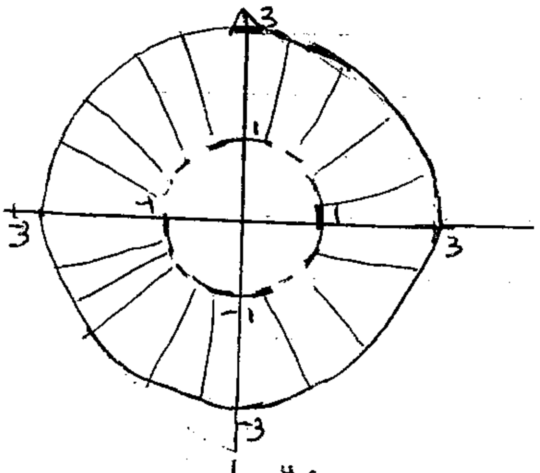
$\therefore x = 1$

For $|x+2| \leq |x-4|$

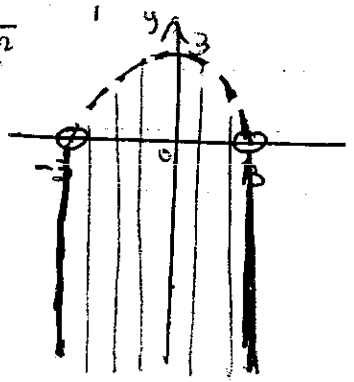
$\therefore x \leq 1$

(b) $x^2+y^2>1$ centre $(0,0)$ radius 1

$x^2+y^2 \leq 9$ centre $(0,0)$ radius 3

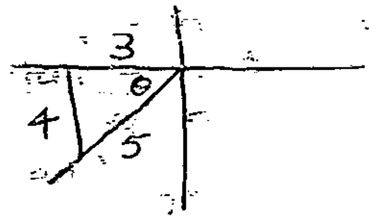


c) $y < \sqrt{9-x^2}$



(d) $\cot\theta = \frac{3}{4}$ $\quad 180° \leq \theta \leq 3$

$\therefore \tan\theta = \frac{4}{3}$



$\sin\theta = -\frac{4}{5}$

$\cos\theta = -\frac{3}{5}$

$\sec\theta = -\frac{5}{3}$

$LHS = \dfrac{9\sec\theta\tan\theta}{1+\sin\theta}$

$= \dfrac{9 \times -\frac{5}{3} \times \frac{4}{3}}{1+ -\frac{4}{5}}$

$= \dfrac{-20}{1/5}$

$= -100$

Question 4

(a) $\dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta$

$LHS \ \dfrac{1+\cos\theta}{1-\cos\theta} \times \dfrac{(1+\cos\theta)}{(1+\cos\theta)}$

$= \dfrac{(1+\cos\theta)^2}{(1-\cos^2\theta)^2}$

$= \dfrac{(1+2\cos\theta+\cos^2\theta)}{\sin^2\theta}$

$= \dfrac{1}{\sin^2\theta} + \dfrac{2\cos\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta + 2\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\sin\theta} + \cot^2\theta$

$= \operatorname{cosec}^2\theta + 2\cot\theta\cdot\operatorname{cosec}\theta + \cot^2\theta$

$= (\operatorname{cosec}\theta + \cot)^2$

$= RHS.$

$\therefore \dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

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Question 4

(b) $x = 2 + 3\cos^2\theta$

$y = 2\sin\theta - 5$

$x = 2 + 3\cos^2\theta$

$\frac{x-2}{3} = \cos^2\theta$

$\frac{y+5}{2} = \sin\theta$

$\frac{(y+5)^2}{4} = \sin^2\theta$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{x-2}{3} + \frac{(y+5)^2}{4} = 1$

$4(x-2) + 3(y+5)^2 = 12$

(c) (i) $\sin\frac{x}{2} = \frac{1}{2}$ $\quad 0 \le x \le 360°$

$0 \le \frac{x}{2} \le \frac{360}{2}$

$0 \le \frac{x}{2} \le 180°$

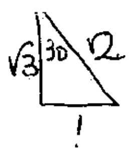
let $\theta = \frac{x}{2}$

$\therefore \sin\theta = \frac{1}{2} \quad 0 \le \theta \le 180°$

sin is pos in Q1 + Q2

$\sin\theta = \frac{1}{2}$

$\theta = 30°$



Quad 1

$\theta = 30°$

Quad 2

$\theta = 180° - 30$

$= 150°$

$\therefore \theta = 30°, 150°$

$\therefore \frac{x}{2} = 30°, 150°$ ✓

$\therefore x = 60°, 300°$ ✓

(c) (ii) $2\cos^2\theta - 3\sin\theta - 3 = 0$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$2(1 - \sin^2\theta) - 3\sin\theta - 3 = 0$

$2 - 2\sin^2\theta - 3\sin\theta - 3 = 0$

$-2\sin^2\theta - 3\sin\theta - 1 = 0$

$2\sin^2\theta + 3\sin\theta + 1 = 0$

let $x = \sin\theta$

$2x^2 + 3x + 1 = 0$

$2x^2 + 2x + x + 1 = 0$

$2x(x+1) + 1(x+1) = 0$

$(2x+1)(x+1) = 0$

$x = -\frac{1}{2}, -1$

$\therefore \sin\theta = -\frac{1}{2}$ or $\sin\theta = -1$

$\sin\theta = -\frac{1}{2}$

sin neg in Q3 + Q4

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

Quad 3

$\theta = 180° + 30$

$= 210°$

Quad 4

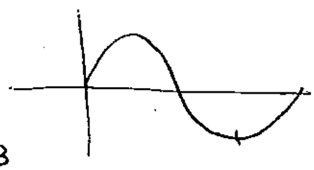
$\theta = 360 - 30$

$= 330°$

$\sin\theta = -1$

$\sin\theta = 1$

$\theta = 90°$



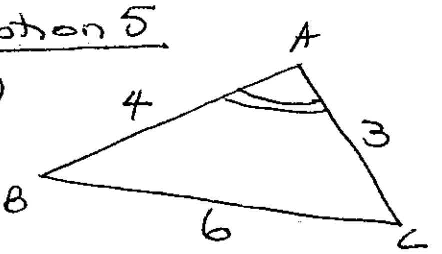
sin neg Q2 + Q3

$\theta = 270°$

$\therefore \theta = 210°, 330°, 270°$

Question 5

a) (i)



cosine rule $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$\cos A = \frac{4^2+3^2-6^2}{2\times 4\times 3}$

$\cos A = -0.458\dot{3}$

$A = 117^\circ 17'$ (n min)

$\therefore$ the size of the largest angle is $117^\circ 17'$

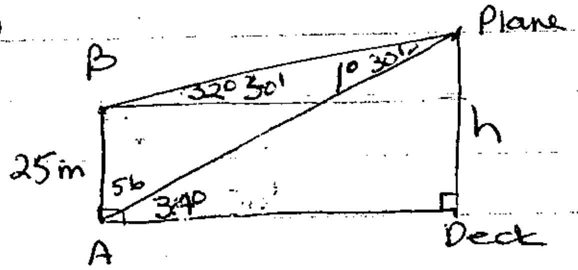
ii) Area $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}\times 4\times 3\times \sin 117^\circ 17'$

$= 5.332508665$

$= 5.3\ cm^2$ (1dp)

(b)



$\angle BAP = 56^\circ$

$\angle ABP = 122^\circ 30'$

$\angle BPA = 1^\circ 30'$

sine rule $\triangle ABP$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{25}{\sin 1^\circ 30'} = \frac{b}{\sin 122^\circ 30'}$

$b = \frac{25\times \sin 122^\circ 30'}{\sin 1^\circ 30'}$

$\therefore AP = 805.4715125$

In $\triangle APD$

$\sin 34^\circ = \frac{h}{805.4715125}$

$h = 805.47\ldots \times \sin 34^\circ$

$h = 450.4139577$

$\therefore h = 450$ m (n 10 m)

$\therefore$ height of the aircraft is 450m

Question 6

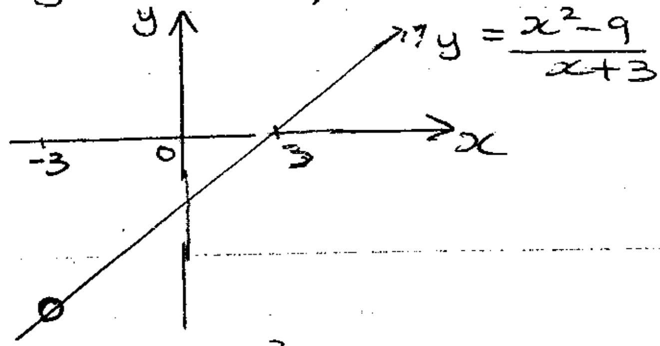
(a) $y = \frac{x^2-9}{x+3}$ $\quad x \neq 3$

$\therefore$ Domain: all real x, $x \neq 3$

$y = \frac{x^2-9}{x+3}$

$y = \frac{(x-3)(x+3)}{x+3}$

$y = x-3$, $x \neq \bar{3}$



(b) $y = \frac{1+2x^2}{1-x^2}$

Domain: $1-x^2 \neq 0$

$(1-x)(1+x) \neq 0$

$x \neq \pm 1$

$\therefore$ Vertical asymptotes are $x = 1$, $x = -1$

Horizontal asymptotes.

$y = \frac{1+2x^2}{1-x^2}$ $\quad (\div x^2)$

$y = \frac{\frac{1}{x^2} + \frac{2x^2}{x^2}}{\frac{1}{x^2} - \frac{x^2}{x^2}}$

$y = \frac{\frac{1}{x^2}+2}{\frac{1}{x^2}-1}$

$\lim_{x\to\infty} \frac{\frac{1}{x^2}+2}{\frac{1}{x^2}-1}$

$y = -\frac{2}{1}$

$y = -2$

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Question 6

b) Intercepts

X intercept $y=0$

$$0 = \frac{1+2x^2}{1-x^2}$$

$$0 = 1+2x^2$$

$$-\frac{1}{2} = x^2 \quad \therefore \text{ no solutions}$$

$\therefore$ no X intercepts.

Y intercept $x=0$

$$y = \frac{1+2(0)^2}{1-(0)^2}$$

$$y = 1$$

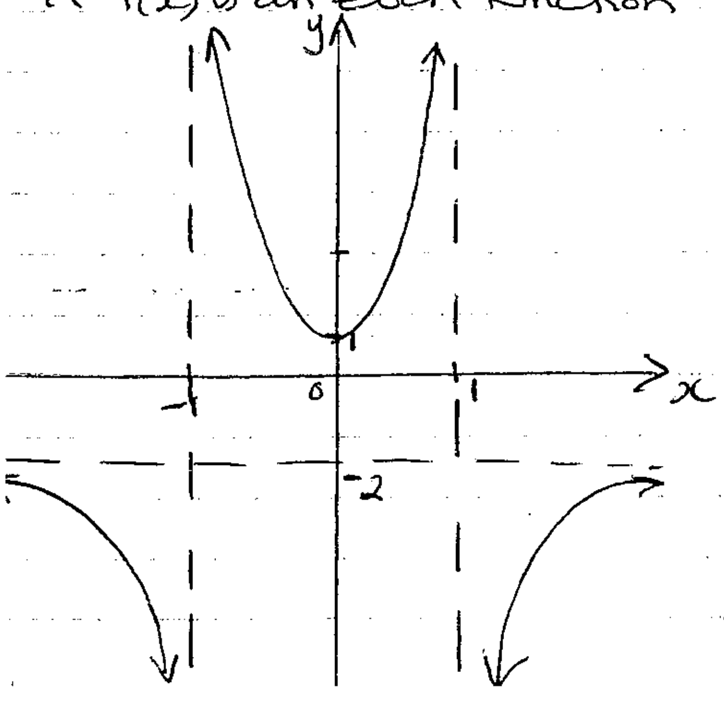
Symmetry

$$f(x) = \frac{1+2x^2}{1-x^2}$$

$$f(-x) = \frac{1+2(-x)^2}{1-(-x)^2} = \frac{1+2x^2}{1-x^2}$$

$\therefore f(-x) = f(x)$

$\therefore f(x)$ is an even function



(c) $y = f(x) = \frac{1}{\sqrt{x-1}}$

Domain: $x-1 > 0$
$x > 1$

X intercept $y=0$

$$0 = \frac{1}{\sqrt{x-1}} \quad \therefore \text{ not possible}$$

$\therefore$ no X intercept

Y intercept $x=0$

$$y = \frac{1}{\sqrt{0-1}} \quad \therefore \text{ not possible}$$

$\therefore$ no y intercept.

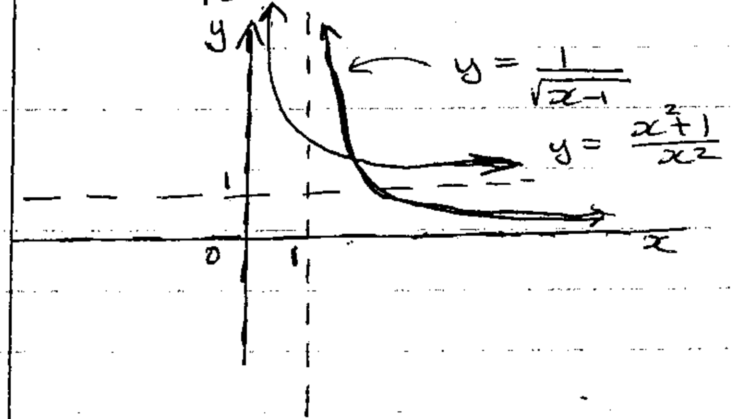
Limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{1}{\sqrt{x-1}} \Rightarrow 0$$

Lim as $x \to 1^+$

$$\sqrt{x-1} \Rightarrow 0$$

$$\frac{1}{\sqrt{x-1}} \Rightarrow \infty$$



(d) Inverse function

$$y = \frac{1}{\sqrt{x-1}}$$

$$x = \frac{1}{\sqrt{y-1}}$$

$$x^2 = \frac{1}{y-1}$$

$$\frac{1}{x^2} = y-1$$

$$\frac{1}{x^2} + 1 = y$$

$$y = \frac{x^2+1}{x^2}, \quad x \neq 0.$$

Domain: $x>0$
Range: $y>1$
No intercepts