QUESTION 1 *(12 MARKS)*

Marks

(a) Factorise $250-2x^3$. **2**

(b) Simplify $\dfrac{x^2}{4x^2+7x+3}-\dfrac{x-3}{4x+3}$. **3**

(c) Solve $x(2-x)>-3$. **3**

(d) If $0^\circ \le \theta \le 360^\circ$, find all values of θ in the following equation: **2**

$$\sin\theta = -\frac{1}{2}.$$

(e) If a and b are rational and $\dfrac{1+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$, find a and b. **2**

QUESTION 2 *(12 MARKS)* **Start this question on a new page.**

(a) Solve simultaneously: **3**

$$2x-y=1$$
$$y=\frac{3}{x}$$

(b) Find the exact value of $\cos\beta$ given that $\sin\beta=\dfrac{4}{9}$ and $\cos\beta<0$. **2**

(c) (i) On the same diagram, sketch $y=x$ and $y=|x+2|$. **3**

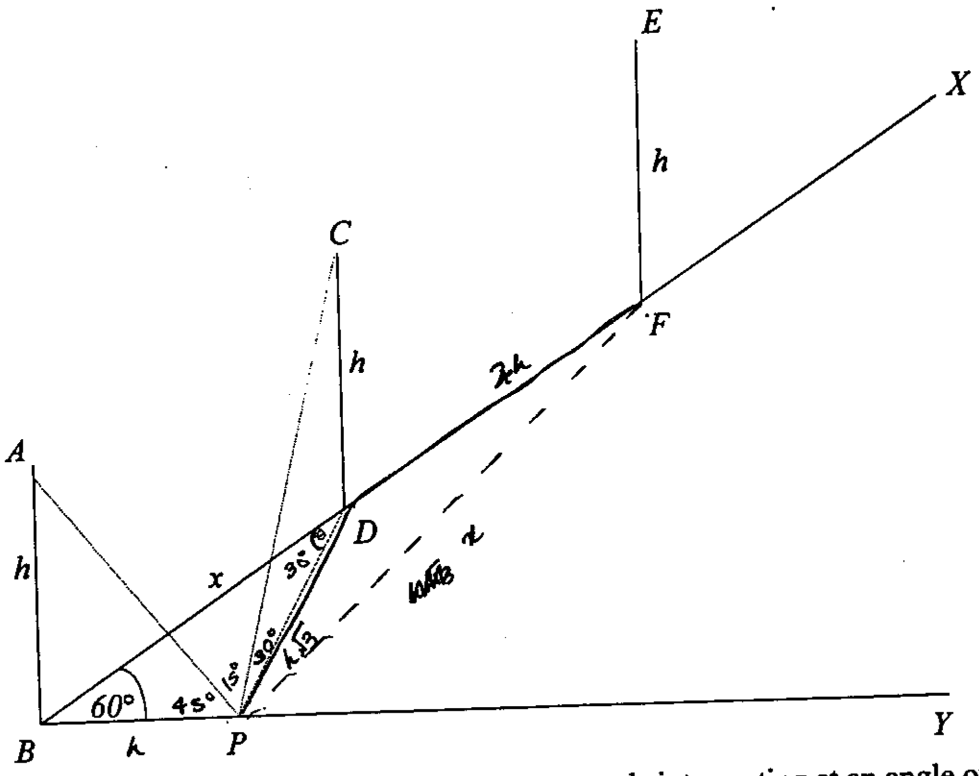
(ii) For what values of x is $|x+2|<x$?

(d) Sketch each of the following functions **4**

(i) $y=\sqrt{9-x^2}$ (ii) $f(x)=4-|1-x|$.

QUESTION 3 *(12 MARKS)* **Start this question on a new page.** **Marks**

(a) **7**



In the above diagram, BX and BY represent two roads intersecting at an angle of $60°$. On the road BX are situated three telegraph poles AB, CD and EF, all of equal height, the same distance, x metres, apart (i.e. $BD = DF = x$). P is a point on the road BY and the angles of elevation of A and C from P are $45°$ and $30°$ respectively.

(i) Show that $BP = h$ and $DP = h\sqrt{3}$.

(ii) By the use of the Sine Rule in triangle BDP, show that angle $BDP = 30°$ and hence that triangle BDP is right angled at P.

(iii) Prove that $x = 2h$.

(iv) By the use of the Cosine Rule in triangle PDF, show that $PF = h\sqrt{13}$ and hence show that the angle of elevation of E from P is approximately $15{\cdot}5°$.

(b) Find the point that divides the interval from (-2,1) to (1,3) **externally** in the ratio 3:2. **2**

(c) Prove that $\dfrac{\sec\theta + \operatorname{cosec}\theta}{\tan\theta + \cot\theta} = \sin\theta + \cos\theta$. **3**

QUESTION 4 *(12 MARKS)* **Start this question on a new page.** **Marks**

(a) For the function: **3**

$$f(x)=\begin{cases} x+2, & x<-2 \\ 2, & -2\le x\le 2 \\ x-2, & x>2 \end{cases}$$

(i) Sketch $f(x)$

(ii) Evaluate $2f(-2)-f(2)+3f(4)$.

(b) **9**

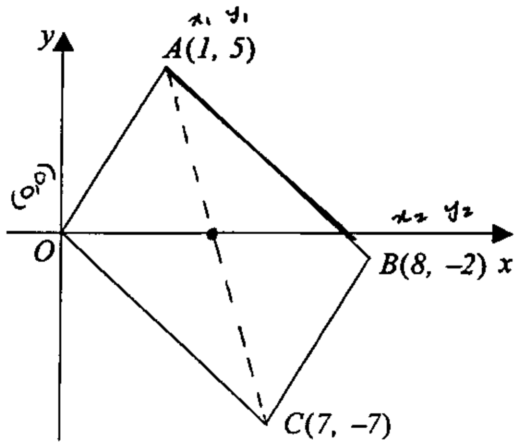


DIAGRAM NOT DRAWN TO SCALE

In the diagram $O(0,\ 0)$, $A(1,\ 5)$, $B(8,\ -2)$ and $C(7,\ -7)$ are the vertices of quadrilateral $OABC$.

(i) Find the midpoint of the interval joining AC.

(ii) Find the gradient of AB.

(iii) Show that the equation of AB is $x + y = 6$.

(iv) Find the exact length of AB.

(v) Show that AB is parallel to OC.

(vi) Explain why $OABC$ is a parallelogram.

(vii) Find the exact perpendicular distance from O to AB.

(viii) Hence find the area of parallelogram $OABC$.

QUESTION 5 *(12 MARKS)* **Start this question on a new page.** **Marks**

(a) For the function $y = \dfrac{4x}{2x-1}$: **6**

(i) Find the equation of the vertical asymptote.

(ii) Find the equation of the horizontal asymptote.

(iii) Find the coordinates of any intercepts for this curve.

(iv) Sketch the curve, showing all the above information.

(v) Hence or otherwise, solve $\dfrac{4x}{2x-1} < 1$.

(b) Sketch the region bounded by the following inequations, the coordinates of the intersections of boundaries are to be shown: **4**

$$y > 3x$$
$$y \le 4 - x^2$$

(c) Find the exact value of $\sec(-210)^\circ$. **2**

QUESTION 6 *(12 MARKS)* **Start this question on a new page**

(a) In a rectangular co-ordinate system with scale 1 cm = 1 unit on each axis, we have $A(0, 8)$, $B(6, 0)$, $C(x, 12)$. If $0 < x < 6$ and area ΔABC is 20 cm^2, determine the value of x. **5**

(b) Solve $|x-2| + |x+2| = 4$ **3**

(c) Find all possible values of θ (to the nearest minute) if $6\cos^2\theta + \sin\theta = 5$ and $0 \le \theta \le 360^\circ$. **4**

QUESTION 7 *(12 MARKS)* **Start this question on a new page.** **Marks**

(a) A yacht sailing due west, turns at A to avoid a treacherous reef and sails on a course bearing $212°20'$ for $2\cdot8$ nautical miles to B. It then turns and sails on a course bearing $330°35'$ to a point C, due west of A. **4**

(i) Draw a diagram, showing the information above.

(ii) Find to the nearest tenth of a nautical mile, the distance BC.

(iii) How much further (to the nearest nautical mile) did the yacht have to sail than if it had maintained its original course?

(b) If $0° \le \theta \le 360°$, find all values of θ for $\tan 2\theta = \dfrac{1}{\sqrt{3}}$. **2**

(c) Solve for x: $2\sin x + \tan x = 0$ for $0 \le x \le 180^0$. **3**

(d) Find all real x such that $\left|\dfrac{x}{2}\right| > \sqrt{x^2 - 9}$. **3**

END OF PAPER

Yr 11 Ext 1 Assess 2 2004

Question 1

a) $250 - 2x^3 = 2(125 - x^3)$

$= 2(5-x)(25+5x+x^2)$

b) $\dfrac{x^2}{4x^2+7x+3} - \dfrac{x-3}{4x+3}$

$= \dfrac{x^2}{(4x+3)(x+1)} - \dfrac{x-3}{4x+3}$

$= \dfrac{x^2-(x-3)(x+1)}{(4x+3)(x+1)}$

$= \dfrac{x^2-(x^2-2x-3)}{(4x+3)(x+1)}$

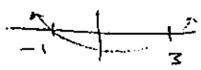
$= \dfrac{2x+3}{(4x+3)(x+1)}$

c) $x(2-x) > -3$

$2x - x^2 > -3$

$0 > x^2 - 2x - 3$

$0 > (x-3)(x+1)$



$\therefore -1 < x < 3$

d) $\sin\theta = -\frac{1}{2}$

related $\theta = 30°$ 3rd + 4th Q.

$\therefore \theta = 210°, 330°$.

e) $a + b\sqrt{7} = \dfrac{1+\sqrt{7}}{3-\sqrt{7}} \times \dfrac{3+\sqrt{7}}{3+\sqrt{7}}$

$= \dfrac{3+4\sqrt{7}+7}{-7}$

$= 5 + 2\sqrt{7}$

Question 2

a) $2x - \frac{3}{x} = 1$

$2x^2 - 3 = x$

$2x^2 - x - 3 = 0$

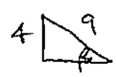
$(2x-3)(x+1) = 0$

$x = \frac{3}{2}$ or -1

$\therefore y = 2$ or -3

$\therefore (\frac{3}{2}, 2)$ or $(-1,-3)$.

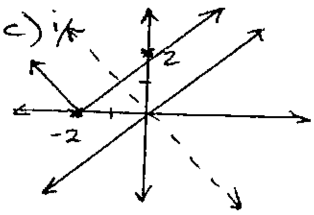
b) $\sin\beta = \frac{4}{9}$ $\quad \cos\beta < 0$



$\sqrt{9^2-4^2} = \sqrt{65}$

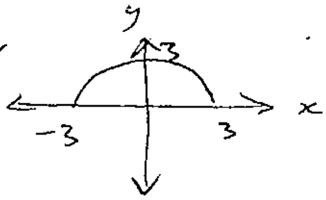
$\cos\beta = -\dfrac{\sqrt{65}}{9}$.

c) i)



ii) $x < 1$

d) i)

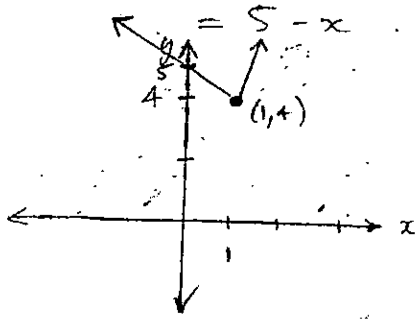


ii) $f(x) = 4 - 1 + x$ for $x \geq 1$

$= 3 + x$

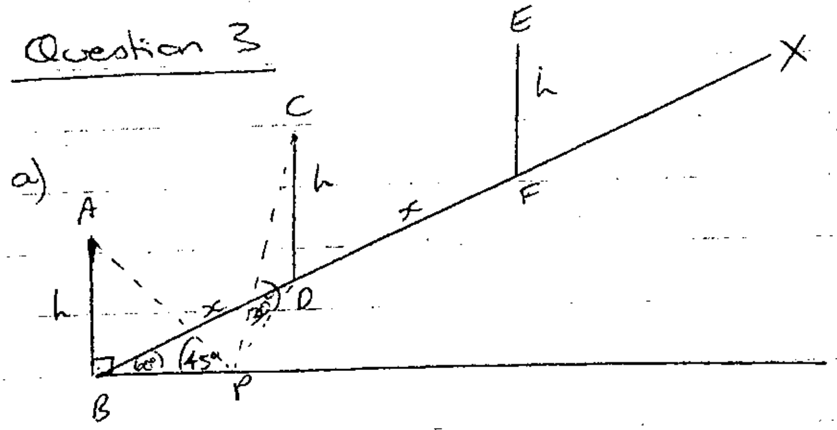
$f(x) = 4 + 1 - x$ for $x < 1$

$= 5 - x$



Question 3

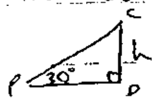
a)



i) $\angle APB = \angle PAB = 45°$ ∴ isosc

∴ $BP = h$

In ΔCDP

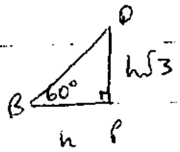


$\frac{h}{DP} = \tan 30°$

$\frac{h}{DP} = \frac{1}{\sqrt{3}}$

∴ $DP = h\sqrt{3}$

ii)



$\frac{h}{\sin \angle BDP} = \frac{h\sqrt{3}}{\sin 60°}$

$\frac{\sqrt{3}}{2} = \sqrt{3} \sin \angle BDP$

$\sin \angle BDP = \frac{1}{2}$

$\angle BDP = 30°$ or $\cancel{150°}$ ($\angle$ sum)

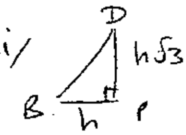
∴ By angle sum

$\angle P = 180 - (60+30)$

$= 90°$

∴ Right angled @ P.

iii)

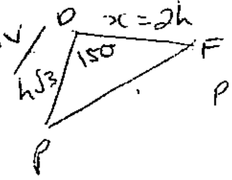


$BD = \sqrt{h^2 + 3h^2}$

$= \sqrt{4h^2}$

$= 2h$

iv)



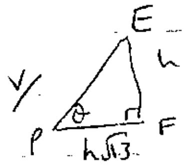
$PF^2 = (2h)^2 + (h\sqrt{3})^2 - 2 \times 2h \times h\sqrt{3} \cos 150°$

$= 4h^2 + 3h^2 - 4\sqrt{3}h^2 \cdot -\frac{\sqrt{3}}{2}$

$= 13h^2$

∴ $PF = h\sqrt{13}$

v)



$\tan\theta = \frac{h}{h\sqrt{13}}$

$\theta = \tan^{-1}\left(\frac{1}{\sqrt{13}}\right)$

$\theta = 15.5°$

b) $(-2,1)$ $(1,3)$ $-3:2$

$x = \frac{2 \times -2 + -3 \times 1}{-3+2}$

$= 7$

$y = \frac{2 \times 1 - 3 \times 3}{-3+2}$

$= 7$

$(7,7)$

c) $\frac{\sec\theta + \operatorname{cosec}\theta}{\tan\theta + \cot\theta} = \sin\theta + c$

$LHS = \frac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} \times \frac{Si}{S}$

$= \frac{\sin\theta + \cos\theta}{\sin^2\theta + \cos^2\theta}$

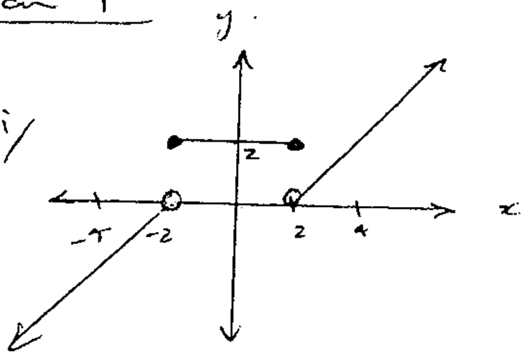
$= \sin\theta + \cos\theta$

$= RHS$

∴ True.

Question 4

a) i/



ii/ $f(-2) = 2$

$f(2) = 2$

$f(-4) = -4 + 2$

$= -2$

$\therefore 2 \times 2 - 2 + 3 \times -2$

$= 4 - 2 - 6$

$= \underline{-4}$

b) i/ $M_{AC} = \left(\frac{1+7}{2}, \frac{5-7}{2}\right)$

$= (4, -1)$

ii/ $m_{AB} = \frac{-2-5}{8-1}$

$= \frac{-7}{7}$

$= -1$

iii/ $eq^n_{AB} \Rightarrow y - 5 = -1(x-1)$

$y - 5 = -x + 1$

$x + y - 6 = 0$

$\therefore x + y = 6$

iv/ $d_{AB} = \sqrt{(8-1)^2 + (-2-5)^2}$

$= \sqrt{49 + 49}$

$= 7\sqrt{2}$

v/ $m_{OC} = \frac{-7}{7}$

$= -1$

$\therefore$ Since $m_{AB} = m_{OC}$

$AB \parallel OC$

vi/ $d_{OC} = \sqrt{7^2 + 7^2}$

$= 7\sqrt{2}$

$\therefore$ OABC is a //gram

$\because$ it has a pair of equal & parallel sid

vii/ $(0,0)$ $x + y - 6 = 0$

$d = \frac{|1 \times 0 + 1 \times 0 - 6|}{\sqrt{1+1}}$

$= \frac{6}{\sqrt{2}}$

$d = 3\sqrt{2}$

viii/ $A = AB \times 3\sqrt{2}$

$= 7\sqrt{2} \times 3\sqrt{2}$

$= 21 \times 2$

$A = 42\ u^2$

Question 5

a) $y = \dfrac{4x}{2x-1}$

i/ $x = \frac{1}{2}$

ii/ $\lim\limits_{x \to \infty} \dfrac{4x}{2x-1}$

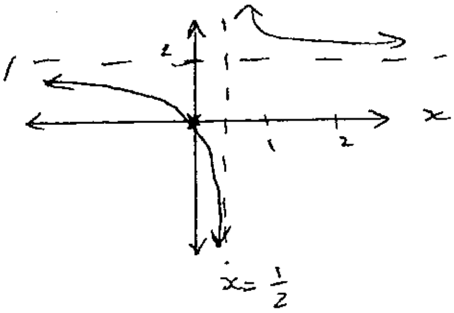
$= \frac{4}{2}$

$y = 2$.

iii/ y int $\to 0$

x int $\to 0$.

iv/



v/ when $y = 1$

$1 = \dfrac{4x}{2x-1}$

$2x - 1 = 4x$.

$-1 = 2x$

$x = -\frac{1}{2}$

$\therefore\ x < -\frac{1}{2}$ or $x > \frac{1}{2}$.

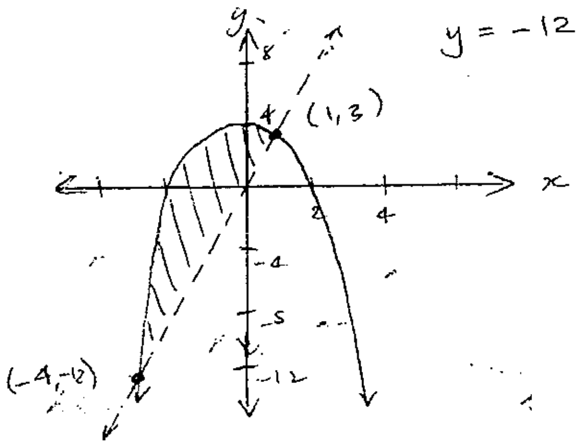
b) Intersection: $3x = 4 - x^2$.

$x^2 + 3x - 4 = 0$.

$(x+4)(x-1) = 0$.

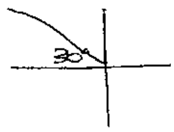
$x = -4, 1$ $\quad (-4, -12)$

$y = -12, 3$. $\quad (1, 3)$



c) $\sec(-210)^\circ$

$= \dfrac{1}{\cos(-210)^\circ}$

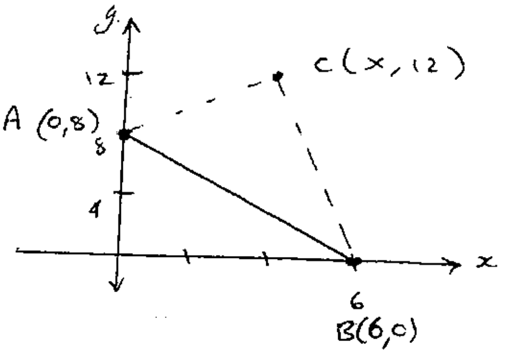


$= \dfrac{1}{-\frac{\sqrt{3}}{2}}$

$= -\dfrac{2}{\sqrt{3}}$.

Question 6

a)



Area $\triangle ABC = 20cm^2 \quad 0<x<6$

$AB = \sqrt{8^2+6^2}$

$= 10$ cm

Perp Ht $\Rightarrow c(x, 12)$

$AB: \ y = -\frac{4}{3}x + 8$

$4x + 3y - 24 = 0$

$= \frac{|4x + 36 - 24|}{\sqrt{4^2+3^2}}$

$= \frac{|4x+12|}{5}$

Area $= \frac{1}{2} \times 10 \times \frac{|4x+12|}{5}$

$20 = |4x+12|$

$\therefore \ 4x + 12 = 20$ or $-4x - 12 = 20$

$4x = 8 \qquad -4x = 32$

$x = 2 \qquad x = -8$ ✗

No negative distances.

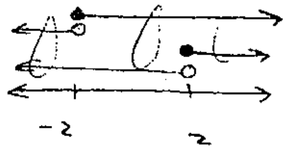
b) $|x-2| + |x+2| = 4$.

$|x-2| = x-2$ for $x \geq 2$

$-x+2$ for $x < 2$

$|x+2| = x+2$ for $x \geq -2$

$-x-2$ for $x < -2$



Case ① $-x+2-x-2 = 4$. for $x <$

$-2x = 4$

$x = -2$

② $-x+2+x+2 = 4$ for $-2 \leq x$

$4 = 4$

$\therefore$ all $x \quad -2 \leq x < 2$.

③ $x-2+x+2 = 4$

$2x = 4$

$x = 2$.

$\therefore$ Solution: $-2 \leq x \leq 2$.

c) $6\cos^2\theta + \sin\theta = 5 \quad 0 \leq \theta \leq 3$

$6(1-\sin^2\theta) + \sin\theta = 5$

$6 - 6\sin^2\theta + \sin\theta = 5$

$6\sin^2\theta - \sin\theta - 1 = 0$.

$(3\sin\theta + 1)(2\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{3}$ or $\sin\theta = \frac{1}{2}$

related $\theta = 19°28'$ $\qquad \theta = 30°, 150°$

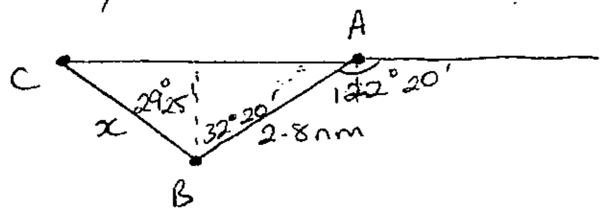
$\theta = 199°28', 340°32'$

$\therefore \ \theta = 30°, 150°, 199°28', 340°32$

Question 7

a)

i/



ii/ $\angle ABC = 29°25' + 32°20'$

$= 61°45'$

$\angle BAC = 57°40'$

$\therefore \angle ACB = 60°35'$

$\therefore \dfrac{x}{\sin 57°40'} = \dfrac{2.8}{\sin 60°35'}$

$x = \dfrac{2.8 \sin 57°40'}{\sin 60°35'}$

$\therefore BC = \underline{2.7 \text{ nm}}$.

iii/ $AC = \dfrac{2.8 \sin 61°45'}{\sin 60°35'}$

$= 2.83$.

$AB + BC = 5.51 \text{ nm}$

$\therefore 5.51 - 2.83 = 2.686 \text{ nm}$

$\therefore$ Yacht travels 2.7 nm further.

b) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$. $0 \le \theta \le 360°$

$\tan u = \frac{1}{\sqrt{3}}$. $0 \le \theta \le 720°$.

related $u = 30°$

$u = 30°, 210°, 390°, 570°$.

$\therefore \theta = 15°, 105°, 195°, 275°$.

c) $2\sin x + \tan x = 0$ $\quad 0 \le x \le 180°$

$2\sin x + \dfrac{\sin x}{\cos x} = 0$.

$\sin x \left(2 + \dfrac{1}{\cos x}\right) = 0$.

$\sin x (2 + \sec x) = 0$.

$\therefore \sin x = 0$ or $\sec x = -2$

$x = 0, 180°$. $\quad \cos x = -\frac{1}{2}$.

$x = 120°$.

$\therefore x = 0, 120°, 180°$.

d) $\left|\dfrac{x}{2}\right| > \sqrt{x^2 - 9}$.

square both sides.

$\dfrac{x^2}{4} > x^2 - 9$

$x^2 > 4x^2 - 36$.

$36 > 3x^2$.

$x^2 > 12$.

$x^2 - 12 > 0$.

$(x - 2\sqrt{3})(x + 2\sqrt{3}) > 0$

$\therefore \underline{x < 2\sqrt{3} \text{ or } x > }$

Check

$\sqrt{x^2 - 9}$ $\quad x^2 - 9 \ge 0$.

$(x-3)(x+3) \ge$

$x \ge 3$ or $x \le -3$

$\therefore$ solution is fine