Question 1 *15 marks*

a) For $f(x) = x^2 + 1 + \frac{1}{x^2}$ show that $f(y) = f\left(\frac{1}{y}\right)$ **2**

b) State the natural domain and range of $y = \sqrt{x-3}$ **2**

c) For the function:

$$f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \le x \le 2 \\ x-5, & x > 2 \end{cases}$$

(i) Sketch the graph of this function **2**

(ii) Evaluate $f(-5) + 3f(-2) - f(2) + 2f(4)$. **2**

d) For the function $y = \frac{2}{x^2 - 9}$

(i) Determine whether the function is odd, even or neither. **2**

(ii) Find the equation of the vertical asymptotes. **1**

(iii) Find the equation of the horizontal asymptote. **1**

(iv) Find the coordinates of any intercepts for this curve. **1**

(v) Sketch the curve, showing all the above information. **2**

Question 2 on page 2.

Question 2 *15 marks* **Start this question on a new page.**

a) Solve for x: $\dfrac{2x+3}{5x-1}=\dfrac{2x+1}{5x-5}$ **3**

b) Solve algebraically the following simultaneously equations. $x+2y=-8$ and $xy=8$ **3**

c) Solve for x: $4+3x-x^2>0$. **2**

c) (i) On the same diagram sketch $y=|2x-5|$ and $y=x+2$ **2**

(ii) For what values of x is $|2x-5|\le x+2$ **2**

d) Solve the following inequation. **3**

$$\frac{2}{x-3}<4,\ x\neq 3$$

Question 3 *15 marks* **Start this question on a new page.**

a) Show that $\sin 45^o.\cos 30^o+\cos 45^o.\sin 30^o=\dfrac{\sqrt{6}+\sqrt{2}}{4}$ **3**

b) Solve for $0^\circ\le x\le 360^\circ$, $2\sin x+\sqrt{3}=0$ **3**

c) If $\tan\theta=\dfrac{7}{24}$ and $180^\circ\le\theta\le 270^\circ$, find

(i) $\sin\theta$ **2**

(ii) $\operatorname{cosec}\theta$ **1**

d) Solve for $0^\circ\le\theta\le 360^\circ$, $\sin 2\theta=\dfrac{1}{2}$ **3**

e) If $\tan^2\theta+2\sec^2\theta=5$, find the value of $\sin^2\theta$ **3**

Question 4 on page 3.

Question 4 *15 marks* **Start this question on a new page.**

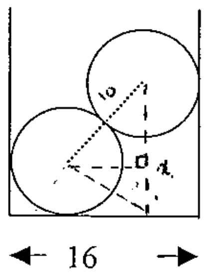
a) Simplify: $\dfrac{(\cos ec\theta + \cot\theta)(\cos ec\theta - \cot\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$ **3**

b) A soccer goal is 7.3m wide. A Newington player shoots for goal when he is16m from one goal post and 18m from the other.

(i) Draw a diagram to illustrate the information. **1**

(ii) Within what angle must a ground-shot be made to score a goal? (Answer to the nearest minute) **2**

c) The diagram shows two identical spheres of radius 5cm in a container. How high is the centre of the higher sphere above the base of the container? Note: The dotted line shown in the diagram joins the centers of the spheres and will also pass through the point of contact of the spheres. **3**



d) Two Newington College Cadet Platoons leave camp at noon in directions $242^{o}T$ and $148^{o}T$ at marching rates of 7 and 8 kilometers per hour respectively.

(i) Draw a diagram showing the information above. **1**

(ii) How far are they apart by 12.45p.m? (Correct to 2 dec.P.). **2**

(iii) What is the bearing of the slower Platoon from the faster Platoon at that time? (Answer to the nearest minute). **3**

Question 5 on page 4.

Question 5 *15 marks* **Start this question on a new page.**

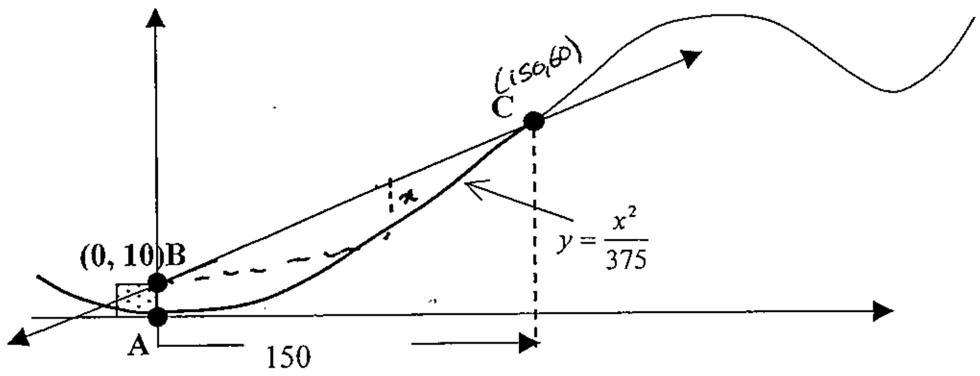
a) Find the co-ordinates of centre and the length of the radius, of a circle whose equation is: $x^2+6x+y^2-4y+4=0$. **3**

✱ b) The line $x+2y-5=0$ is at most $\sqrt{5}$ units from the point $(k,3)$. What range of values may k take. **3**

c) Find the coordinates of the point M which divides the interval joining $(2,-3)$ and $(4,6)$ **externally** in the ratio 2:1 **3**

d) A ski resort plans to open a new slope. The diagram below shows the proposed slope running from the high point, **C**, down to point **A**. The straight line **BC** represents the proposed chair lift. Point **B**, the start of the chair lift, is to be situated on the roof of an existing building.

The situation can be modeled mathematically if part of the curve $y=\frac{x^2}{375}$ is used to represent the slope **AC**, point **A** has coordinates (0, 0), **B** has coordinates (0, 10), **C** has coordinates (150, 60) and **BC** is a straight line.



(i) Find the length of the straight line **BC**. (To the nearest metre.) **1**

(ii) Find the equation of the straight line **BC**. **2**

(iii) How high is the chair lift above the snow at the mid point of **BC**? **2**

(iv) Find the angle of inclination of the chair lift. (To the nearest minute). **1**

END OF PAPER

Newington College Extension 1 Year 11 Ass 2 2005

Question 1 (15 marks)

(a) $f(x) = x^2 + 1 + \frac{1}{x^2}$

$f(y) = y^2 + 1 + \frac{1}{y^2}$ 1

$f\left(\frac{1}{y}\right) = \left(\frac{1}{y}\right)^2 + 1 + \frac{1}{(1/y)^2}$

$= \frac{1}{y^2} + 1 + y^2$ 1

$\therefore f(y) = f\left(\frac{1}{y}\right)$

b) $y = \sqrt{x-3}$

main: $x - 3 \geq 0$

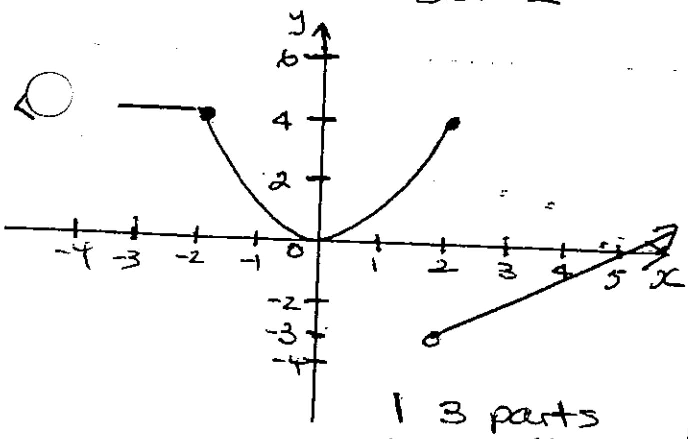
$x \geq 3$

$\therefore$ domain: $x \geq 3$ 1

range: $y \geq 0$ 1

c)(i) Sketch

$$f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \leq x \leq 2 \\ x - 5 & x > 2 \end{cases}$$



1 3 parts
1 details

(ii) $f(-5) + 3f(-2) - f(2) + 2(f(4))$

$f(-5) = 4$

$f(-2) = 4$

$f(2) = 4$ 1

$f(4) = -1$

$= 4 + 3 \times 4 - 4 + 2 \times -1$

$= 10$ 1

(d)(i) $y = \frac{2}{x^2 - 9}$

$f(x) = \frac{2}{x^2 - 9}$

f(x) connec must be there somewh

$f(-x) = \frac{2}{(-x)^2 - 9}$ 1

$f(-x) = \frac{2}{x^2 - 9}$

$\therefore f(-x) = f(x)$

$\therefore$ it is an even function 1

(ii) Vertical asymptotes

$y = \frac{2}{x^2 - 9}$

$x^2 - 9 \neq 0$

$(x-3)(x+3) \neq 0$

$\therefore x \neq \pm 3$

$\therefore$ Vertical asymptotes are $x = 3$ and $x = -3$ 1

equations

(iii) Horizontal asymptotes

$y = \frac{2}{x^2 - 9}$

$y = \dfrac{\frac{2}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}}$

$\lim_{x \to \infty} y = \frac{0}{1 - 0}$

$y = 0$

equation.

$\therefore$ horizontal asymptote at $y = 0$ 1

Intercepts

X int $y = 0$ 1

$0 = \frac{2}{x^2 - 9}$

$0 \neq 2$

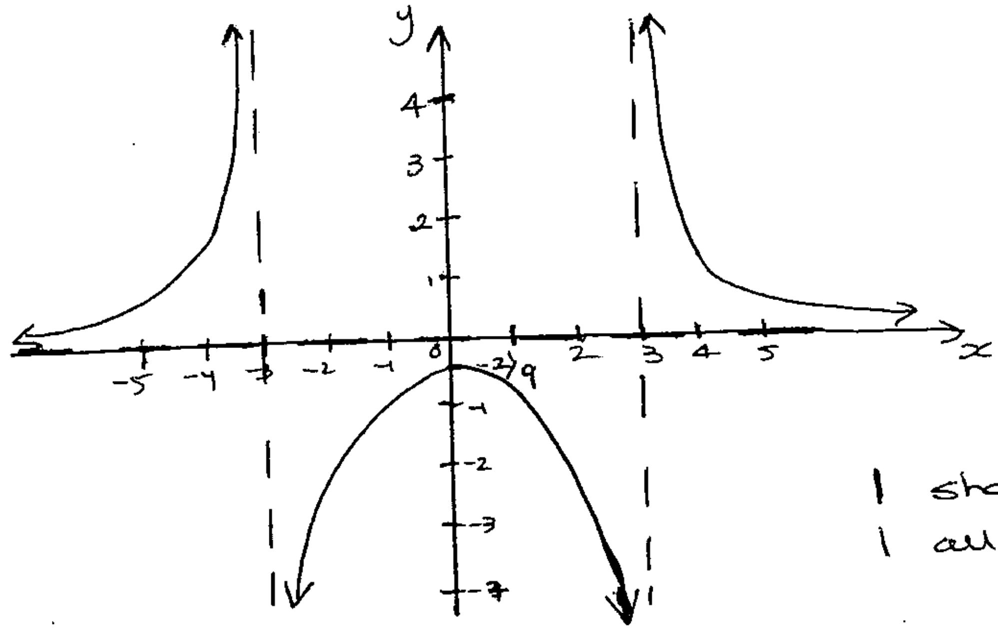
$\therefore$ no x intercepts

Y int $x = 0$

$y = \frac{2}{-9}$

$\left(0, -\frac{2}{9}\right)$ 1

$$y = \frac{2}{x^2-9}$$



1 shape
1 all information

2005 | Yr 11 3 unit Question 2 Term 2

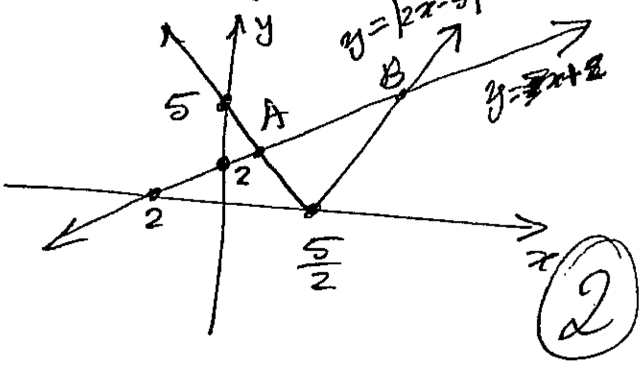
$$\frac{2x+3}{5x-1} = \frac{2x+1}{5x-5}$$

$(2x+3)(5x-5) = (2x+1)(5x-1)$

$10x^2 - 10x + 15x - 15$
$= 10x^2 - 2x + 5x - 1$

$2x = 14$

$\underline{\underline{x = 7}}$ (3)



(2)

Either A or B

$-(2x-5) = x+2$

$-2x+5 = x+2$

$\underline{\underline{x=1}}$

$2x-5 = x+2$

$\underline{\underline{x=7}}$ (2)

$\cancel{}\, 1 \le x \le 7$

$x + 2y = -8,\ xy = 8$

$y = \frac{8}{x}$

$x + 2\left(\frac{8}{x}\right) = -8$ (1)

$x^2 + 16 = -8x$

$x^2 + 8x + 16 = 0$

$(x+4)(x+4) = 0$ (1)

$\underline{\underline{x = -4}}$

Sub $y = \frac{8}{(-4)} = -2$

ANS $\left.\begin{matrix} x = -4 \\ y = -2 \end{matrix}\right\}$ (1)

$\frac{2}{x-3} < 4$

$\frac{2}{(x-3)}(x-3)^2 < 4(x-3)^2$ (1)

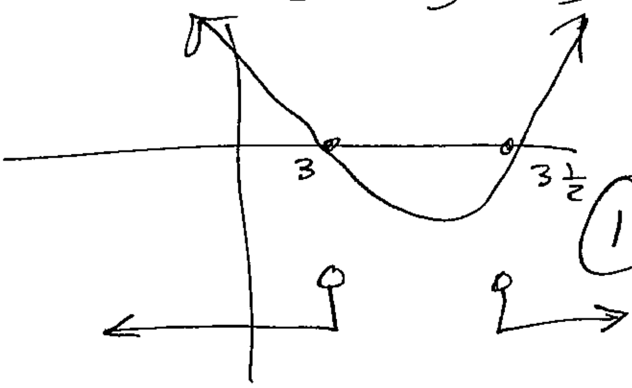
$2(x-3) < 4(x^2-6x+9)$

$2x-6 < 4x^2-24x+36$

$0 < 4x^2-26x+42$

$0 < 2(2x^2-13x+21)$ (1)

$0 < (2x-7)(x-3)$



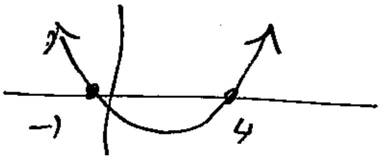
(1)

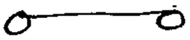
$x < 3 \quad x > 3\frac{1}{2}$

$4 + 3x - x^2 > 0$

$0 > x^2 - 3x - 4$

$0 > (x-4)(x+1)$ (1)





$-1 < x < 4$ (1)

Solutions

Q3

(a) $\sin 45 \cos 30 + \cos 45 \sin 30 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ 2 mks (correct exac[t] values)

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$.

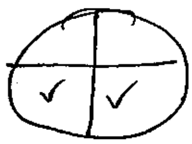
$= \frac{\sqrt{2}(\sqrt{3}+1)}{4}$ 1mk. (rationalising den[ominator])

$= \frac{\sqrt{6}+\sqrt{2}}{4}$.

(b) $2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$ 1 mk (quadrant)

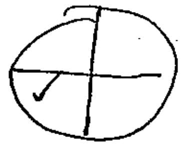


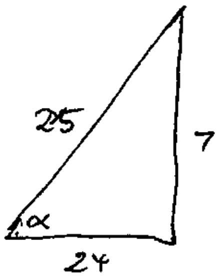
let $\sin \alpha = \frac{\sqrt{3}}{2}$ $\therefore \alpha = 60°$ 1mk (related angle).

$\therefore x = 180 + 60$ or $360 - 60$

$\therefore x = 240°$ or $300°$. 1mk (solution).

(c) $\tan \theta = \frac{7}{24}$, $180° \le \theta \le 270°$





let $\theta = 180 + \alpha$

$\therefore x^2 = 24^2 + 7^2$

$\therefore x = 25$ 1mk (Pythag).

$\therefore \sin \alpha = \frac{7}{25}$ and $\csc \alpha = \frac{25}{7}$.

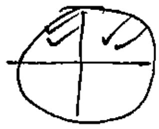
$\therefore$ (i) $\sin \theta = \sin(180 + \alpha)$ (ii) $\csc \theta = -\frac{25}{7}$.

$= -\sin \alpha$

$= -\frac{7}{25}$ 1mk (sin) 1mk (inverse).

d) $\sin 2\theta = \frac{1}{2}$; for $0° \le \theta \le 360°$

$\therefore 0° \le 2\theta \le 720°$



let $\sin \alpha = \frac{1}{2}$ 1mk (domain).

$\therefore \alpha = 30°$ 1mk (related angle).

$2\theta = \alpha,\ 180 - \alpha,\ 360 + \alpha,\ 540 - \alpha$

$\therefore 2\theta = 30°, 150°, 390°, 510°$ $\therefore \theta = 15°, 75°, 195°, 255°$.

) $\tan^2\theta + 2\sec^2\theta = 5$

$\therefore$ ~~$\tan^2\theta +$~~

$\sec^2\theta - 1 + 2\sec^2\theta = 5$ (1 mk $\underline{\text{sub}}$ identity).

$\therefore 3\sec^2\theta = 6$

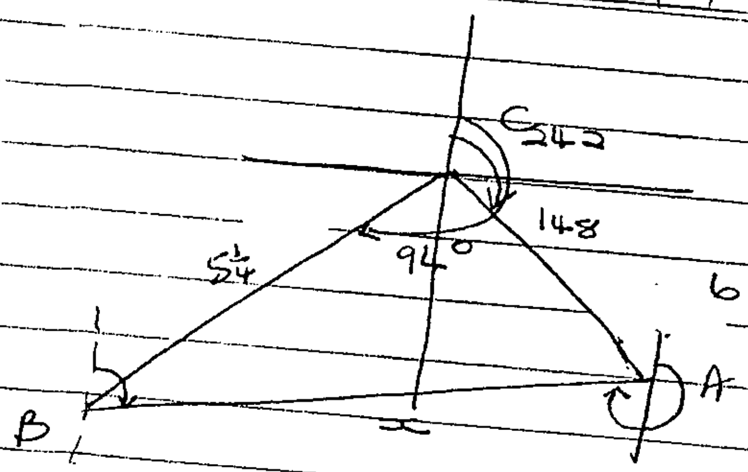
$\therefore \sec^2\theta = 2$

$\therefore \cos^2\theta = \frac{1}{2}$. (1 mk $\cos^2\theta$).

but $\sin^2\theta = 1 - \cos^2\theta$.

$\therefore \sin^2\theta = \frac{1}{2}$. (1 mk $\sin^2\theta$).

Q4



(1)

$\frac{3}{4} \times 7 = 5\frac{1}{4}$

$\frac{3}{4} \times 8 = 6$

$x^2 = 6^2 + (5\tfrac{1}{4})^2 - 2 \times 6 \times 5\tfrac{1}{4} \cos 94$ (1)

$= 67.95715785$

$x \doteq 8.24361315$

$= 8.24$ to 2dp (1)

$\frac{\sin A}{5\frac{1}{4}} = \frac{\sin C}{x}$

$\frac{\sin A}{5\frac{1}{4}} = \frac{\sin 94}{8.24}$

$\sin A = 5\tfrac{1}{4} \times \frac{\sin 94^\circ}{8.24}$

rounded

$= 0.635583891$ (1)

$A = 39°27'47.89''$

exact

0.635305316

$39°26'46.26''$

(1)

Bearing $= 360 - 32 - A$ (1)

$288°$ 321

$288°33'$

(Q4)

a) $\dfrac{(\text{Cosec}\,\theta + \text{Cot}\,\theta)(\text{Cosec}\,\theta + \text{Cot}\,\theta)}{(\text{Sec}\,\theta + \tan\theta)(\text{Sec}\,\theta - \tan\theta)}$

$= \dfrac{\text{Cosec}^2\theta - \text{Cot}^2\theta}{\text{Sec}^2\theta - \tan^2\theta}$ 1

$\text{Sin}^2\theta + \text{Cos}^2\theta = 1$

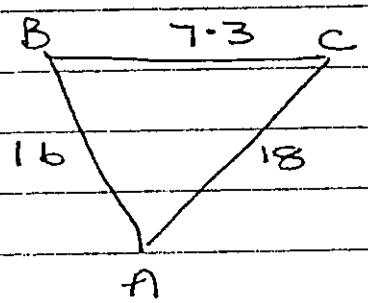
$1 + \text{Cot}^2\theta = \text{Co}$

$\text{Tan}^2\theta + 1 = \text{Sec}$

$= \dfrac{1 + \text{Cot}^2\theta - \text{Cot}^2\theta}{\text{Tan}^2\theta + 1 - \tan^2\theta}$ 1

$= 1$ 1

b)



1

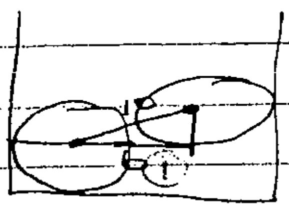
$\text{Cos}\,A = \dfrac{b^2 + c^2 - a^2}{2bc}$

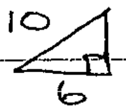
$= \dfrac{18^2 + 16^2 - 7{\cdot}3^2}{2 \times 18 \times 16}$ 1

$= \dfrac{526{\cdot}71}{576}$

$A = 23^\circ\ 52'\ 31{\cdot}811''$

$A = 23^\circ\ 53'$ 1 to nearest min





1

$d = 8 + 5$

$= 13$ 1

Q5 - Solution [out of 12]

$\boxed{\;/13}$

a) $x^2+6x+y^2-4y+4=0$

$(x+3)^2+(y-2)^2 = -4+9+4$

$(x+3)^2+(y-2)^2 = 9$

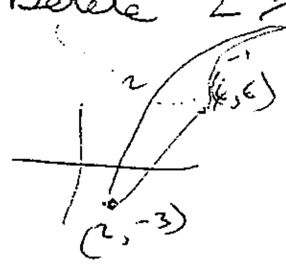
Centre $(-3,2)$

radius 3

$\boxed{3}$

b) Delete [~~3~~ marks for all candidates] [Answer (= 4, 7 $\boxed{\cancel{3}}$

c)



$$P(x,y) = P\left(\frac{2\times4-2}{2-1},\ \frac{2\times6+3}{2-1}\right)$$

$$=(6,15)$$

$\boxed{3}$

(d) (i) $BC = \sqrt{(60-10)^2+(150-0)^2}$

$= \sqrt{50^2+150^2}$

$= \underline{158}$ m [nearest m]

$\boxed{1}$

ii) $m = \dfrac{60-10}{150} = \dfrac{1}{3}$

$y-10 = \frac{1}{3}(x-0)$

$3y-30 = x$

$x-3y+30 = 0$ $\{y = \frac{1}{3}x+10\}$

$\boxed{2}$

(iii) Mid pt $BC = \left(\frac{150}{2}, \frac{70}{2}\right)$

$=(75,35)$

$\therefore y = \dfrac{75^2}{375}$

$= \underline{15}$ m

$\therefore$ 20 m above snow.

$\boxed{2} + 1 = \boxed{3}$

(iv) Angle of inclination $= \tan^{-1}\left(\frac{50}{150}\right)$

$= \underline{18^\circ 26'}$

$\boxed{1}$