Marks

Question 1 (14 Marks)

(a) Solve $x^2 - 6x + 3 = 0$ by completing the square. Give your answer in surd form. 3

(b) Solve simultaneously $y - 2x = 1$ and $x^2 + y^2 = 10$ 3

(c) Solve

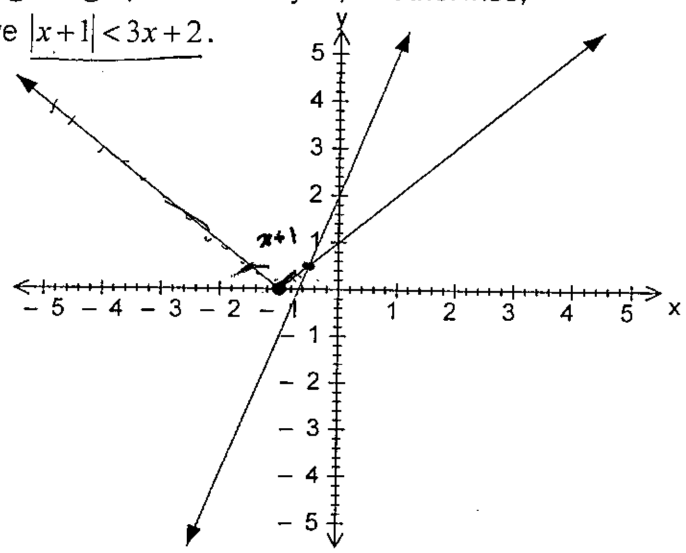
(i) $x^2 \geq 16$ 1

(ii) $|2x - 1| < 3$ 2

(iii) $\dfrac{x+3}{x-3} \geq 3$ 3

(d) The graph below shows the functions $y = |x+1|$ and $y = 3x + 2$. 2

Using the graph to assist you, or otherwise, solve $|x+1| < 3x + 2$.



Question 2 (16 Marks) START ON A NEW PAGE — Marks

(a) Consider the function $f(x)=\sqrt{x+1}$. — 6

(i) State the domain and range of $f(x)$

(ii) Sketch the function

(iii) Find the inverse $f^{-1}(x)$

(b) Sketch the graph of $y=-(x-2)^3$ showing its key features. — 2

(c) Show whether the $f(x)=\dfrac{2x}{3(1+x^2)}$ is an odd function, an even function or neither. — 2

(d) Consider the function $f(x)=\dfrac{2x}{x^2+2x-3}$ — 6

(i) Find the x and y intercepts

(ii) Find the equations of any vertical asymptotes

(iii) Find the equation of the horizontal asymptote

(iv) Sketch the graph of the function.

Question 3 (23 Marks) START ON A NEW PAGE

(a) Find the exact value of $\cot 120^\circ$ — 1

(b) If $\tan\theta=2$ and $\sin\theta<0$, find the exact value of $\cos\theta$. — 2

(c) Solve the following equations for $0^\circ\le\theta\le360^\circ$ — 7

(i) $2\sin\theta=-\sqrt{3}$

(ii) $3\cos^2\theta+2\cos\theta=0$ (nearest minute)

(iii) $\sec\dfrac{\theta}{2}=-2$

(d) Eliminate θ from the pair of equations to find a relationship between x and y: — 2

$x=1+\sin\theta$

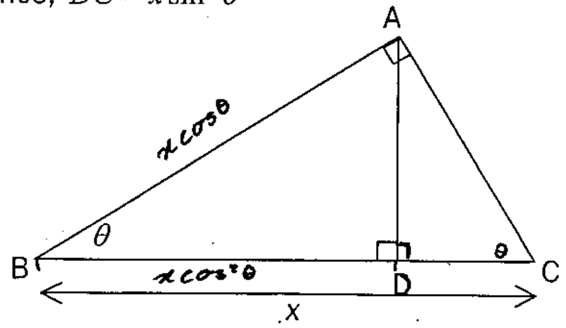
$y=1+\cos\theta$

(e) Show that $\dfrac{\cot\theta}{\operatorname{cosec}\theta-1}+\dfrac{\cot\theta}{\operatorname{cosec}\theta+1}=2\sec\theta$ — 3

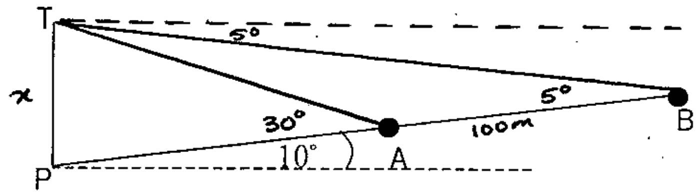
Q3 continued on next page

Question 3 continued **Marks**

*(f) ΔABC and ΔABD are right-angled. Show that 4
(i) $AB = x\cos\theta$
(ii) $BD = x\cos^2\theta$
(iii) Hence, $DC = x\sin^2\theta$



(g) P, A, and B are points on a ramp which makes an angle of 10° to the horizontal as shown on the diagram below. From A and B, the angles of elevation of the top T of a flagpole at P were 30° and 5° respectively. The distance AB is 100 metres. 4
(i) Copy and complete the diagram below.
(ii) Calculate the height of the tower to the nearest centimetre.

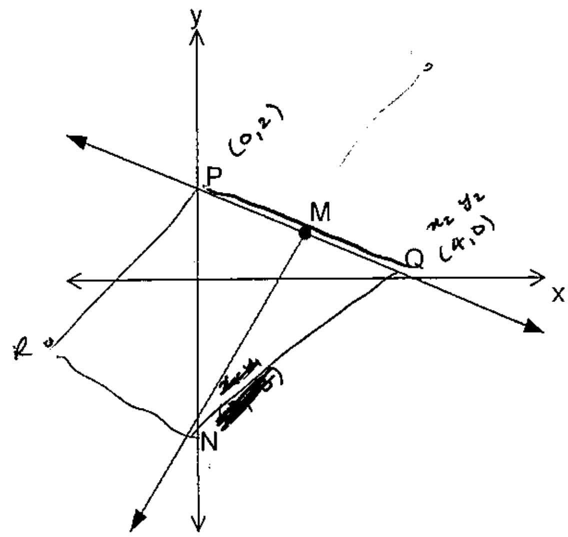


Q4 on next page

Question 4 (16 marks) START ON A NEW PAGE **Marks**

(a) Find the coordinates of the point P that divides the interval AB, where A is (-3,5) and B is (-6,-10), externally in the ratio 2:3. 3

(b)



x_1 y_1 x_2 y_2

The diagram above shows the points P (0,2) and Q (4,0). The point M is the midpoint of PQ. The line MN is perpendicular to PQ and meets the y-axis at N.

(i) Show that the gradient of PQ is $-\frac{1}{2}$. 1

(ii) Find the coordinates of M. 1

(iii) Find the equation of the line MN. 2

(iv) Show that N has coordinates (0,-3). 1

(v) Find the distance NQ. 1

(vi) Find the equation of the circle with centre N and radius NQ. 2

(vii) Show that this circle passes through the point P. 1

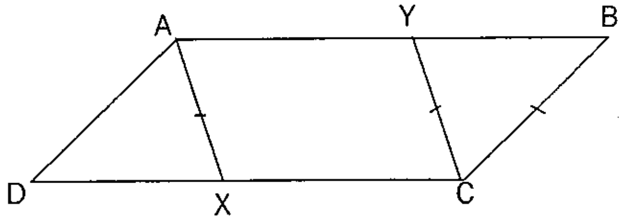
(viii) Find the coordinates of the point R such that PRQN is a rhombus. 1

(c) The perpendicular distance between the point (a,-2) and the line $3y = 4x + 2$ is 8 units. Find any possible values of a. 3

Q5 on next page

Question 5 (16 Marks) START ON A NEW PAGE Marks

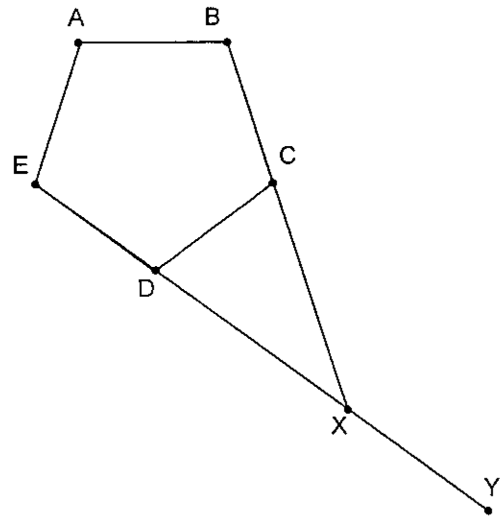
(a) 6



ABCD is a parallelogram. The point X lies on CD and the point Y lies on AB. AX=YC=BC as shown on the diagram.

(i) Explain why $A\hat{D}X = C\hat{B}Y$.
(ii) Show that *AD*=*AX*
(iii) Show that triangles *ADX* and *CBY* are congruent.
(iv) Hence prove that *AYCX* is a parallelogram.

(b)



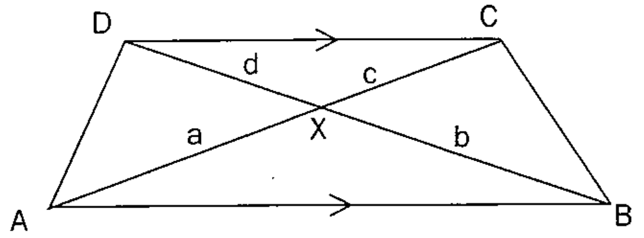
In the diagram ABCDE is a regular pentagon and BC and ED produced meet at X. The point Y lies on EDX produced. 3

(i) Find the size of $B\hat{C}D$.
(ii) Find the size of $C\hat{X}Y$ giving reasons.

Q5 continued on next page

Question 5 continued Marks

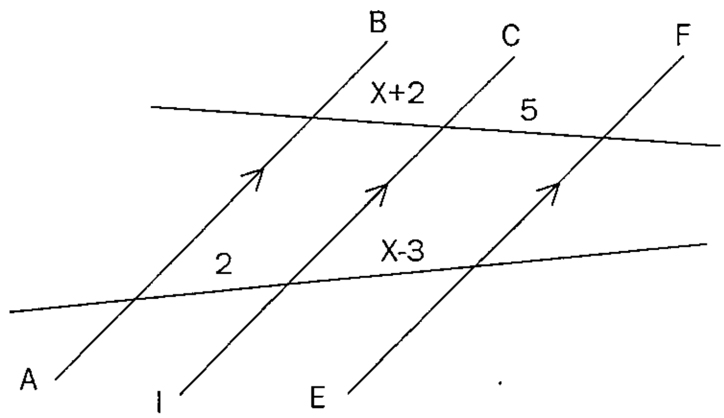
(c) 4



(i) Prove that $\Delta DXC \parallel\!\mid \Delta BXA$.

(ii) Hence, prove that ΔDAX and ΔCBX have the same area.

(d) 3



Given that $AB \parallel CD \parallel EF$, find the value of x giving reasons.

END OF PAPER

Q1

a) $x^2 - 6x = -3$

$x^2 - 6x + \left(\frac{-6}{2}\right)^2 = -3 + \left(\frac{-6}{2}\right)^2$

$x^2 - 6x + 9 = -3 + 9$

$(x-3)^2 = 6$

$x - 3 = \pm\sqrt{6}$

$x = 3 \pm \sqrt{6}$

b) $x^2 + y^2 = 10$ ①

$y - 2x = 1$ ②

From ② $y = 2x + 1$

sub $y = 2x+1$ in ①

$x^2 + (2x+1)^2 = 10$

$x^2 + 4x^2 + 4x + 1 = 10$

$5x^2 + 4x - 9 = 0$

$(5x+9)(x-1) = 0$

$x = -9/5$ or 1

$x = -9/5 \quad y = 2(-9/5) + 1$

$= -13/5$

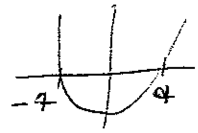
$(-9/5, -13/5)$

$x = 1 \quad y = 3$

$(1, 3)$

c) i) $x^2 \geq 16$

$x^2 - 16 \geq 0$



$(x-4)(x+4) \geq 0$

$x \leq -4$ or $x \geq 4$

ii) $|2x-1| < 3$

$\pm(2x-1) < 3$

$2x - 1 < 3$	$-(2x-1) < 3$
$2x < 4$	$-2x + 1 < 3$
$x < 4$	$-2x < 2$
	$x > -1$

$-1 < x < 2$

iii) $\frac{x+3}{x-3} \geq 3 \quad x \neq 3$

$\times(x-3)^2 \quad \times(x-3)^2$

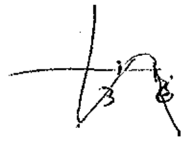
$(x+3)(x-3) \geq 3(x-3)^2$

$(x+3)(x-3) - 3(x-3)^2 \geq 0$

$(x-3)\left(x+3-3(x-3)\right) \geq 0$

$(x-3)(-2x+12) \geq 0$

$3 \leq x \leq 6$



but $x \neq 3$

$\therefore 3 < x \leq 6$

d) $y = x + 1$ ① (positive branch of $y = |x+1|$)

$y = 3x + 2$ ②

① − ②

$0 = -2x - 1$

$x = -1/2$

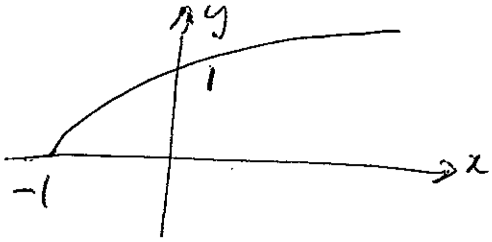
$\therefore x > -\frac{1}{2}$

Q2

a) i) domain $x \geq -1$

range $y \geq 0$

ii)

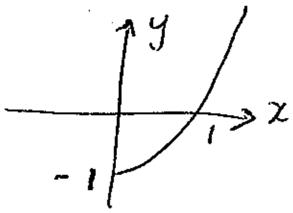


iii) $x = \sqrt{y+1}$

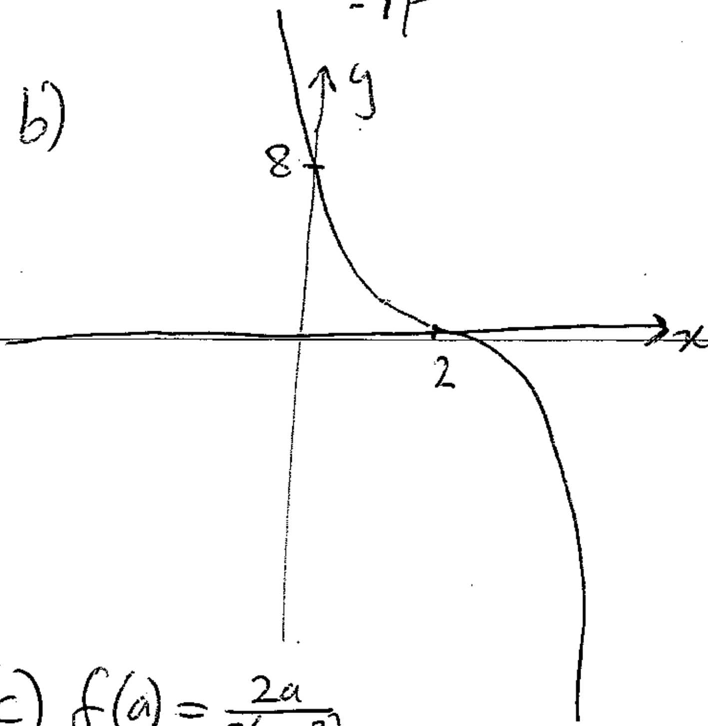
$x^2 = y + 1$

$y = x^2 - 1$

for $x \geq 0$



b)



c) $f(a) = \frac{2a}{3(1+a^2)}$

$f(-a) = \frac{2(-a)}{3(1+(-a)^2)} = \frac{-2a}{3(1+a^2)}$

$= -f(a)$

$\therefore$ ODD

d) $f(x) = \frac{2x}{x^2+2x-3}$

$= \frac{2x}{(x+3)(x-1)}$

i) $x=0 \quad y=0 \quad (0,0)$

x & y intercept is the origin

ii) $x = -3$ and $x = 1$

iii) $\lim_{x \to \infty} \frac{2x}{x^2+2x-3}$ C

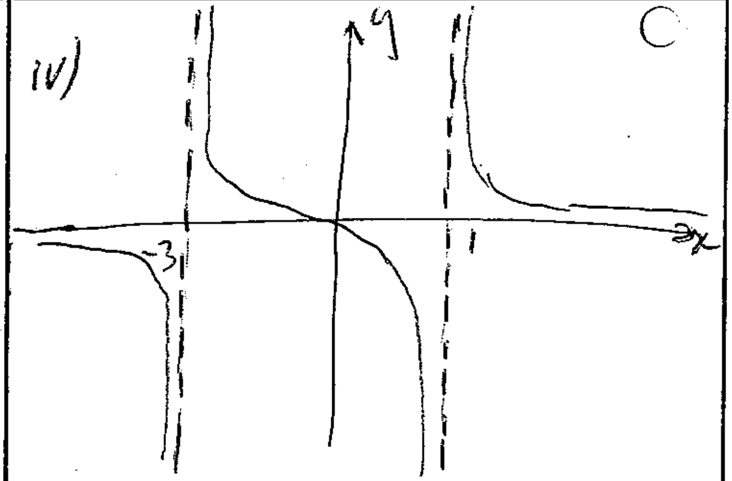
$= \lim_{x \to \infty} \frac{\frac{2x}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}}$

$= \lim_{x \to \infty} \frac{\frac{2}{x}}{1 + \frac{2}{x} - 3/x^2}$

$= \frac{0}{1} = 0$

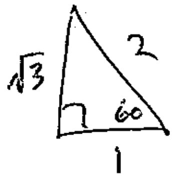
$\therefore y = 0$

iv) C



x	-4	-1	$1/2$	2
y	-1.6	0.5	-0.2	0.8

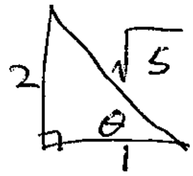
Q3 a) $\cot 120 = \frac{-1}{\tan 60}$



$= \frac{-1}{\sqrt{3}}$

b) $\tan\theta = 2$

$\sin\theta < 2$



$\cos\theta = \frac{1}{\sqrt{5}}$

c) i) $\sin\theta = -\sqrt{3}/2$

$\theta = 180+60,\ 360-60$

$= 240,\ 300°$

ii) $3\cos^2\theta + 2\cos\theta = 0$

$\cos\theta(3\cos\theta+2) = 0$

$\cos\theta = 0$ or $\cos\theta = \frac{-2}{3}$

$\theta = 90°$ or $270°$ $\theta = 180 - 48°11'$ or $180 + 48°11'$

$\therefore \theta = 90°, 270°, 131°49, 228°11$

iii) $\sec\frac{\theta}{2} = -2$

$\cos\frac{\theta}{2} = -\frac{1}{2}$

$\frac{\theta}{2} = 180-60,\ 180+60$

$= 120,\ 240$

$\theta = 240$ or 480

But $0 \le \theta \le 360$

$\therefore \theta = 240°$

d) $x = 1 + \sin\theta$ ①

$y = 1 + \cos\theta$ ②

From ① $\sin\theta = x - 1$

From ② $\cos\theta = y - 1$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore (x-1)^2 + (y-1)^2 = 1$

e) $LHS = \frac{\cot\theta}{\text{cosec}\,\theta - 1} + \frac{\cot\theta}{\text{cosec}\,\theta + 1}$

$= \frac{\cot\theta(\text{cosec}\,\theta + 1) + \cot\theta(\text{cosec}\,\theta - 1)}{\text{cosec}^2\theta - 1}$

$= \frac{2\cot\theta\,\text{cosec}\,\theta}{\text{cosec}^2\theta - 1}$

$\boxed{\begin{array}{l}\text{From } \sin^2\theta + \cos^2\theta = 1 \\ 1 + \cot^2\theta = \text{cosec}^2\theta \\ \cot^2\theta = \text{cosec}^2\theta - 1\end{array}}$

$= \frac{2\cot\theta\,\text{cosec}\,\theta}{\cot^2\theta}$

$= \frac{2\,\text{cosec}\,\theta}{\cot\theta}$

$= \frac{2}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$

$= \frac{2}{\cos\theta}$

$= 2\sec\theta = RHS.$

Q 3 continued

f) i) $\cos\theta = \frac{AB}{x}$

$\therefore AB = x\cos\theta$

ii) $\cos\theta = \frac{BD}{AB}$

$\cos\theta = \frac{BD}{x\cos\theta}$

$BD = x\cos^2\theta$

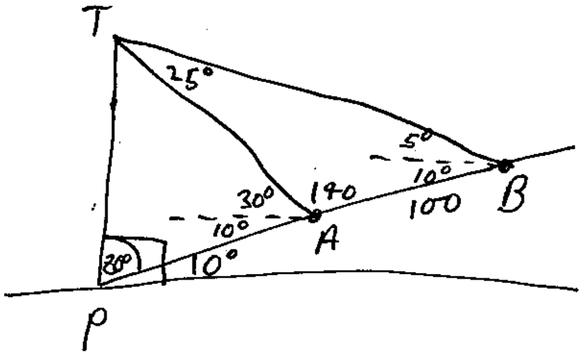
iii) $BC = BC - BD$

$= x - x\cos^2\theta$

$= x(1-\cos^2\theta)$

$= x\sin^2\theta$

g)



In Δ ATB

$\frac{100}{\sin 25°} = \frac{AT}{\sin 15°}$

$AT = \frac{100\sin 15}{\sin 25}$

In Δ APT

$\frac{AT}{\sin 8\theta} = \frac{TP}{\sin 40°}$

$TP = \frac{\sin 40\left(\frac{100\sin 15}{\sin 25}\right)}{\sin 80}$

$= 39.97\,m$

Q4

a) $(-3,5)$ $(-6,-10)$

$-2:3$

$\left(\frac{3\times -3 + -2\times -6}{-2+3}, \frac{3\times 5 + -2\times -10}{-2+3}\right)$

$= \frac{-9+12}{1}, \frac{15+20}{1}$

$= (3, 35)$

b) i) $m = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{0-2}{4-0}$

$= \frac{-2}{4}$

$= -\frac{1}{2}$

ii) $Midpt = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{0+4}{2}, \frac{2+0}{2}\right)$

$= (2,1)$

iii) $m = \frac{-1}{-\frac{1}{2}} = 2$ $(2,1)$

$y - y_1 = m(x-x_1)$

$y - 1 = 2(x-2)$

$y = 2x - 4 + 1$

$y = 2x - 3$

iv) $x = 0$ $y = 2(0) - 3$

$= -3$

$\therefore N$ is $(0,-3)$

v) $(0,-3)$ $(4,0)$

$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(0-4)^2 + (-3-0)^2}$

$= \sqrt{16+9} = 5$

Q4 continued

b)

vi) $(x-0)^2+(y+3)^2=25$

vii) $x=0$

$(0-0)^2+(y+3)^2=25$

$(y+3)^2=25$

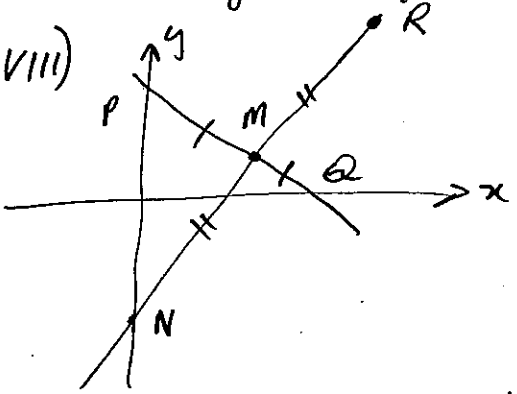
$y+3=\pm\sqrt{25}$

$y=-3\pm5$

$=2$ or -8

$P(0,2)$ $\therefore$ circle passes through the point P

viii)



M is the mid point of NR if PRQN is a rhombus.

Let R be (x,y)

Then $(2,1)=\left(\frac{x+0}{2},\frac{y+-3}{2}\right)$

$\frac{x}{2}=2$

$x=4$

$\frac{y-3}{2}=1$

$y-3=2$

$y=5$

$\therefore$ R is $(4,5)$

c) $D=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$4x-3y+2=0$

$D=8$ $\quad(a,-2)$

$8=\frac{|4a+-3(-2)+2|}{\sqrt{4^2+(-3)^2}}$

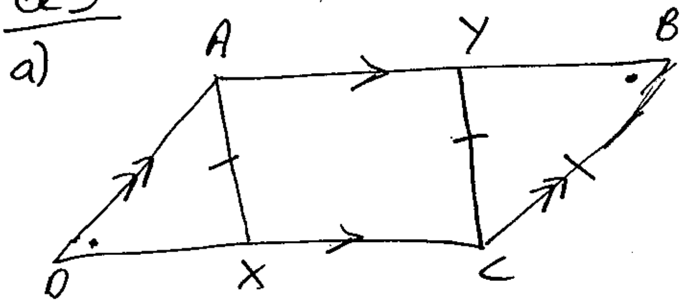
$8=\frac{|4a+8|}{5}$

$40=|4a+8|$

$4a+8=40$	$-(4a+8)=40$
$4a=32$	$-4a-8=40$
$a=8$	$-4a=48$
	$a=-6$

Q5

a)



i) $\angle ADX=\angle CBY$ (opposite angles in a parallelogram)

ii) $AD=BC$ (opposite sides of a parallelogram)

$BC=AX$ (given)

$\therefore AD=AX$

iii) In Δs ADX and CBY

$\angle ADX=\angle YBC$ (above)

$\angle ADX=\angle AXD$ (base angles of an isosceles Δ)

Q5 continued

$\angle CYB = \angle CBY$ (base angles of an isosceles Δ)

$\therefore \angle AXD = \angle BYC$

$AX = CY$ (given)

$\therefore \Delta ADX \equiv \Delta CBY$ (AAS.

iv) $DX = BY$ (corresponding sides of congruent Δs)

$AB = DC$ (opposite sides of a parallelogram)

$XC = DC - DX$

$AY = AB - BY$

$\therefore XC = AY$

$AX = YC$ given

$\therefore AYCX$ is a parallelogram (opposite sides equal)

b) $\angle BCD = \dfrac{180 \times (5-2)}{5}$

$= 108^\circ$

$\angle XCD = 180 - 108$

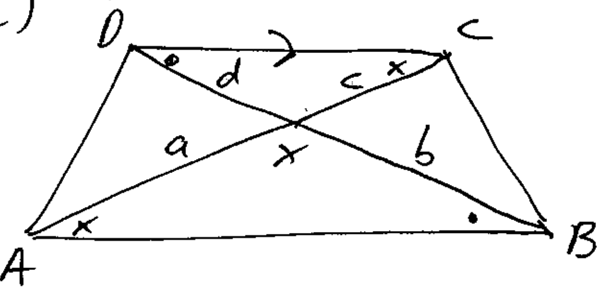
$= 72^\circ$ (supplementary $\angle$s)

similarly $\angle CDX = 72^\circ$

$\therefore \angle CXY = 72 + 72^\circ = 144^\circ$

(exterior angle of a triangle.)

c)



i) In Δs DXC and BXA

$\angle CDX = \angle XBA$ (alternate $\angle$s $DC \parallel AB$)

$\angle DCX = \angle XAB$ (alternate $\angle$s $DC \parallel AB$)

$\therefore \Delta DXC \mid\mid\mid \Delta BXA$ (equiangular)

ii) $\dfrac{b}{d} = \dfrac{a}{c}$ (corresponding sides in same ratio)

$\therefore bc = ad$

Area $\Delta DAX = \frac{1}{2} ad \sin \angle AXD$

$\Delta CXB = \frac{1}{2} bc \sin \angle CXB$

$\angle CXB = \angle AXD$ (vert. opp)

$\therefore$ areas are equal.

d) $\dfrac{2}{x-3} = \dfrac{x+2}{5}$

$10 = (x-3)(x+2)$

$10 = x^2 - x - 6$

$0 = x^2 - x - 16$

$x = \dfrac{--1 \pm \sqrt{1^2 - 4 \times 1 \times -16}}{2}$

$= \dfrac{1 \pm \sqrt{1+64}}{2}$

$= \dfrac{1 \pm \sqrt{65}}{2}$

(parallel lines cut intercepts in the same ratio).