Question 1 (18 Marks) **Marks**

(a) Factorise fully

(i) $16x^3 - 54y^3$ **2**

(ii) $x^2 - y^2 + y - x$ **2**

(iii) $a^5 - 2a^3 + a$ **3**

(b) Simplify $\dfrac{1}{y-x} + \dfrac{x}{(x-y)^2}$ **3**

(c) Express $2.03\dot{9}$ as a rational number in its simplest form. **3**

(d) Express $2\sqrt{27} - \dfrac{\sqrt{12}}{2} + 3\sqrt{48}$ in its simplest form. **2**

(e) Solve the equations $x - y = 6$ and $x^2 + y^2 = 18$ simultaneously and explain the geometric significance of your answer. **3**

Question 2 (18 Marks) (START QUESTION IN A NEW BOOKLET)

(a) Solve

(i) $|x-3| = 11$ **2**

(ii) $|2x-11| = 3x-4$ **3**

(iii) $|1-2x| \le 5$ **2**

(iv) $\dfrac{1-y}{6} - \dfrac{y-2}{3} = \dfrac{y+1}{4}$ **2**

(b) Express $1 + \dfrac{2}{\sqrt{3}-1}$ in the form $a + b\sqrt{3}$ where a and b are rational. **3**

Question 2 Continued. **Marks**

(c) If $x - y + \sqrt{x+y} = \sqrt{6}$, find the value of x and y. **2**

(d) Solve $\dfrac{x}{x-1} < -2$ **4**

Question 3 (18 Marks) (START QUESTION IN A NEW BOOKLET)

(a) If $f(x) = x^2 + 2$ and $g(x) = 2x + 3$, find the value of $f(g(3))$. **2**

(b) Write down the natural domain of $f(x) = \dfrac{1}{\sqrt{x^2 - 9}}$ **1**

(c) Sketch the function $y = \dfrac{2x-3}{x-1}$, stating the equations of its vertical and horizontal asymptotes? **5**

(d) Noting any restrictions on the domain of the function, make a neat sketch of the graph of **2**

$$y = \frac{x^2 - 9}{x+3}$$

(e) Solve $|x-2| + |x+1| = 3$. **3**

(f) Sketch the graphs of $y = |x+2|$ and $y = |x-4|$ on the same set of axes and use your graphs to solve $|x+2| \le |x-4|$. **5**

Question 4 (18 Marks) (START QUESTION IN A NEW BOOKLET)

(a) Without using a calculator, prove that $\dfrac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \cos 60^\circ$. **3**

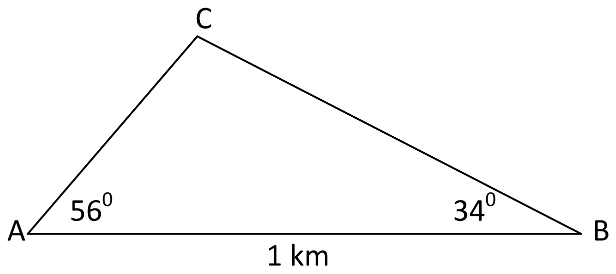
(b) If θ is acute and $\sec\theta = \dfrac{\sqrt{5}}{2}$, find the exact value of: **3**

(i) $\sin\theta$

(ii) $\tan\theta$

Question 4 Continued. **Marks**

(c) From two observation points A and B 1 km apart, a balloon C is observed at angles of elevation of 56° and 34° respectively. Find to the nearest metre, the height of the balloon above the ground. **4**



(d) Find the exact value of $\dfrac{\sin 135^{\circ} - \cos 240^{\circ}}{\sin 225^{\circ} + \cos 120^{\circ}}$. **2**

(e) If $\tan\theta = -\dfrac{5}{12}$ and $270^{\circ} < \theta < 360^{\circ}$, find the exact value of $\cos\theta$. **2**

(f) Prove $\dfrac{1}{\sec\theta + \tan\theta} = \dfrac{\cos\theta}{1+\sin\theta}$. **2**

(g) Prove $\dfrac{2\cos^2\theta - 1}{\cos^2\theta - \sin^2\theta} = 1$. **2**

END OF PAPER

Q1 (a) (i) $2\left(8x^3 - 27y^3\right) = 2(2x-3y)(4x^2+6xy+9y^2)$ ✓ ✓

(ii) $(x-y)(x+y) - (x-y) = (x-y)(x+y-1)$ ✓ ✓

(iii) $a\left(a^4 - 2a^2 + 1\right) = a\left(a^2-1\right)^2 = a(a-1)^2(a+1)^2$ ✓ ✓ ✓

(b) $-\dfrac{1}{x-y} + \dfrac{x}{(x-y)^2} = \dfrac{-(x-y)+x}{(x-y)^2} = \dfrac{y}{(x-y)^2}$ ✓ ✓

(c) Let $x = 2.03\dot{9}$

$\therefore\ 10x = 20.39\dot{9}$ ✓

$\therefore\ 9x = 18.36$ ✓

$x = \dfrac{1836}{900}$

$= 2\dfrac{1}{25}$ ✓

(d) $6\sqrt{3} - \sqrt{3} + 12\sqrt{3} = 17\sqrt{3}$ ✓ ✓

(e) $y = x - 6$

$\therefore\ x^2 + (x-6)^2 = 18$

$x^2 + x^2 - 12x + 36 = 18$

$2x^2 - 12x + 18 = 0$ ✓

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$\left.\begin{aligned} \therefore\ x &= 3 \\ y &= -3 \end{aligned}\right\}$ ✓

Since there is only one solution the line is a tangent to the circle. ✓

Q2

(a) (i) $x - 3 = 11$ $\qquad$ $x - 3 = -11$

$x = 14$ $\qquad$ $x = -8$

(ii) $2x - 11 = 3x - 4$ $\qquad$ $2x - 11 = -(3x - 4)$

$x = -7$ $\qquad$ $2x - 11 = -3x + 4$

$5x = 15$

$x = 3$

Testing solutions

$x = -7$: LHS = 25 , RHS = −25

$x = 3$: LHS = 5 , RHS = 5

$\therefore$ $x = -7$ is not a solution

$\therefore$ Solution is $x = 3$

(iii) $1 - 2x \leq 5$ $\qquad$ $1 - 2x \geq -5$

$-2x \leq 4$ $\qquad$ $-2x \geq -6$

$x \geq -2$ $\qquad$ $x \leq 3$

$\therefore$ Solution is $-2 \leq x \leq 3$

(iv) $2(1-y) - 4(y-2) = 3(y+1)$

$2 - 2y - 4y + 8 = 3y + 3$

$10 - 6y = 3y + 3$

$9y = 7$

$y = \frac{7}{9}$.

(b) $$\frac{\sqrt{3}-1+2}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}+1)^2}{3-1}$$

$$= \frac{3 + 2\sqrt{3} + 1}{2}$$

$$= \frac{4 + 2\sqrt{3}}{2}$$

$$= 2 + \sqrt{3}$$

(c)

$$\left.\begin{aligned} x - y &= 0 \\ x + y &= 6 \end{aligned}\right\}$$

$$2x = 6$$

$$\left.\begin{aligned} x &= 3 \\ y &= 3 \end{aligned}\right\}$$

(d)

$$(x-1)^2 \cdot \frac{x}{(x-1)} < -2 \cdot (x-1)^2$$

$$x(x-1) < -2(x-1)^2$$

$$2(x-1)^2 + x(x-1) < 0$$

$$(x-1)(2x-2+x) < 0$$

$$(x-1)(3x-2) < 0$$

$$\therefore \quad \frac{2}{3} < x < 1$$

Q3

(a) $g(3) = 9$ ✓

$\therefore f(9) = 81 + 2 = 83$ ✓

(b) $x < -3, \quad x > 3$ ✓

(c) Vertical asymptote is $x = 1$ ✓

Horizontal asymptote:

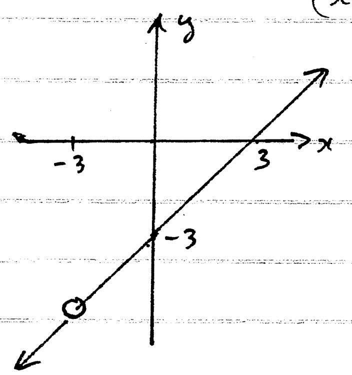
$$y = \frac{2 - \frac{3}{x}}{1 - \frac{1}{x}}$$ ✓

as $x \to \infty$, $y \to \frac{2}{1} = 2$

$\therefore \quad y = 2$ ✓

(d) Function is undefined for $x = -3$

Otherwise $y = \dfrac{(x-3)(x+3)}{(x+3)} = x - 3$ ✓



✓

(e) (i) $(x-2)^2 + (y-3)^2 = 16$ ✓

(ii) Solve $\left.\begin{array}{l} y = x \\ (x-2)^2 + (y-3)^2 = 16 \end{array}\right\}$ as $y = x$ passes through the points of intersection ✓

$(x-2)^2 + (x-3)^2 = 16$

$x^2 - 4x + 4 + x^2 - 6x + 9 = 16$

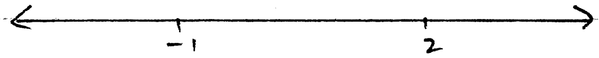
$2x^2 - 10x - 3 = 0$ ✓

$$x = \frac{10 \pm \sqrt{100 + 24}}{4}$$

$$= \frac{10 \pm \sqrt{124}}{4}$$ ✓

$$\therefore x = \frac{5 \pm \sqrt{31}}{2}, \quad y = \frac{5 \pm \sqrt{31}}{2}$$

e ~~(f)~~



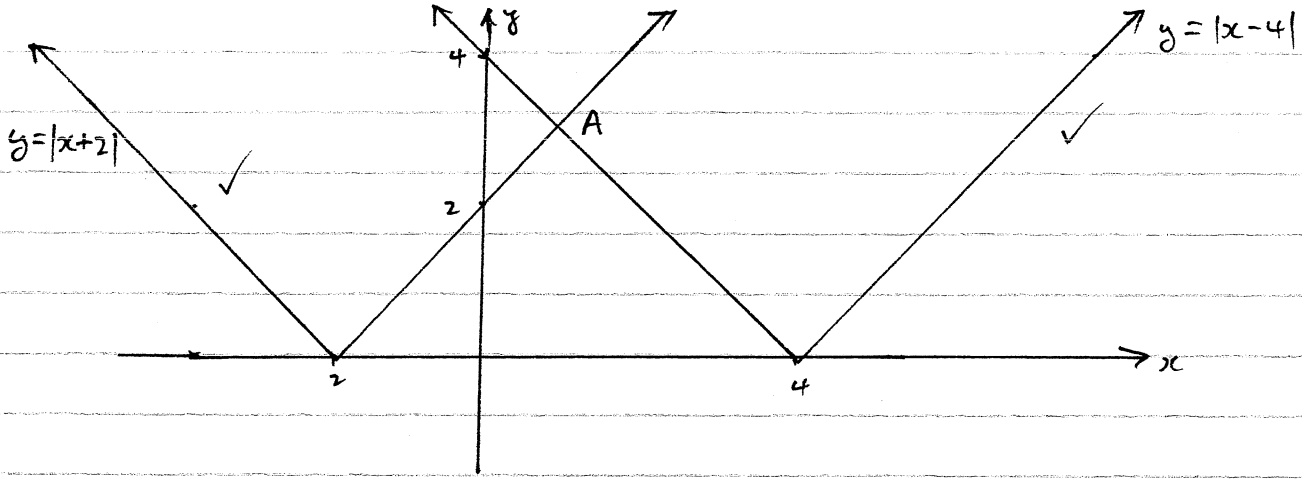
Defining values of x are -1 and 2 ✓

For $-1 < x \quad 2$

$$\text{Expression} = -(x-2) + (x+1)$$
$$= -x + 2 + x + 1$$
$$= 3$$ ✓

$\therefore \quad -1 < x < 2$

f (g)



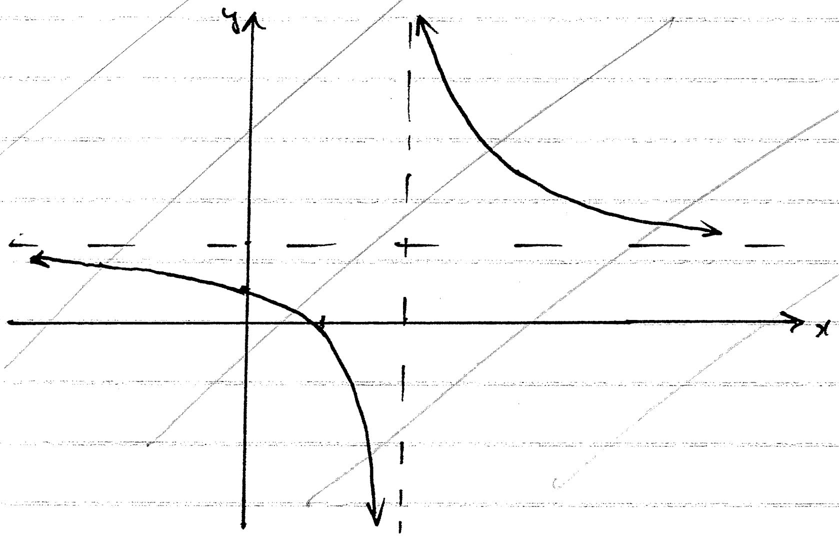
To find the point A

Solve
$$\left.\begin{aligned} y &= -(x-4) \\ y &= x+2 \end{aligned}\right\}$$ ✓

$$y = -x + 4$$
$$y = x + 2$$
$$-x + 4 = x + 2$$
$$2x = 2$$ ✓
$$x = 1$$

$\therefore$ From the graph solution is $x \leq 1$ ✓

Q4 (a) $(y-1) = \frac{1}{(x-2)}$ ✓



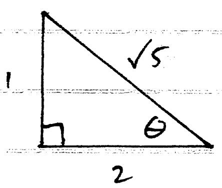
✓✓

(a) ~~(b)~~ LHS $= \dfrac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$ ✓ $= \dfrac{1-\frac{1}{3}}{1+\frac{1}{3}} = \dfrac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$ ✓

RHS $= \cos 60^\circ = \frac{1}{2}$ ✓

$\therefore$ LHS = RHS Hence result.

(b) ~~(c)~~



Using Pythagoras, missing side is 1 ✓

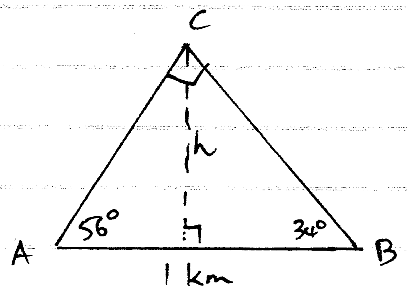
$\therefore$ (i) $\sin\theta = \frac{1}{\sqrt{5}}$ ✓

(ii) $\tan\theta = \frac{1}{2}$ ✓

$\therefore \sqrt{(1-\sin^2\theta)(1+\tan^2\theta)} = \sqrt{\left(1-\frac{1}{5}\right)\left(1+\frac{1}{4}\right)}$ ✓

$= \sqrt{\frac{4}{5}\times\frac{5}{4}} = \sqrt{1} = 1$

C ~~(d)~~



$\angle ACB = 90^\circ$ ✓

$\therefore \cos 56^\circ = \frac{AC}{1000}$ ✓

$AC = 1000\cos 56^\circ$

$\sin 56^\circ = \frac{h}{AC}$ ✓

$\therefore h = (1000\cos 56^\circ)\sin 56^\circ$

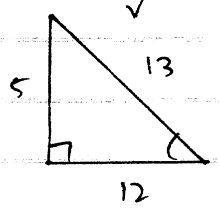
$\therefore h = 464$ m to nearest metre ✓

d ~~(e)~~

$$\frac{\frac{1}{\sqrt{2}} - \left(-\frac{1}{2}\right)}{-\frac{1}{\sqrt{2}} + \left(-\frac{1}{2}\right)} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{2}}{-\frac{1}{\sqrt{2}} - \frac{1}{2}}$$ ✓

$$= \frac{\frac{1}{\sqrt{2}} + \frac{1}{2}}{-\left(\frac{1}{\sqrt{2}} + \frac{1}{2}\right)} = -1$$ ✓

~~(f)~~ e



✓

Using the acute equivalent of θ and the fact that $\cos\theta$ is positive in the range

$$\cos\theta = \frac{12}{13}.$$ ✓

f)

$$LHS = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{1}{\frac{1+\sin\theta}{\cos\theta}} \begin{matrix} \times \cos\theta \\ \times \cos\theta \end{matrix}$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

$$= RHS$$

g)

$$LHS = \frac{2\cos^2\theta - 1}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2\cos^2\theta - 1}{\cos^2\theta - (1 - \cos^2\theta)}$$

$$= \frac{2\cos^2\theta - 1}{\cos^2\theta + \cos^2\theta - 1}$$

$$= \frac{2\cos^2\theta - 1}{2\cos^2\theta - 1}$$

$$= 1$$

$$= RHS$$