Question 1 *(15 Marks)* START QUESTION ON A NEW PAGE **Marks**

(a) Write $4.0\dot{5}\dot{2}$ as a fraction in simplest terms **2**

(b) Simplify $\dfrac{3}{1+\sqrt{3}}-\dfrac{2}{1-\sqrt{3}}$ **3**

(c) If the two shorter sides of a right angled triangle are $\sqrt{5}+1$ and $\sqrt{5}-1$, find the exact length of the hypotenuse **3**

(d) Factorise **4**

(i) $27x+x^4$

(ii) $9a^2+12ab+4b^2$

(e) Simplify $\dfrac{2a^2b+10ab}{(b+3)(b^2-3b+9)}\div\dfrac{a^2-25}{4(b+3)}$ **3**

Question 2 *(12 Marks)* START QUESTION ON A NEW PAGE

(a) Solve **7**

(i) $\dfrac{2x}{3}-\dfrac{x-1}{2}=\dfrac{2}{9}$

(ii) $x=\dfrac{x+3}{x}$ (answer in surd form)

(iii) $|4r+1|=1-r$

(b) Solve $x^6-9x^3-8=0$ **3**

(c) If $x=\dfrac{y+1}{y}$ express with y as the subject **2**

Question 3 *(12 Marks)* *START QUESTION ON A NEW PAGE*

(a) Solve the following Inequations: **8**

(i) $3-2x>5$

(ii) $x^2\le 9$

(iii) $\dfrac{2x+1}{x}\ge 1$

(iv) $|x-3|<1$

Question 3 Continued **Marks**

(b) **4**

(i) Show that the graphs of $y = 2^x$ and $y = x + 2$ intersect at the points (0,2) and (2,4).

(ii) Draw a neat sketch, indicating clearly the region represented by the intersection of the following inequalities: $y > 2^x$ and $y \le x + 2$

Question 4 *(20 Marks)* START QUESTION ON A NEW PAGE

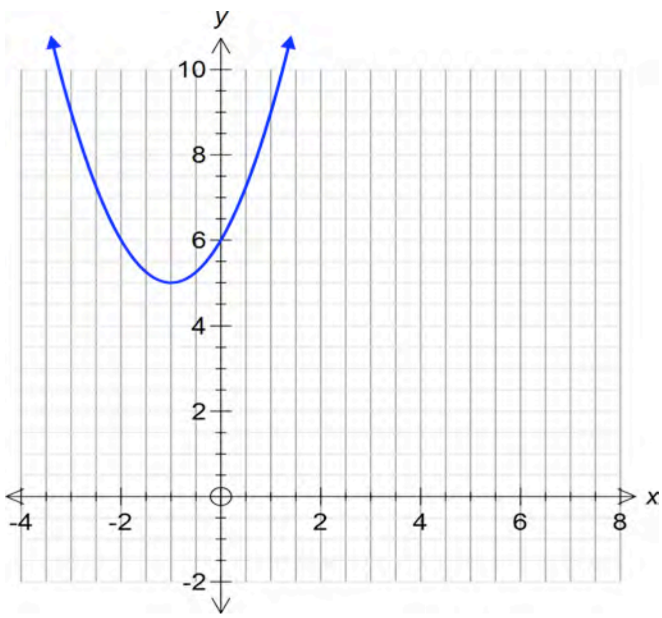
(a) Sketch the graph of the function $f(x) = |2x + 1|$ clearly showing any intercepts with the axes. **2**

(b) For the graph below **5**

(i) State if it is a function

(ii) Write down the domain and range

(iii) On the copy of this graph attached to the end of this paper, sketch the inverse.

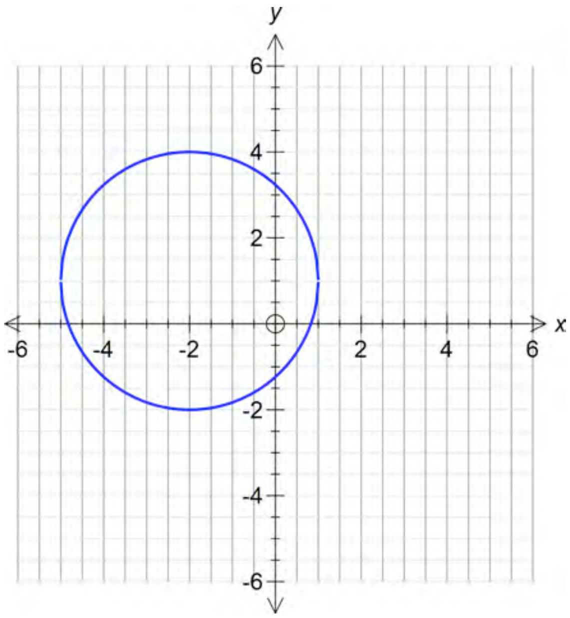


(c) Find the inverse of the function $f(x) = 1 + \log_2 x$ **2**

(d) Sketch the graph of $y = 2 - \dfrac{1}{x+1}$ clearly showing clearly any asymptotes or intercepts with the axes. **3**

Question 4 Continued **Marks**

(e) Write down the equation of the circle below **2**



(f) Consider the function $f\left(x\right)=\dfrac{-2x^2+1}{x^2-1}$ **6**

(i) Show if the function is odd, even or neither.

(ii) Write down the equations of any vertical or horizontal asymptotes.

(iii) Find any intercepts

(iv) Draw a neat large sketch of this function showing all the above information clearly

Question 5 *(12 Marks)* START QUESTION ON A NEW PAGE

(a) Solve the following trigonometric equations if $0^\circ \le \theta \le 360^\circ$: **9**

(i) $\tan\theta = 1$

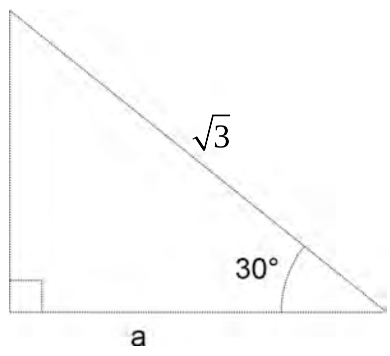
(ii) $\sin 2\theta = \dfrac{1}{2}$

(iii) $4\cos^2\theta = \cos\theta$ (answer to the nearest minute where necessary)

(b) If θ is obtuse and $\sin\theta = \dfrac{5}{7}$, without finding θ find the exact value of $\cos\theta$ and $\sec\theta$. **3**

Question 6 *(12 Marks)* START QUESTION ON A NEW PAGE **Marks**

(a) Find the exact value of *a* in the triangle below **2**



(b) Find the exact value of $\cot 330^{\circ}$ **1**

(c) Prove that: **5**

(i) $\cos^2(90-\theta)^{\circ}\cot\theta = \sin\theta\cos\theta$

(iI) $\dfrac{\sin\theta}{1+\cos\theta}+\dfrac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

(d) Two ships A and B start from the same position P and sail in different directions. **4**

Ship A sails on a bearing of $350^{\circ}T$ at 23 knots. Ship B sails on a bearing of $260^{\circ}T$ at 18 knots. (note 1 knot is 1 nautical mile per hour)

(i) Draw a neat diagram showing the path of both ships and their location after two hour's sailing.

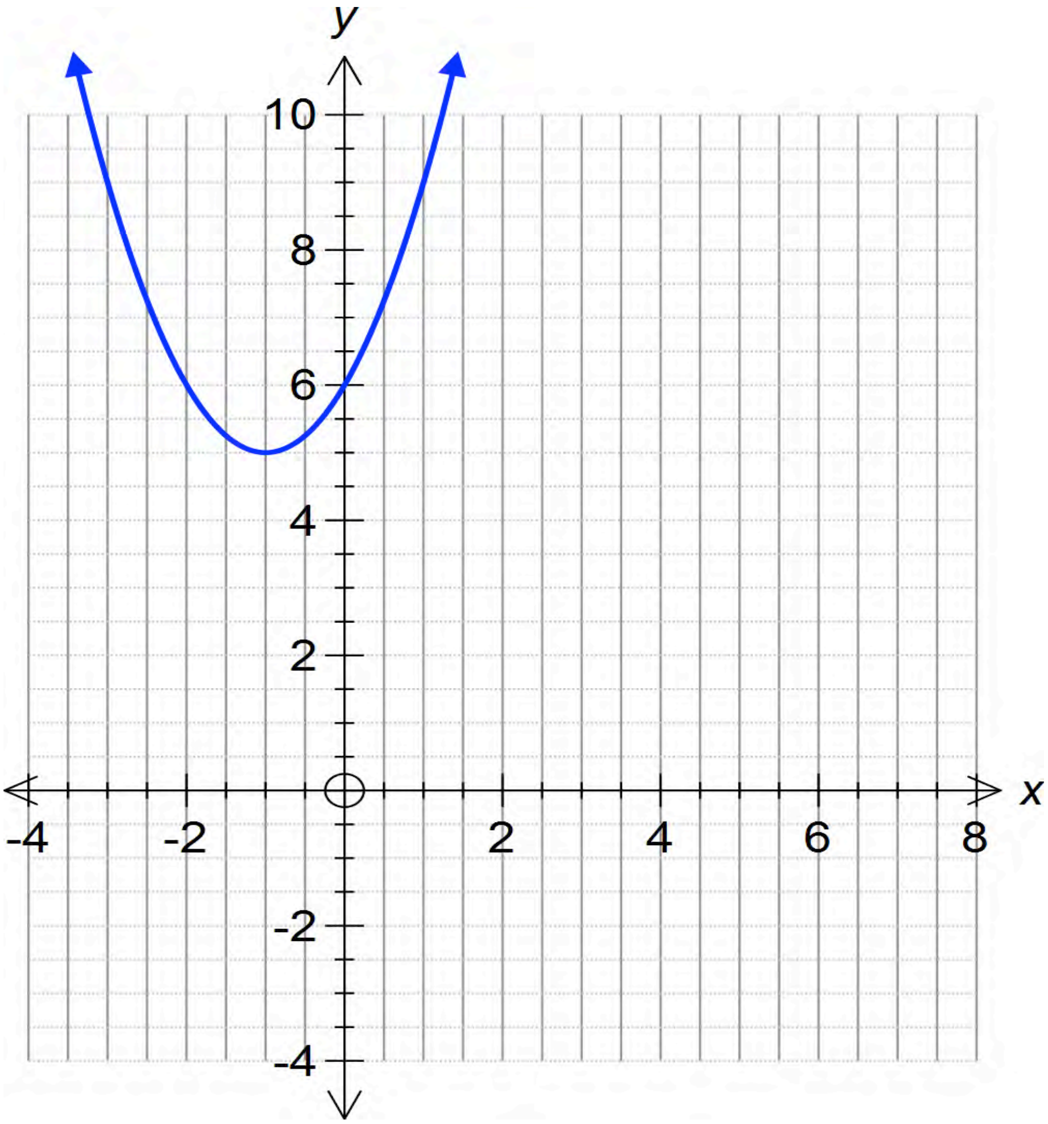
(ii) Show that $< APB = 90^{\circ}$

(iii) If ship B remains stationary, find the bearing on which ship A will need to sail in order to rejoin ship B

END OF PAPER

Computer Number: ___________________

Use this sheet to answer Question 4 (b) (iii), hand in with the rest of Question 4.



2012

Year 11 Ext 1

Question 1

(a) let $x = 4.05252\ldots$

$10x = 40.5252\ldots$ ✓ (1)

$1000x = 4052.5252\ldots$ (2)

$990x = 4012$

$x = \dfrac{4012}{990}$ ✓

$x = 4\dfrac{26}{495}$ ✓

$x = \dfrac{2006}{495}$

1 mark either line 2 or 3

1 correct answer.

(in any way)

OR $x = 4.052525\ldots$

$100x = 405.2525\ldots$

$99x = 401.2$ ✓

(2) $x = \dfrac{401.2}{99}$

$x = \dfrac{4012}{990}$ ✓

$x = \dfrac{2006}{495}$

(b) $\dfrac{3}{1+\sqrt{3}} - \dfrac{2}{1-\sqrt{3}}$

$= \dfrac{3(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} - \dfrac{2(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$ ✓

$= \dfrac{3(1-\sqrt{3}) - 2(1+\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$

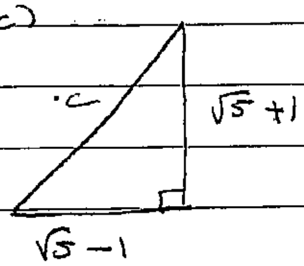
$= \dfrac{3-3\sqrt{3}-2-2\sqrt{3}}{1-3}$ ✓ both lines must be expanded correctly

$= \dfrac{1-5\sqrt{3}}{-2}$

$= \dfrac{5\sqrt{3}-1}{2}$ ✓ (3)

Question 1

(c)



$c^2 = (\sqrt{5}+1)^2 + (\sqrt{5}-1)^2$ ✓

$= 5 + 2\sqrt{5} + 1 + 5 - 2\sqrt{5} + 1$ ✓

$c^2 = 12$

$c = \sqrt{12}$ ✓ (3)

$c = 2\sqrt{3}$

(d)(i) $27x + x^4$

$= x(27 + x^3)$ ✓

$= x(3 + x)(9 - 3x + x^2)$ ✓ (2)

(ii) $9a^2 + 12ab + 4b^2$

$= (3a + 2b)(3a + 2b)$ ✓✓ (2)

$= (3a + 2b)^2$

(e) $\dfrac{2a^2b + 10ab}{(b+3)(b^2 - 3b + 9)} \div \dfrac{a^2 - 25}{4(b+3)}$ ✓

$= \dfrac{2ab(a+5)}{(b+3)(b^2-3b+9)} \times \dfrac{4(b+3)}{(a+5)(a-5)}$ ✓ Fully factorised

$= \dfrac{8ab}{(b^2-3b+9)(a-5)}$ ✓ (3)

Question 2 ⑫

(a) (i) $\frac{2x}{3} - \frac{x-1}{2} = \frac{2}{9}$ $(\times 18)$

②
$12x - 9(x-1) = 4$ ✓
$12x - 9x + 9 = 4$
$3x = -5$ ✓
$x = \frac{-5}{3}$ ✓

(ii) $x^2 = x + 3$ $(x \neq 0)$

②
$x^2 - x - 3 = 0$ ✓
$x = \frac{1 \pm \sqrt{1 - 4.1.(-3)}}{2}$ ✓
$x = \frac{1 \pm \sqrt{13}}{2}$

(iii) $4r + 1 = 1 - r$ or $-(4r+1) = 1 - r$

③
$5r = 0$ ✓ $-4r - \cancel{4} = 1 - r$ $-\cancel{3r} = 5$
$r = 0$ $-3r = \cancel{-5}\ 2$ ✓ $\cancel{r = \frac{-5}{3}}$
$r = \cancel{-5}/3$

test solution: $r = 0$ is ok
$r = -5/3$ is invalid ✓ ✓

(b) $\cancel{(x^3 - 8)(x^3 + 1) = 0}$ | $(x^3 - 8)(x^3 - 1) = 0$

③
$\cancel{x^3 = 8 \quad x^3 = -1}$ | $x^3 = 8 \quad x^3 = 1$
$\cancel{x = 2 \quad x = -1}$ | $x = 2 \quad x = 1$ ✓ ✓

(c) $xy = y + 1$ ✓

②
$xy - y = 1$ ✓
$y(x-1) = 1$
$y = \frac{1}{x-1}$

Question 3

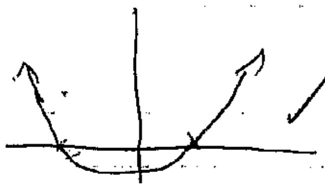
(a) (i) $3 - 2x > 5$

$2x < -2$

$x < -1$.

(ii) $x^2 \leq 9$

$(x+3)(x-3) \leq 0$



$-3 \leq x \leq 3$.

(iii) $\dfrac{2x+1}{x} \geq 1$ $(\times x^2)$ $(x \neq 0)$.

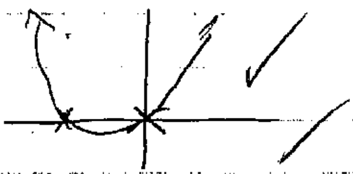
$x(2x+1) \geq x^2$

$2x^2 + \cancel{2x} - x^2 \geq 0$.

$x^2 + x \geq 0$

$x(x+1) \geq 0$

$x \leq -1$ $x > 0$.



(iv) $|x-3| < 1$

$x-3 < 1$ or $-x+3 < 1$.

$x < 4$ $x > 2$

$\therefore 2 < x < 4$.

(b). $y = 2^x$ sub $x = -1$ $y = 2^{-1} = 1/2$

$6y - 7x = 10$ $6y + 7 = 10$, $y = \frac{1}{2}$.

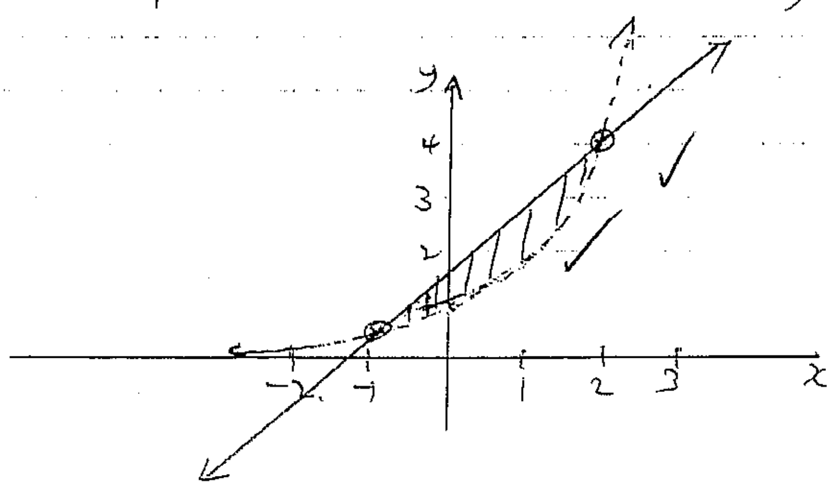
sub $x = 2$ $y = 2^2 = 4$.

$6y - 7 \times 2 = 10$.

$6y = 24$

$y = 4$.

since points lies on both lines they are points of intersection

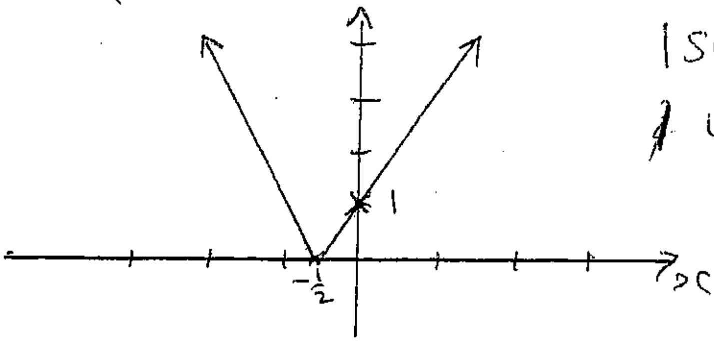


1 for boundary

1 for area.

stion 4.

a)



1 shape
1 intercepts

(b) (i) yes
(ii) domain $x \in \mathbb{R}$
range $y \geq 5$

(iii) see attached.

c) ~~let $y = 1 + \log_2 x$~~
~~swap x, y for inverse~~
~~$x = 1 + \log_2 y$~~
~~$\log_2 y = x - 1$~~
~~$y = 2^{x-1}$~~

let $y = \frac{x+1}{2x}$

$x = \frac{y+1}{2y}$

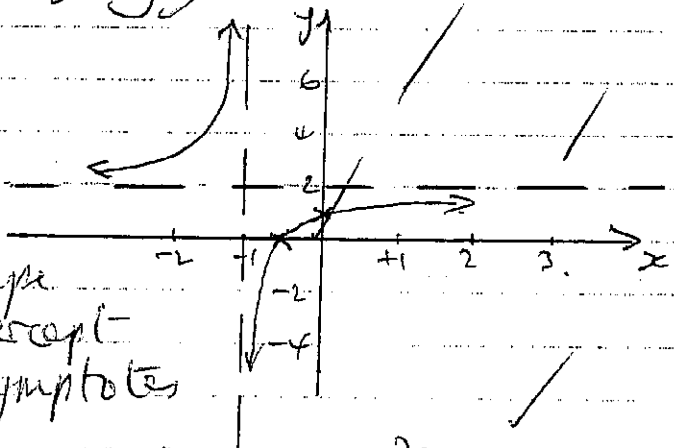
$2xy = y+1$

$2xy - y = 1$

$y(2x-1) = 1$

$y = \frac{1}{2x-1}$

d)



1 shape
intercept
1 asymptotes

$y - 2 = -\frac{1}{x+1}$

if $x = 0$ $y = 1$

if $y = 0$ $\frac{1}{x+1} = 2$

$x = -\frac{1}{2}$

e) $(x+2)^2 + (y-1)^2 = 9$.

1 $x^2 + y^2 = 9$

1 $(x+2)^2 + (y-1)^2 = \boxed{}$

$f(-x) = \dfrac{-2(-x)^2+1}{(-x)^2-1}$

$= \dfrac{-2x^2+1}{x^2-1}$

$= f(x)$

$\therefore$ even. ✓

(ii) $f(x) = \dfrac{-2x^2+1}{(x+1)(x-1)}$

$\therefore x = \pm 1$ are ✓ asymptotes. (both needed)

also $f(x) = \dfrac{-\frac{2x^2}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$

$= \dfrac{-2 + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$

as $x \to \infty$ $y \to -2$ ✓

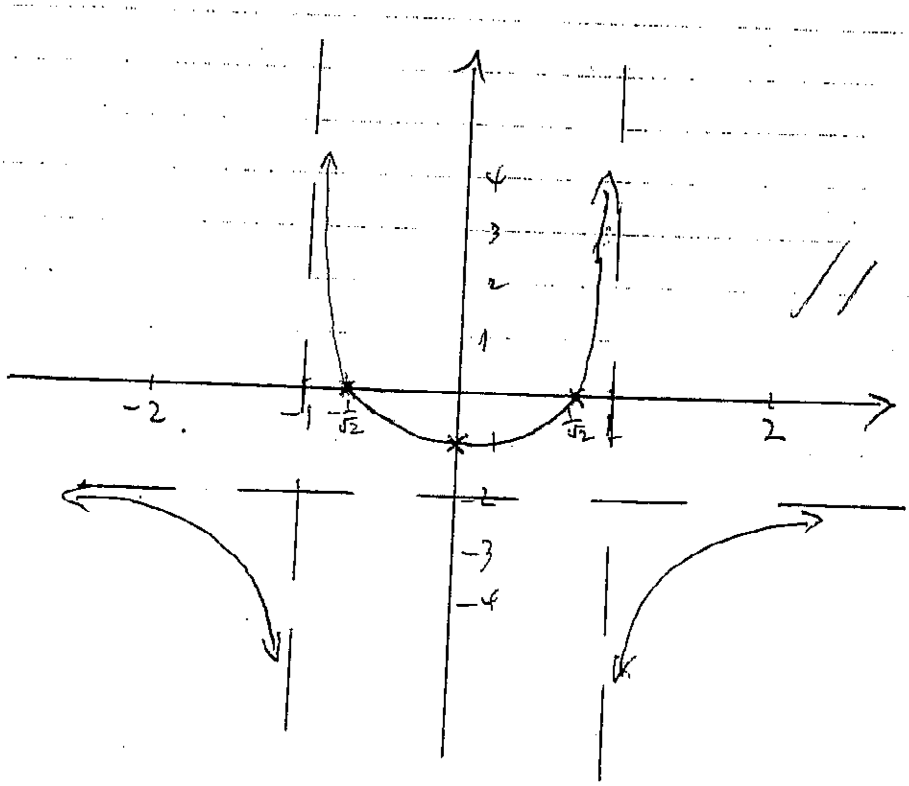
$\therefore$ $y=-2$ is horizontal ✓asymptote.

(iii) when $x=0$. $f(0) = -1$.

when $f(x)=0$ $-2x^2+1 = 0$ ✓ (both x & y intercepts required)

$x^2 = \frac{1}{2}$.

$x = \pm\frac{1}{\sqrt{2}}$ $(\doteqdot \pm 0.7)$.



// Must make sense

estion 5

S | A
T | C

(a) (i) $\tan\theta = 1$

$0 \le \theta \le 360$

related angle $45°$

Quadrants 1, 4 ✓ or similar

$\therefore \theta = 45, 225$ ✓

(2)

(ii) $\sin 2\theta = -\frac{1}{2}$.

$0 \le 2\theta \le 720$.

related angle $30°$

Quadrants ~~3, 4~~ 1, 2

~~$2\theta = 210, 330, 570, 690$~~ $2\theta = 30, 150, 390, 510$

~~$\theta = 105, 165, 285, 345$~~. $\theta = 15, 75, 195, 255$.

3 (3 marks – all solutions / 2 marks – 2 solutions / 1 mark – working OK)

(iii) $4\cos^2\theta - \cos\theta = 0$.

$\cos\theta(4\cos\theta - 1) = 0$. ✓

$\cos\theta = 0$ $\quad\cos\theta = \frac{1}{4}$.

$0 \le \theta \le 360$

4

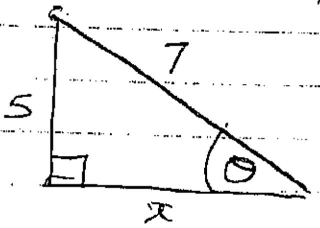
$\theta = 90, 270$ ✓ related < $75°31'$ ✓

Quadrants 1, 4 ✓

$\theta = \boxed{75°31', 284°29'}$

$\therefore \theta = 90, 75°31', 270, 284°29'$

(b)



By Pythag $x = \sqrt{49-25}$

$= \sqrt{24}$. ✓

cos θ and sec θ are negative in Q2 (obtuse angle)

$\therefore \cos\theta = \frac{-\sqrt{24}}{7}$ $\left(\frac{2\sqrt{6}}{7}\right)$ ✓

$\sec\theta = \frac{-7}{\sqrt{24}}$. $\left(\frac{7\sqrt{6}}{12}\right)$ ✓

[one mark if $\sec\theta = \frac{1}{\cos\theta}$ even if $\cos\theta$ is incorrect)

Year 11 Ext 1 Q6

6(a) $\cos 30^\circ = \dfrac{a}{\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ — $\frac{1}{2}$ $\cos 30^\circ$

$a = \frac{3}{2}$ — $\frac{2}{2}$ correct answer

(b) $-\sqrt{3}$ — 1

(c) (i) $LHS = \cos^2(90^\circ - \theta)\cot\theta$ — $\frac{1}{2}$ $\cos(90^\circ - \theta) = \sin\theta$ or $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \sin^2\theta \cdot \dfrac{\cos\theta}{\sin\theta}$ — $\frac{2}{2}$ correct solution

$= \sin\theta\cos\theta$

$= RHS$

(ii) $LHS = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$ — $\frac{1}{3}$ common denom: correct

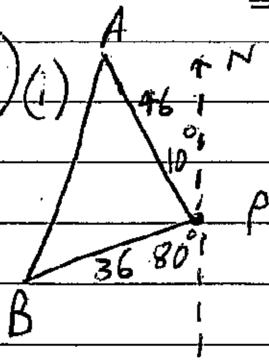
$= \dfrac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$ — $\frac{2}{3}$ using $\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$ — $\frac{3}{3}$ correct solution

$= 2\operatorname{cosec}\theta$

$= RHS$

(d)(i)

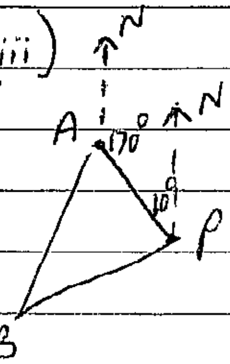


1 correct diagram

(ii) $\angle APB = 180^\circ - (10^\circ + 80^\circ)$ — 1 correct method

$= 90^\circ$

(iii)



$\tan\angle PAB = \dfrac{36}{46}$ — $\frac{1}{2}$

$\angle PAB = 38^\circ$ (nearest degree)

Bearing $= 38^\circ + 170^\circ$ — $\frac{2}{2}$

$= 208^\circ$