NORTH SYDNEY BOYS' HIGH SCHOOL
2009 Year 11 Assessment Task 2

MATHEMATICS
(Extension 1)

General instructions

- Working time – 50 minutes.
- Write on your own A4 paper.
  Each question is to commence on a new page.
- Write using blue or black pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question (Insufficient/illegible working may cause a deduction of marks).
- Attempt **all** questions.

Class teacher (please ✔)

- ◯ Mr Barrett
- ◯ Mr Lowe
- ◯ Mr Rezcallah
- ◯ Mr Trenwith
- ◯ Mr Ireland
- ◯ Mr Weiss
- ◯ Mr Lam

NAME: ..... **PAGES USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	**Total**	%
MARKS	/9	/13	/18	/11	/17	/68	

Question 1 (9 Marks) Commence a NEW page. **Marks**

Solve, giving exact values:

(a) $x + 6 = 3 - x$. **1**

(b) $\dfrac{2}{x+5} - \dfrac{3}{x-5} = \dfrac{1}{x^2-25}$. **2**

(c) $(x+1)^2 = 8$. **2**

(d) $x(x+1) = 1$. **2**

(e) $|3x + 2| = 1 - x$. **2**

Question 2 (13 Marks) Commence a NEW page. **Marks**

(a) Write the exact values of

i. $\cos 180°$. **1**

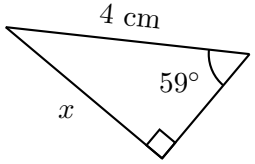
ii. $\cot 240°$. **1**

iii. $\tan 315°$. **1**

(b) i. Write $75°$ in radians as an exact value. **1**

ii. Evaluate $\cos^2 2.3$ correct to 3 decimal places. **2**

(c) Find the value of the pronumeral in each of the following diagrams.

i. 

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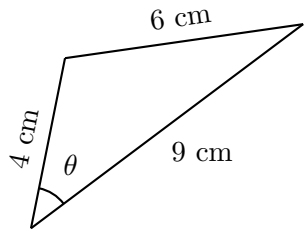
ii. 

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(d) A ship leaves port A on a bearing of 120°T travelling for 2 hours at 30 knots (nautical miles/hr). It changes to a new bearing of 220°T for 1 hour at 20 knots. **3**

How far is it from port A?

Question 3 (18 Marks) Commence a NEW page. **Marks**

(a) Solve the following inequalities:

i. $\dfrac{3x}{4} \leq \dfrac{7x}{3} - 2.$ **2**

ii. $x^2 + x > 2.$ **2**

iii. $|2x + 1| < 3.$ **3**

iv. $\dfrac{x+2}{2x-3} \leq 1.$ **3**

(b) Form an equation and solve:

i. There are 450 students at a particular school. If there are 40 more boys than girls, how many boys and girls are there? **2**

ii. A box contains 50 coins made up of 10 cent coins and 20 cent coins. How many of each are in the box if the value of the coins is \$7? **3**

(c) Find the coordinates of the points of intersection of the graphs **3**

$$y = x - 3 \qquad \& \qquad x^2 + y^2 - 4y - 13 = 0$$

Question 4 (11 Marks) Commence a NEW page. **Marks**

(a) If $\tan\theta = -\dfrac{4}{3}$ and $90^\circ < \theta < 180^\circ$, find the exact value of

i. $\sin\theta.$ **2**

ii. $\cot\theta.$ **1**

(b) Simplify fully

i. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}.$ **2**

ii. $1 - \sin^2(180^\circ - \theta).$ **2**

(c) Prove the following identities:

i. $\tan\theta\left(1 - \cot^2\theta\right) + \cot\theta\left(1 - \tan^2\theta\right) = 0.$ **2**

ii. $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}.$ **2**

Question 5 (17 Marks) Commence a NEW page. **Marks**

(a) Consider the function $y = 4\sin 3x$, where x is in degrees.

i. Find the period. **1**

ii. Find the amplitude. **1**

iii. Sketch the function for $0° \leq x \leq 360°$. **2**

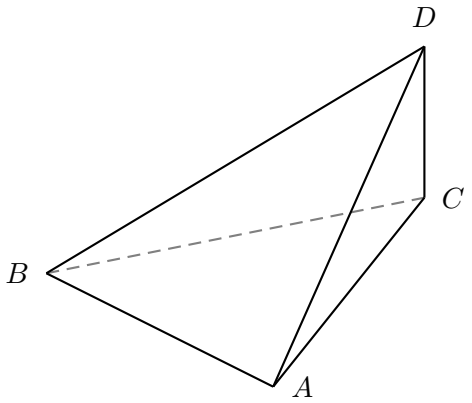
(b) Solve for $0° \leq x \leq 360°$

i. $\cos x = -\dfrac{1}{2}$. **2**

ii. $\tan 2x = \dfrac{1}{\sqrt{3}}$ **3**

iii. $3\cos^2 x + 5\sin x = 1$. **4**

(c) The points A and B are 500 m apart on the ground and D is the top of a tower. $\angle BAD$ and $\angle DBA$ are 59° and 54° respectively. The elevation of D from A is 5°.



Copy the diagram into your writing booklet and mark on the figure all the angles stated above.

i. Show that the height h metres of the tower is given by **3**

$$h = \frac{500\sin 5°\sin 54°}{\sin 67°}$$

ii. Find h to the nearest metre. **1**

End of paper.

Suggested Solutions

Question 1

(a) (1 mark)

$$\begin{aligned} x+6 &= 3-x \\ 2x &= -3 \\ x &= -\frac{3}{2} \end{aligned}$$

(b) (2 marks)

✓ [1] for removing denominators.
✓ [1] for $x=-26$.

$$\underset{\times(x-5)(x+5)}{\frac{2}{x+5}} - \underset{\times(x-5)(x+5)}{\frac{3}{x-5}} = \underset{\times(x-5)(x+5)}{\frac{1}{x^2-25}}$$

$$\begin{aligned} 2(x-5)-3(x+5) &= 1 \\ 2x-10-3x-15 &= 1 \\ -x-25 &= 1 \\ x &= -26 \end{aligned}$$

(c) (2 marks)

$$\begin{aligned} (x+1)^2 &= 8 \\ x+1 &= \pm\sqrt{8} \\ x &= -1 \pm \sqrt{8} \\ &= -1 \pm 2\sqrt{2} \end{aligned}$$

(d) (2 marks)

$$\begin{aligned} x^2+x-1 &= 0 \\ x &= \frac{-1\pm\sqrt{1+4}}{2} \\ &= \frac{-1\pm\sqrt{5}}{2} \end{aligned}$$

(e) (2 marks)

$$|3x+2| = 1-x$$

$$|3x+2| = \begin{cases} 3x+2 & 3x+2 \geq 0 \\ -(3x+2) & 3x+2 < 0 \end{cases}$$

$$\begin{aligned} 3x+2 &= 1-x \\ 4x &= -1 \\ x &= -\frac{1}{4} \end{aligned} \qquad \begin{aligned} -(3x+2) &= 1-x \\ -3x-2 &= 1-x \\ -2x &= 3 \\ x &= -\frac{3}{2} \end{aligned}$$

Test values:

- $x=-\frac{1}{4}$.

$$\begin{aligned} \left|3\left(-\tfrac{1}{4}\right)+2\right| &= \left|-\tfrac{3}{4}+2\right| \\ &= \left|\tfrac{5}{4}\right| \ \checkmark \\ 1-\left(-\frac{1}{4}\right) &= \frac{5}{4} \ \checkmark \end{aligned}$$

- $x=-\frac{3}{2}$.

$$\begin{aligned} \left|3\left(-\tfrac{3}{2}\right)+2\right| &= \left|-\tfrac{9}{2}+2\right| \\ &= \left|-\frac{5}{2}\right| = \frac{5}{2} \ \checkmark \\ 1-\left(-\frac{3}{2}\right) &= \frac{5}{2} \ \checkmark \end{aligned}$$

Question 2

(a) i. (1 mark)

$$\cos 180^\circ = -1$$

ii. (1 mark)

$$\cot 240^\circ = \frac{1}{\tan(180^\circ+60^\circ)} = \frac{1}{\sqrt{3}}$$

iii. (1 mark)

$$\tan 315^\circ = \tan(360^\circ-45^\circ) = -1$$

(b) i. (1 mark)

$$75^\circ \Rightarrow \frac{75^\circ}{180^\circ}\times\pi = \frac{5\pi}{12}$$

ii. (2 marks)

$$(\cos 2.3)^2 = 0.444$$

(c) i. (2 marks)

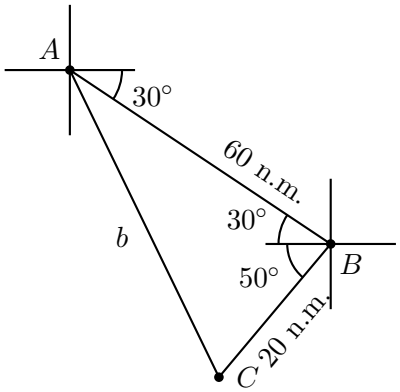
$$\begin{aligned} \frac{x}{4} &= \sin 59^\circ \\ x &= 4\sin 59^\circ = 3.43 \text{ (2 d.p.)} \end{aligned}$$

ii. (2 marks)

$$\begin{aligned} \cos\theta &= \frac{b^2+c^2-a^2}{2bc} \\ &= \frac{4^2+9^2-6^2}{2\times 4\times 9} \\ &= \frac{61}{72} \\ \theta &= 32^\circ 5' \end{aligned}$$

(d) (3 marks)

✓ [1] for correct diagram.
✓ [1] for correct application of cosine rule.
✓ [1] for $b = 59.9$ to 1 d.p.



$$\begin{aligned} b^2 &= a^2 + c^2 - 2ac\cos B \\ &= 3\,600 + 400 - 2\,400\cos 80 \\ &= 3\,583.24 \\ \therefore b &= 59.9 \text{ n.m. (1 d.p.)} \end{aligned}$$

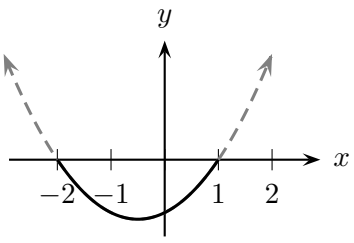
Question 3

(a) i. (2 marks)

$$\begin{aligned} \frac{3x}{4} &\leq \frac{7x}{3} - 2 \\ 9x &\leq 28x - 24 \\ 24 &\leq 19x \\ x &\geq \frac{24}{19} \end{aligned}$$

ii. (2 marks)

$$\begin{aligned} x^2 + x - 2 &> 0 \\ (x+2)(x-1) &> 0 \end{aligned}$$



$\therefore x < -2$ or $x > 1$.

iii. (3 marks)

$$|2x+1| < 3$$

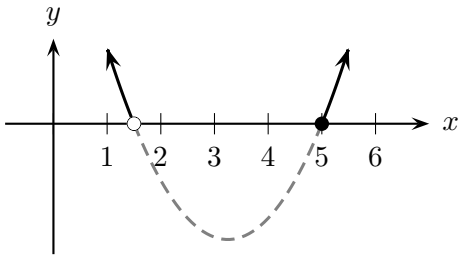
$$|2x+1| = \begin{cases} 2x+1 & 2x+1 \geq 0 \\ -(2x+1) & 2x+1 < 0 \end{cases}$$

$2x+1 < 3$	$-(2x+1) < 3$
$2x < 2$	$-2x < 4$
$x < 1$	$x > -2$

$$\therefore -2 < x < 1$$

iv. (3 marks)

$$\begin{aligned} \frac{x+2}{2x-3} &\leq \frac{1}{\times(2x-3)^2} \\ \times(2x-3)^2 & \\ (x+2)(2x-3) &\leq (2x-3)^2 \\ (2x-3)^2 - (x+2)(2x-3) &\geq 0 \\ (2x-3)\left((2x-3) - (x+2)\right) &\geq 0 \\ (2x-3)(x-5) &\geq 0 \end{aligned}$$



$$x < \frac{3}{2} \text{ or } x \geq 5$$

(b) i. (2 marks)

✓ [1] for solving equation, resulting in $x = 205$.
✓ [1] for conclusion.

Let the # of girls be x,

$$\begin{aligned} (x+40) + x &= 450 \\ 2x &= 410 \\ x &= 205 \end{aligned}$$

Hence there are 205 girls & 245 boys.

ii. (3 marks)

✓ [1] for setting up simultaneous equations.
✓ [1] for solving simultaneous equations.
✓ [1] for correct final conclusion.

Let the # of 10c coins be x, and # of 20c coins be y.

$$\begin{cases} x + y = 50 & (1) \\ 10x + 20y = 700 & (2) \end{cases}$$
$$(2) - [(1) \times 10]$$
$$10y = 200$$
$$y = 20$$
$$\therefore x = 30$$

Hence there are thirty 10c coins and twenty 20c coins.

(c) (3 marks)

✓ [1] for correct substitution
✓ [1] for solution of $x = 1$, $x = 4$.
✓ [1] for correct points of intersection.

$$\begin{cases} y = x - 3 & (1) \\ x^2 + y^2 - 4y - 13 = 0 & (2) \end{cases}$$

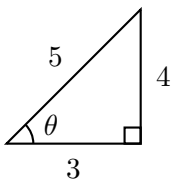
Substitute (1) to (2)

$$x^2 + (x-3)^2 - 4(x-3) - 13 = 0$$
$$x^2 + x^2 - 6x + 9 - 4x + 12 - 13 = 0$$
$$2x^2 - 10x + 8 = 0$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 1 \text{ or } 4$$

Points of intersection are $(1, -2)$ and $(4, 1)$.

Question 4

(a) i. (2 marks)



$$\sin\theta = \frac{4}{5}$$

ii. (1 mark)

$$\cot\theta = -\frac{3}{4}$$

(b) i. (2 marks)

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + (1 + 2\cos\theta + \cos^2\theta)}{\sin\theta(1+\cos\theta)}$$
$$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$
$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}}$$
$$= \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$$

ii. (2 marks)

$$1 - \sin^2(180^\circ - \theta) = 1 - \sin^2\theta = \cos^2\theta$$

(c) i. (2 marks)

$$\tan\theta\left(1 - \cot^2\theta\right) + \cot\theta\left(1 - \tan^2\theta\right)$$
$$= \tan\theta - \frac{1}{\tan\theta} + \frac{1}{\tan\theta} - \tan\theta$$
$$= 0$$

ii. (2 marks)

$$\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \frac{1+\sin\theta}{\cos\theta}$$

Question 5

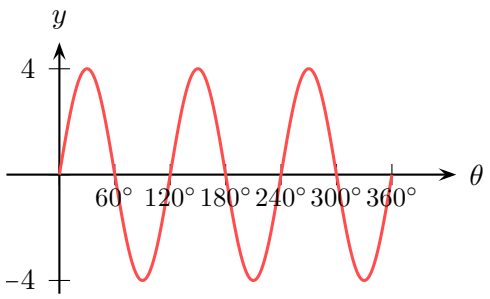
(a) i. (1 mark)

$$T = \frac{360^\circ}{3} = 120^\circ$$

ii. (1 mark)

$$a = 4$$

iii. (2 marks)



(b) i. (2 marks)

$$\cos x = -\frac{1}{2}$$
$$x = 120^\circ, 240^\circ$$

ii. (3 marks)

✓ [1] for $0 \le 2x \le 720^\circ$.
✓ [1] for finding the values of $2x$.
✓ [1] for finding the values of x.

$$0^\circ \le 2x \le 720^\circ$$
$$\tan 2x = \frac{1}{\sqrt{3}}$$
$$2x = 30^\circ, 210^\circ, 390^\circ, 570^\circ$$
$$\therefore x = 15^\circ, 105^\circ, 195^\circ, 285^\circ$$

iii. (4 marks)

✓ [1] for changing $\cos^2 x \to 1 - \sin^2 x$.
✓ [1] for rearranging to the quadratic $3\sin^2 x - 5\sin x - 2 = 0$.
✓ [1] for solving the quadratic.
✓ [1] for final answer.

$$3\cos^2 x + 5\sin x = 1$$
$$3(1 - \sin^2 x) + 5\sin x = 1$$
$$3 - 3\sin^2 x + 5\sin x = 1$$
$$3\sin^2 x - 5\sin x - 2 = 0$$

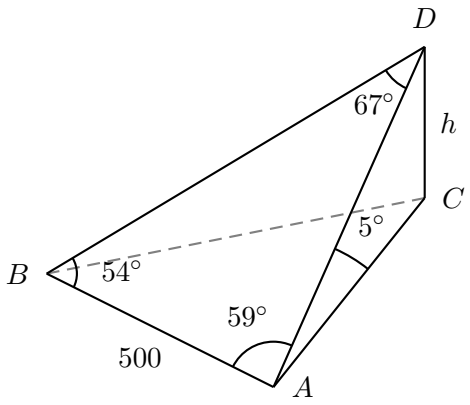
Let $m = \sin x$,

$$3m^2 - 5m - 2 = 0$$
$$(3m + 1)(m - 2) = 0$$
$$\therefore m = -\frac{1}{3}, m = 2$$

$\therefore \sin x = -\frac{1}{3}$ as $\sin x = 2$ has no solution

$$\therefore x = 199^\circ 28' \text{ or } 340^\circ 32'$$

(c) i. (3 marks)



$$\frac{h}{AD} = \sin 5^\circ$$
$$\therefore AD = \frac{h}{\sin 5^\circ}$$

Applying the sine rule to $\triangle DBA$,

$$\frac{AD}{\sin 54^\circ} = \frac{500}{\sin 67^\circ}$$
$$\frac{h}{\sin 5^\circ \sin 54^\circ} = \frac{500}{\sin 67^\circ}$$
$$\therefore h = \frac{500 \sin 5^\circ \sin 54^\circ}{\sin 67^\circ}$$

ii. (1 mark)

$$h = 38.3\,\text{m} = 38\,\text{m} \text{ (nearest m)}$$