Ext 1

NORTH SYDNEY BOYS HIGH SCHOOL

2010 Year 11 Task 2

Mathematics
Extension 1

General Instructions

- Working time – 50 minutes
- Write on the lined paper in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Each new question is to be started on a **new page.**
- Attempt all questions

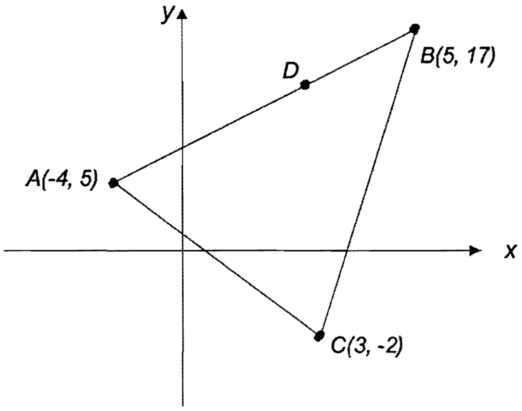
Class Teacher:
(Please tick or highlight)
- O Mr Berry
- O Mr Ireland
- O Mr Fletcher
- O Mr Lam
- O Mr Rezcallah
- O Mr Trenwith
- O Mr Weiss

Student Name:______________________________

(To be used by the exam markers only.)

Question No	1	2	3	4	5	Total	Total %
Mark	/10	/9	/17	/10	/8	/54	/100

Question 1 (10 marks) **Marks**



The triangle ABC has vertices $A(-4,5)$, $B(5,17)$ and $C(3,-2)$.

(a) Show that the equation of the straight line through A and C is
$$x+y-1=0$$ **2**

(b) Find the length of side AC. **1**

(c) Point D divides side AB internally in the ratio 2 : 1.
Find the co-ordinates of D. **2**

(d) Find the perpendicular distance of B from AC. **2**

(e) Hence calculate the area of triangle ΔABC **2**

(f) The line through D parallel to AC meets BC at point E.
Calculate the area of triangle ΔBDE. **1**

Question 2 (9 marks)

(a) Find the equation of the line which passes through the point of intersection of $15x-6y-10=0$ and $x+y-3=0$, and through the point $P(1,3)$. **3**

(b) Find the angle that the line $2x+3y+1=0$ makes with the positive direction of the x-axis. **2**

(c) Sketch the region where $y \leq x^2-4$ and $y > -3x$ are true simultaneously. Show any boundary points clearly. **4**

Question 3 (17 marks)

(a) Express in simplest form as a single fraction, with no negative indices:

(i) $4x^{-2}$ (ii) $\dfrac{x^{-2}-1}{x^{-1}-1}$ **3**

(b) Solve for x:

(i) $9^x = 27$ (ii) $x^{\frac{-5}{4}} = 32$ **4**

(c) Evaluate:

(i) $\log_a \frac{1}{\sqrt{a}}$ (ii) $\log_3 63 - \log_3 7$ **3**

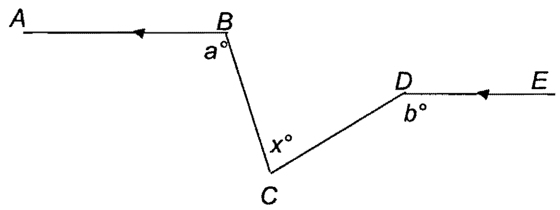
(d) Evaluate $\log_4 27$ to two decimal places. **2**

(e) Solve for x:

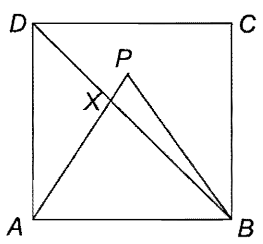
(i) $\log_{11} 2 + \log_{11} x = \log_{11} 7$ (ii) $2\log_5 x = \log_5(4x+5)$ **5**

Question 4 (10 marks)

(a) Express x in terms of a and b. *(no reasons needed)* **2**



(b)



ABCD is a square. **3**

PAB is an equilateral triangle.

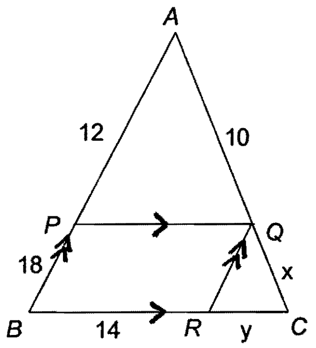
Calculate the size of $\angle DXA$, giving geometrical reasons.

(c) Find the size of each interior angle in a regular 20-sided polygon. **2**

(d) Find the values of x and y. **3**

Give the geometrical reason.

(Not to scale)

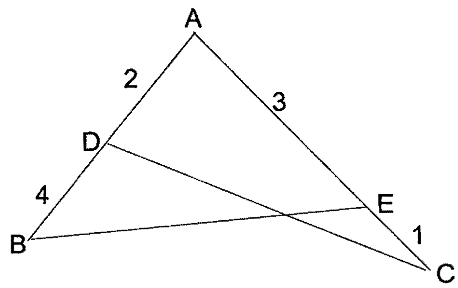


Question 5 (8 marks)

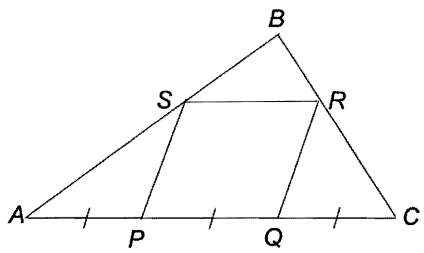
(a) (i) State the geometrical reason why ΔABE is similar to ΔACD in the diagram below. **1**

(ii) Given that $BE = 5$, calculate DC. **2**

(Not to scale)



(b) In the following diagram $PQRS$ is a rhombus.



(i) Prove $\angle SPQ = 2 \times \angle SAP$ **2**

(ii) Prove $\angle ABC = 90°$ **3**

END OF EXAMINATION

Suggested Solutions

Question 1

(a) (2 marks)

- ✓ [1] for correct gradient $m = -1$.
- ✓ [1] for setting up equation.

$$
\begin{aligned}
&A(-4,5) \qquad C(3,-2)\\
&\frac{y+2}{x-3} = \frac{5+2}{-4-3}\\
&\qquad = \frac{7}{-7} = -1\\
&\therefore y+2 = -x+3\\
&y = -x+1\\
&x+y-1=0
\end{aligned}
$$

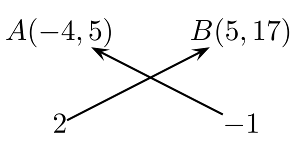
Alternatively, substitute coordinates of points A and C into equation.

(b) (1 mark)

$$
\begin{aligned}
d_{AC} &= \sqrt{(5-(-2))^2 + (-4-3)^2}\\
&= \sqrt{7^2+7^2}\\
&= \sqrt{49\times 2} = 7\sqrt{2}
\end{aligned}
$$

(c) (2 marks)

- ✓ [1] for each value of x and y correctly found.



$$
\begin{aligned}
x_D &= \frac{1(-4)+2(5)}{2+1} & y_D &= \frac{1(5)+2(17)}{2+1}\\
&= \frac{-4+10}{3} = 2 & &= \frac{5+34}{3} = 13
\end{aligned}
$$

$$\therefore D(2,13)$$

(d) (2 marks)

- ✓ [1] for correct substitution into ⊥ dist. formula.
- ✓ [1] for final answer.

$$
\begin{aligned}
&AC: x+y-1=0 \qquad B(5,17)\\
d_\perp &= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}\\
&= \frac{|1(5)+1(17)-1|}{\sqrt{1^2+1^2}}\\
&= \frac{21}{\sqrt{2}} = \frac{21\sqrt{2}}{2}
\end{aligned}
$$

(e) (2 marks)

- ✓ [1] for correct substitution.
- ✓ [1] for final answer.

$$
\begin{aligned}
A &= \frac{1}{2}bh\\
&= \frac{1}{2}\times 7\cancel{\sqrt{2}}\times\frac{21}{\cancel{\sqrt{2}}}\\
&= 73.5
\end{aligned}
$$

(f) (1 mark)

$$
\begin{aligned}
\frac{A_{\triangle BDE}}{A_{\triangle ABC}} &= \frac{1^2}{3^2} = \frac{1}{9}\\
\therefore A_{\triangle BDE} &= \frac{1}{9}\times\frac{147}{2} = \frac{147}{18}\\
&= \frac{49}{6} = 8\frac{1}{6}
\end{aligned}
$$

Other methods exist, though this one shown here would be the easiest.

Question 2

(a) (3 marks)

- ✓ [1] for correctly placing equations into k method equation.
- ✓ [1] for $k = 13$.
- ✓ [1] for final factorised equation.

$$15x - 6y - 10 = 0 \qquad x + y - 3 = 0$$

$$15x - 6y - 10 + k(x + y - 3) = 0\Big|_{x=1,y=3}$$

$$\begin{aligned} 15 - 18 - 10 + k(1 + 3 - 3) &= 0 \\ k &= 18 + 10 - 15 \\ &= 13 \\ 15x - 6y - 10 + 13x + 13y - 39 &= 0 \\ 28x + 7y - 49 &= 0 \\ 7(4x + y - 7) &= 0 \\ \therefore 4x + y - 7 &= 0 \end{aligned}$$

(b) (2 marks)

- ✓ [1] for $\tan\theta = -\frac{2}{3}$.
- ✓ [1] for final answer.

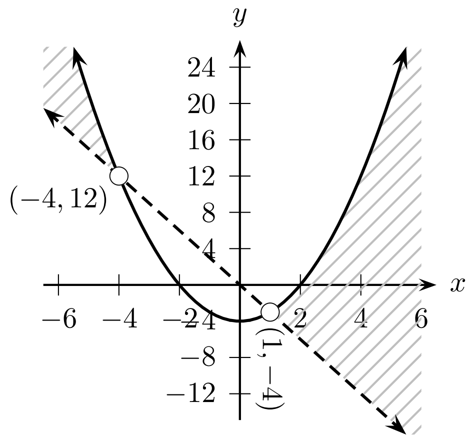
$$\begin{aligned} 2x + 3y + 1 &= 0 \\ 3y &= -2x - 1 \\ y &= -\frac{2}{3}x - \frac{1}{3} \\ \therefore m &= -\frac{2}{3} = \tan\theta \\ \theta &= 180^\circ - 33^\circ 41' \\ &= 146.31^\circ / 146^\circ 19' \end{aligned}$$

(c) (4 marks)

- ✓ [1] for dotted line $y = -3x$
- ✓ [1] for parabola $y = x^2 - 4$.
- ✓ [1] for boundary points
- ✓ [1] for correct regions.

$$\begin{cases} y = x^2 - 4 \\ y = -3x \end{cases}$$

$$\begin{aligned} x^2 - 4 &= -3x \\ x^2 + 3x - 4 &= 0 \\ (x + 4)(x - 1) &= 0 \\ \therefore x &= -4, 1 \end{aligned}$$



Question 3

(a) i. (1 mark)

$$4x^{-2} = \frac{4}{x^2}$$

ii. (2 marks)

- ✓ [1] for correctly multiplying numerator & denominator by x^2 to obtain $\frac{1-x^2}{x-x^2}$.
- ✓ [1] for final answer.

$$\begin{aligned} \frac{x^{-2} - 1}{x^{-1} - 1}\frac{\times x^2}{\times x^2} &= \frac{1 - x^2}{x - x^2} \\ &= \frac{(1 - x)(1 + x)}{x(1 - x)} \\ &= \frac{1 + x}{x} \end{aligned}$$

(b) i. (2 marks)

- ✓ [1] for $3^{2x} = 3^3$.
- ✓ [1] for final answer.

$$\begin{aligned} 9^x &= 27 \\ 3^{2x} &= 3^3 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

ii. (2 marks)

- ✓ [1] for $x = 32^{-4/5}$.
- ✓ [1] for final answer.

$$\begin{aligned} x^{-\frac{5}{4}} &= 32 \\ \left(x^{-\frac{5}{4}}\right)^{-\frac{4}{5}} &= (32)^{-\frac{4}{5}} \\ x = 2^{-4} &= \frac{1}{16} \end{aligned}$$

(c) i. (1 mark)

$$\log_a \frac{1}{\sqrt{a}} = \log_a a^{-\frac{1}{2}} = -\frac{1}{2}$$

ii. (2 marks)

$$\log_3 63 - \log_3 7 = \log_3 \left(\frac{63}{7}\right)$$
$$= \log_3 9 = 2$$

(d) (2 marks)

✓ [1] for correct change of base.

✓ [1] for final answer.

$$\log_4 27 = \frac{\log_{10} 27}{\log_{10} 4}$$
$$= 2.38 \text{ (2 d.p.)}$$

(e) i. (2 marks)

✓ [1] for correct use of addition rule for logarithms.

✓ [1] for final answer.

$$\log_{11} 2 + \log_{11} x = \log_{11} 7$$
$$\log_{11}(2x) = \log_{11} 7$$
$$\therefore 2x = 7$$
$$x = \frac{7}{2}$$

ii. (3 marks)

✓ [1] for $x^2 = 4x + 5$.

✓ [1] for solutions of x.

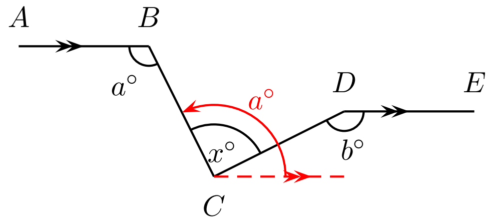
✓ [1] for justification of $x = 3$.

$$2\log_5 x = \log_5(4x+5)$$
$$\log_5\left(x^2\right) = \log_5(4x+5)$$
$$x^2 = 4x + 5$$
$$x^4 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$x = 5, -1$$

But as $x > 0$ in the domain of the logarithmic function. Hence $x = 5$.

Question 4

(a) (2 marks)

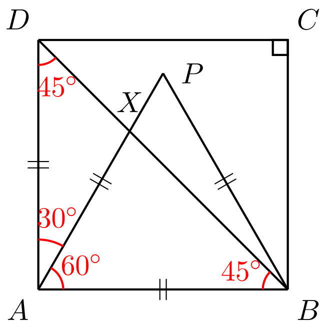


$$a = x + (180° - b)$$
$$x = a + b - 180°$$

(b) (3 marks)

✓ [2] for correct reasoning.

✓ [1] for final answer.



- $\angle PAB = 60°$ ($\angle$ in equilateral $\triangle$)
- $\therefore \angle XAD = 30°$.
- $\angle ADB = 45°$
(diagonals of a square bisect the $\angle$)
- By the $\angle$ sum of $\triangle DXA$,

$$\angle DXA = 180° - 30° - 45° \quad = 105°$$

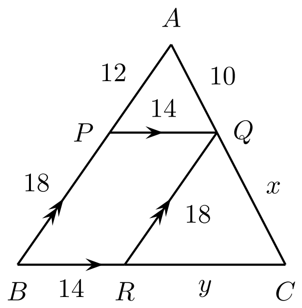
(c) (2 marks)

$$\sum \angle = (n-2) \times 180°$$
$$= 18 \times 180°$$
$$= 3\,240°$$
$$\therefore \angle = \frac{3\,240°}{20} = 162°$$

(d) (3 marks)

✓ [1] each for correct x and y.

✓ [1] for correct reasoning.



- By the intercepts of transversals over parallel lines PQ and BC,

$$\frac{\cancel{12}^{\,2}}{\cancel{18}^{\,3}} = \frac{10}{x}$$

$$\therefore x = \frac{10 \times 3}{2} = 15$$

- By the same reason as above,

$$\frac{y}{14} = \frac{x}{10}$$

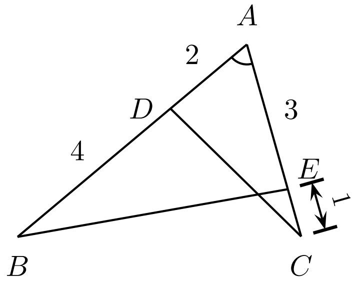
$$y = \frac{15 \times 14}{10} = \frac{\cancel{5} \times 3 \times \cancel{2} \times 7}{\cancel{10}} = 21$$

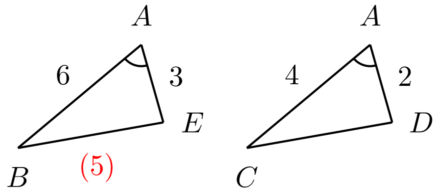
Question 5

(a) i. (1 mark)

✓ [1] for final statement. Proof shown for completeness.

- Splitting the triangles into separate shapes:





$$\frac{AC}{AB} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{AD}{AE} = \frac{2}{3}$$

- $\angle BAE = \angle DAC$ (common from diagram)

Hence $\triangle ABE \parallel\!\!| \triangle ACD$ (two matching sides in same ratio plus included angle equal)*.

ii. (2 marks)

$$BE = 5$$

$$\frac{CD}{BE} = \frac{2}{3} \quad \Rightarrow \quad \frac{CD}{5} = \frac{2}{3}$$

$$CD = \frac{10}{3}$$

(b) i. (2 marks)

✓ [2] for correct proof shown.

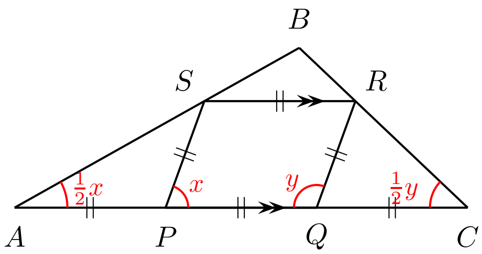
- Let $\angle SPQ = \alpha$.
- Since $\angle PAS = \angle PSA$ (base $\angle$ of isos $\triangle$), and $\angle PAS + \angle PSA = \angle SPQ$ (exterior $\angle$ of $\triangle$)

$$2\angle PAS = \angle SPQ$$

$$\therefore \angle PAS = \frac{1}{2}\angle SPQ = \frac{1}{2}x$$

ii. (3 marks)

✓ [1] for each matching bullet point shown.



- By the same reasoning as the previous part, if $\angle RQP = y$ then $\angle RCQ = \frac{1}{2}y$.
- Since $\angle SPQ + \angle RQP = 180°$ (cointerior $\angle$, $SR \parallel PQ$), then

$$\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2}(x+y)$$
$$= 90°$$

- By the $\angle$ sum of $\triangle ABC$, if $\angle BAC + \angle BCA = 90°$, then

$$\angle ABC = 90°$$

* "SAS" will not be accepted for this part