North Sydney Boys High School

MATHEMATICS (EXTENSION 1)

2011 Preliminary Course Assessment Task 2

General instructions

- Working time – 50 minutes.
- Commence each new question on a new page.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question. Marks may *not* be awarded for incomplete or untidy solutions.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets used in the correct order within this paper and hand to examination supervisors.

Class (please ✔)

- ◯ 11M3A – Mr Weiss
- ◯ 11M3B – Mr Ireland
- ◯ 11M3C – Mr Berry
- ◯ 11M3D – Mrs Collins
- ◯ 11M3E – Mr Lam
- ◯ 11M3F – Mr Fletcher/Ms Everingham

NAME .... **# BOOKLETS USED**: ....

Marker's use only.

QUESTION	1	2	3	4	5	Total	%
MARKS	/11	/15	/8	/10	/5	/49	

Question 1 (11 Marks) Commence a NEW page. **Marks**

(a) Write $\dfrac{4\pi}{15}$ in degrees. **1**

(b) Evaluate $\tan 1.5$ correct to two decimal places. **1**

(c) Write down the exact value of

i. $\cot 330^\circ$ **1**

ii. $\cos(-180^\circ)$ **1**

(d) Given $\tan\theta = \frac{3}{5}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$. **2**

(e) Simplify $\sin(90^\circ - \theta) \times \tan(360^\circ - \theta)$. **2**

(f) Given the function $y = 5\cos 2x$

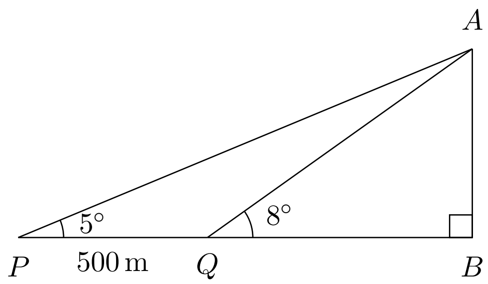
i. State the amplitude **1**

ii. State the period. **1**

iii. Sketch the graph of the function for $0 \le x \le 180^\circ$. **1**

Question 2 (15 Marks) Commence a NEW page. **Marks**

(a) Steve observes a cliff AB from a point P. The angle of elevation to the top of the cliff is 5°. From Q, which is 500 m closer to the cliff, the angle of elevation to the top of the cliff is 8°.



i. Calculate AQ to the nearest metre. **3**

ii. Hence, find QB to the nearest 10 metres. **2**

(b) Solve for $0^\circ \le x \le 360^\circ$:

i. $\sin 2x = -\dfrac{1}{\sqrt{2}}$. **2**

ii. $2\sin^2 x = 3\cos x$. **3**

(c) Show that $\dfrac{\cot^2\theta}{1+\cot^2\theta} = \cos^2\theta$. **2**

(d) In $\triangle ABC$, $AB = 25$ cm and $BC = 28$ cm. The area of $\triangle ABC$ is $340\,\text{cm}^2$. **3**

Find the value(s) of $\angle ABC$ to the nearest degree.

Question 3 (8 Marks) Commence a NEW page. **Marks**

The coordinates of the point A, B and C are $(0, 2)$, $(4, 0)$ and $(6, -4)$ respectively.

(a) Find the length of AB. **1**

(b) Find the gradient of AB. **1**

(c) Find the equation of the line ℓ, drawn through C parallel to AB. **2**

(d) Find the coordinates of D, the point where ℓ intersects the x axis. **1**

(e) Find the perpendicular distance of the point A from the line ℓ. **1**

(f) Find the area of the quadrilateral $ABCD$. **2**

Question 4 (10 Marks) Commence a NEW page. **Marks**

(a) Find the coordinates of the points $P(x, y)$ which divides the interval AB joining the points $A(-5, 11)$ and $B(7, 3)$ externally in the ratio $3 : 1$. **2**

(b) Find the equation of the line through the point of intersection of the lines $3x - 2y + 6 = 0$ and $2x - 3y + 1 = 0$ that is perpendicular to the line $x - y + 1 = 0$. **4**

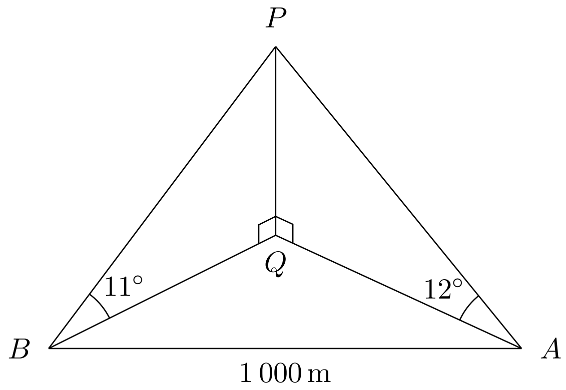
(c) Sketch the region of the number plane where the following inequalities hold simultaneously: **4**

$$\begin{cases} y \geq x^2 - 4 \\ x + y < 2 \\ x \geq 0 \end{cases}$$

Question 5 (5 Marks) Commence a NEW page. **Marks**

The angle of elevation of a tower PQ of height h metres from a point A due east of Q is $12°$. From another point B, the bearing of the tower is $051°$T and the angle of elevation is $11°$.

$AB = 1\,000\,\text{m}$ and AB is on the same level as the base Q.



(a) Show that $\angle AQB = 141°$. **1**

(b) Show that **3**

$$h^2 = \frac{1\,000\,000}{\tan^2 78° + \tan^2 79° - 2\tan 78° \tan 79° \cos 141°}$$

(c) Find h, correct to the nearest metre. **1**

End of paper.

Question 1

(a) (1 mark)

$$\frac{4\pi}{15} = \frac{4 \times 180^\circ}{15} = 48^\circ$$

(b) (1 mark)

$$\tan 1.5 = 14.1$$

(c) i. (1 mark)

$$\cot 330^\circ = \frac{1}{\tan 330^\circ} = -\sqrt{3}$$

ii. (1 mark)

$$\cos(-180^\circ) = \cos 180^\circ = -1$$

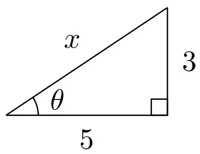
(d) (2 marks)

✓ [1] for correct fraction.

✓ [1] for correct sign.

$$\tan\theta = \frac{3}{5} \qquad \cos\theta < 0$$

Hence θ is in the 3rd quadrant.



$$x^2 = 3^2 + 5^2 = 34$$
$$\therefore x = \sqrt{34}$$
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = -\frac{\sqrt{34}}{3}$$

(e) (2 marks)

✓ [1] for $\cos\theta \times -\tan\theta$

✓ [1] for final answer

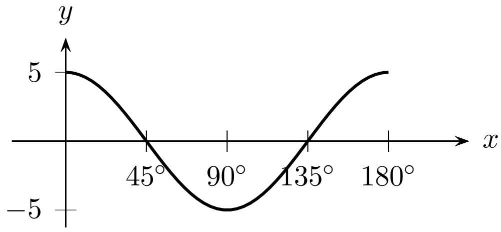
$$\begin{aligned}\sin(90^\circ - \theta) \times \tan(360^\circ - \theta) &= \cos\theta \times -\tan\theta \\ &= \cos\theta \times \frac{-\sin\theta}{\cos\theta} \\ &= -\sin\theta\end{aligned}$$

(f) $y = 5\cos 2x$

i. (1 mark) – $a = 5$.

ii. (1 mark) – $T = \frac{360^\circ}{2} = 180^\circ$.

iii. (1 mark)



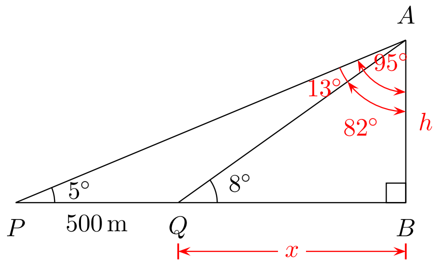
Question 2

(a) i. (3 marks)

✓ [1] for $\angle PAC = 3^\circ$

✓ [1] for applying the sine rule

✓ [1] for final answer



$$\angle QAB = 90^\circ - 8^\circ = 82^\circ$$
$$\angle PAB = 90^\circ - 5^\circ = 85^\circ$$
$$\therefore \angle PAQ = 85^\circ - 82^\circ = 3^\circ$$

Applying the sine rule on $\triangle PAQ$,

$$\frac{AQ}{\sin 5^\circ} = \frac{500}{\sin 3^\circ}$$
$$AQ = \frac{500\sin 5^\circ}{\sin 3^\circ} = 833\,\text{m}$$

ii. (2 marks)

✓ [1] for $\frac{QB}{AQ} = \cos 8^\circ$

✓ [1] for final answer

$$\frac{QB}{AQ} = \cos 8^\circ$$
$$QB = AQ\cos 8^\circ$$
$$= \frac{500\sin 5^\circ \cos 8^\circ}{\sin 3^\circ} = 820\,\text{m}$$

(b) i. (2 marks)

✓ [1] for solutions in $2x$

✓ [1] for final solutions in x.

$$\sin 2x = -\frac{1}{\sqrt{2}} \qquad 0^\circ \le 2x \le 720^\circ$$

$$\begin{aligned} 2x &= 225^\circ, 315^\circ, 360^\circ + 225^\circ, 360^\circ + 315^\circ \\ &= 225^\circ, 315^\circ, 585^\circ, 675^\circ \end{aligned}$$

$$\therefore x = 112.5^\circ, 157.5^\circ, 292.5^\circ, 337.5^\circ$$

ii. (3 marks)

✓ [1] for $2 - 2\cos x = 3\cos x$.

✓ [1] for $2\cos x = 1$ or $\cos x = -2$.

$$\begin{aligned} 2\sin^2 x &= 3\cos x \\ 2\left(1 - \cos^2 x\right) &= 3\cos x \\ 2 - 2\cos^2 x &= 3\cos x \\ 2\cos^2 x + 3\cos x - 2 &= 0 \end{aligned}$$

Let $m = \cos x$

$$\begin{aligned} 2m^2 + 3m - 2 &= 0 \\ (2m-1)(m+2) &= 0 \\ \therefore m = \cos x &= \frac{1}{2}, -2 \end{aligned}$$

However, $|\cos x| \le 1$. $\therefore \cos x = \frac{1}{2}$ only

$$x = 60^\circ, 300^\circ$$

(c) (2 marks)

✓ [1] for $\frac{\cot^2\theta}{\operatorname{cosec}^2\theta}$

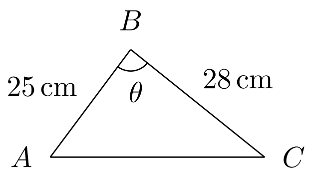
✓ [1] for final answer

$$\frac{\cot^2\theta}{1 + \cot^2\theta} = \frac{\frac{\cos^2\theta}{\sin^2\theta}}{\operatorname{cosec}^2\theta} = \frac{\frac{\cos^2\theta}{\cancel{\sin^2\theta}}}{\frac{1}{\cancel{\sin^2\theta}}} = \cos^2\theta$$

(d) (3 marks)

✓ [1] for applying area of triangle formula via sine

✓ [2] for final solutions.



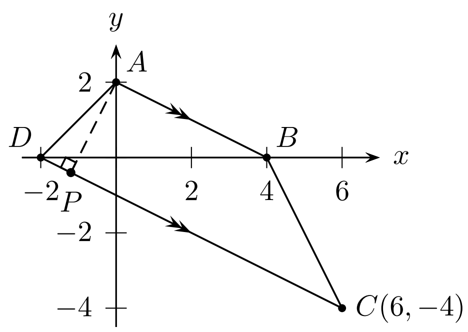
$$A = 340\,\text{cm}^2 = \frac{1}{2}ac\sin\angle ABC$$

$$\frac{1}{2} \times 28 \times 25 \sin\angle ABC = 340$$

$$\begin{aligned} \sin\angle ABC &= \frac{34}{35} \\ \angle ABC &= 76^\circ 16, 103^\circ 44 \\ &= 76^\circ, 104^\circ \end{aligned}$$

Question 3

(a) (1 mark)



$$AB = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$$

(b) (1 mark) – $m_{AB} = \frac{-2}{4} = -\frac{1}{2}$

(c) (2 marks)

✓ [1] for applying the point gradient formula

✓ [1] for final answer

$$m_\ell = m_{AB} = -\frac{1}{2}$$

Applying the point gradient formula

$$\begin{aligned} y + 4 &= -\frac{1}{2}(x - 6) \\ 2y + 8 &= -x + 6 \\ x + 2y + 2 &= 0 \qquad \left(y = -\frac{1}{2}x - 1\right) \end{aligned}$$

(d) (1 mark)

$$x + 2y + 2\Big|_{y=0} = 0$$

$$x + 2 = 0 \quad \Rightarrow \quad x = -2$$

$$\therefore D(-2, 0)$$

(e) (1 mark)

$$d_{\perp} = \frac{|1(0) + 2(2) + 2|}{\sqrt{1^2 + 2^2}} = \frac{6}{\sqrt{5}}$$

(f) (2 marks)

$$CD = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5}$$

As $ABCD$ is a trapezium,

$$\begin{aligned} A &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2} \times \frac{6}{\sqrt{5}} \left(2\sqrt{5} + 4\sqrt{5}\right) \\ &= \frac{3}{\cancel{\sqrt{5}}} \times 6\cancel{\sqrt{5}} = 18 \end{aligned}$$

Question 4

(a) (2 marks)

$$\begin{aligned} P &= \left(\frac{3(7) + (-5)(-1)}{3-1}, \frac{3(3) + 11(-1)}{3-1}\right) \\ &= (13, -1) \end{aligned}$$

(b) (4 marks)

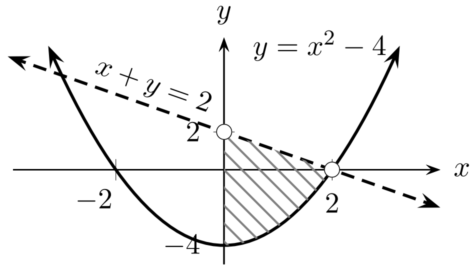
- ✓ [1] for placing two equations in k form
- ✓ [1] for $\frac{3+2k}{2+3k} = -1$
- ✓ [1] for $k = -1$
- ✓ [1] for $x + y + 5 = 0$

$$\begin{aligned} 3x - 2y + 6 + k(2x - 3y + 1) &= 0 \\ (3 + 2k)x + (-2 - 3k)y + (k + 6) &= 0 \end{aligned}$$

The gradient of the line $x - y + 1 = 0$ is $m = 1$. Hence required gradient is $m = -1$.

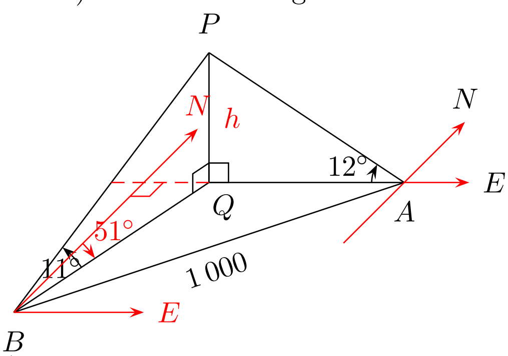
$$\begin{aligned} \therefore \frac{3+2k}{2+3k} &= -1 \\ 3 + 2k &= -2 - 3k \\ -5k = 5 \quad &\Rightarrow \quad k = -1 \\ \therefore (3-2)x + (-2+3)y + (-1+6) &= 0 \\ x + y + 5 &= 0 \end{aligned}$$

(c) (4 marks)



Question 5

(a) (1 mark) Redraw the diagram:



$\angle AQB = 90^\circ + 51^\circ = 141^\circ$
(exterior $\angle$ of $\triangle$ is the sum of the interior opposite angles.)

(b) (3 marks)

- ✓ [1] for $BQ = h\tan 78^\circ$ (or for AQ)
- ✓ [1] for applying the cosine rule
- ✓ [1] for final answer

In $\triangle BQP$,

$$\begin{aligned} \frac{BQ}{h} &= \tan(90^\circ - 11^\circ) = \tan 79^\circ \\ \therefore BQ &= h\tan 79^\circ \end{aligned}$$

Similarly in $\triangle AQP$,

$$\therefore AQ = h\tan 78^\circ$$

Applying the cosine rule in $\triangle ABQ$,

$$\begin{aligned} 1\,000^2 &= (h\tan 79^\circ)^2 + (h\tan 78^\circ)^2 \\ &\quad - 2(h\tan 78^\circ)(h\tan 79^\circ)\cos 141^\circ \\ &= h^2\left(\tan^2 79^\circ + \tan^2 78^\circ\right. \\ &\quad \left. -2\tan 78^\circ \tan 79^\circ \cos 141^\circ\right) \end{aligned}$$

$$\therefore h^2 = \frac{1\,000\,000}{\tan^2 79^\circ + \tan^2 78^\circ - 2\tan 78^\circ \tan 79^\circ \cos 141^\circ}$$

(c) (1 mark)

$$h = 108\,\text{m}$$