SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2003

Year 11

HALF YEARLY EXAMINATION

Mathematics Extension

General Instructions

- Reading time – 5 minutes.
- Working time – 90 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may **NOT** be awarded for messy or badly arranged work.
- Hand in your answer booklets in 4 sections. Section A (Questions 1 & 2), Section B (Questions 3 & 4), Section C (Question 5) and Section D (Questions 6 & 7).
- Start each **NEW** section in a separate answer booklet.

Total Marks - 90 Marks

- Attempt Sections A - D
- All questions are NOT of equal value.

Examiner: *E. Choy*

SECTION A

Question 1 (18 marks) **Marks**

(a) Evaluate correct to four significant figures **2**

$$5 \cdot 6729 \times 10^{18} \div 2 \cdot 17407 \times 10^{-7}$$

(b) Convert $\frac{7\pi}{15}$ radians to degrees **1**

(c) State the exact value of $\operatorname{cosec} 135^\circ$ **1**

(d) Simplify:

(i) $8^{-2/3} + 81^{3/4}$ **2**

(ii) $\frac{x^{-1}}{y^{-1}} - \left(\frac{x}{y}\right)^{-1}$ **2**

(e) Express $\frac{2}{4-\sqrt{11}}$ with a rational denominator **2**

(f) A function is defined by $f(x) = x(x^2 - 2)$. **2**
Find the value of $f(a)$ and $f(-a)$.

(g) Find in general form the equation of the straight line that passes through the point $(1,-3)$ and is perpendicular to the line $y = \frac{2}{3}x + 4$ **2**

(h)

7

3

B

M

A

Use the diagram above to write down the ratio in which

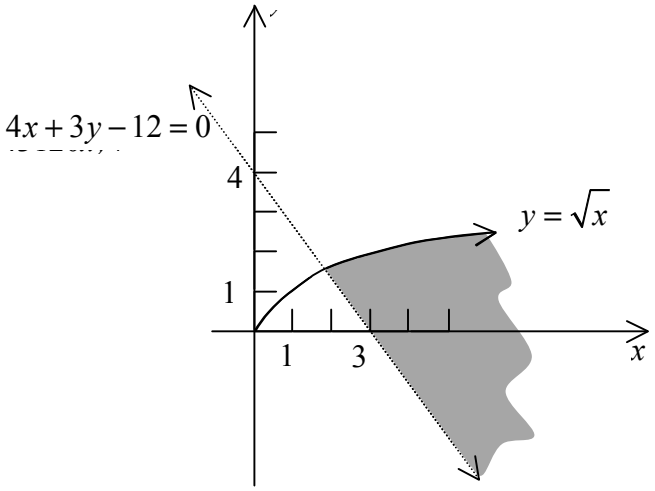
(i) *M* divides the interval *AB* **2**

(ii) *B* divides the interval *AM* **2**

SECTION A (continued)

Question 2 (10 marks) **Marks**

(a) Write down the inequations that correspond to the region shaded in the diagram below **2**



(b) Find, without using a calculator, the value of **3**

$$\left(\sin 22\tfrac{1}{2}^{\circ} - \cos 22\tfrac{1}{2}^{\circ}\right)^2$$

(c) Prove $\dfrac{\sin^2\theta}{1-\cos\theta} = 1+\cos\theta$ **2**

(d) If $\cos\theta = \frac{4}{5}$, find the value of $\tan^2\theta - \sec\theta$ **3**

SECTION B (Start a NEW booklet)

Question 3 (9 marks) **Marks**

(a) Solve and graph, on separate number lines, the solutions of

(i) $2(4-7x) \le 30$ **2**

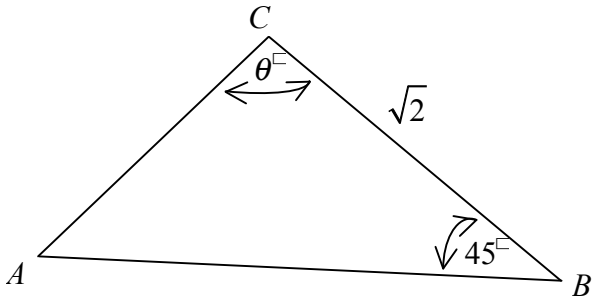
(ii) $|3-x| = 17$ **2**

(iii) $\frac{2}{x} \ge 5$ **2**

(b) The lines $4x-3y+2=0$ and $7x-y-6=0$ meet at the point P. **3**
WITHOUT finding the coordinates of P, find in general form the equation of the line that passes through P and the point $(0,1)$.

Question 4 (12 marks)

(a) **5**



In the figure, $BC = \sqrt{2}$ metres, $\angle ABC = 45^\circ$ and $\angle ACB = \theta^\circ$.

Show that $AC = \dfrac{\sqrt{2}}{\sin\theta + \cos\theta}$ metres

(b) (i) Find the perpendicular distance from the origin to the line $x+y+c=0$ **2**

(ii) The line $x+y+c=0$ cuts the circle $x^2+y^2=9$ in two distinct points. **3**
Prove that $-3\sqrt{2} < c < 3\sqrt{2}$

(iii) Explain what happens when $c = \pm 3\sqrt{2}$ **2**

SECTION C (Start a NEW booklet)

Question 5 (17 marks) **Marks**

(a) On separate number planes, sketch the following graphs, giving all the intercepts with the x and y axes.

(i) $y = 3^x + 1$ **2**

(ii) $y = \dfrac{3}{|x|}$ **3**

(b) If $f(t) = \dfrac{t^2 + t + 1}{t}$ show that $f\left(\dfrac{a}{b}\right) = f\left(\dfrac{b}{a}\right)$ for **3**

(c) A function $f(x)$ is defined by

$$f(x) = \begin{cases} 2 - x, & x \geq 0 \\ 2 - x^2, & x < 0 \end{cases}$$

(i) Draw a neat sketch of the function giving *ALL* intercepts with the x and y axes. **3**

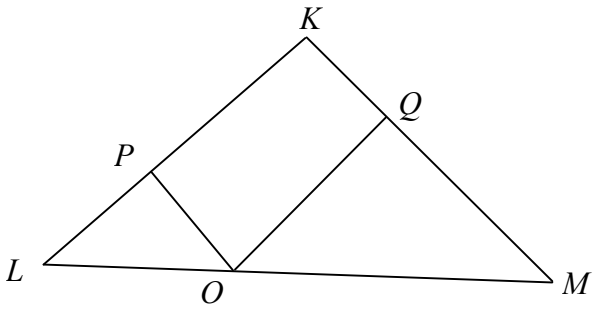
(ii) Find the values of a for which $f(a^2) + f(-1) = 0$ **3**

(d) Sketch on a Cartesian number plane the *locus* of all points equidistant from the x and y axes. **3**

SECTION D (Start a NEW booklet)

Question 6 (19 marks) **Marks**

(a) **5**



In ΔKLM , $KL = KM$, $OP \perp LK$ and $OQ \perp KM$.

Prove that $LP.OQ = OP.MQ$

(b) If $\sin x = 2\sin(\theta - x)$, prove that $\tan x = \dfrac{2\sin\theta}{1+2\cos\theta}$ **4**

(c) Solve $\dfrac{3x-1}{x+2} \geq 1$ **4**

(d) (i) Draw the graphs of $y = |2x-1|$ and $y = \frac{2}{3}x + 1$ for $-2 \leq x \leq 3$ **4**

(ii) Hence solve the equation $3|2x-1| = 2x+3$ **2**

Question 7 (5 marks)

(i) Show that $\dfrac{1}{\sqrt{n-1}+\sqrt{n}} = \sqrt{n} - \sqrt{n-1}$, **3**
for any positive integer n.

(ii) Hence evaluate **2**

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \square + \frac{1}{\sqrt{8}+\sqrt{9}} + \frac{1}{\sqrt{9}+\sqrt{10}}$$

THIS IS THE END OF THE PAPER