SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2004

YEAR 11

HALF YEARLY EXAMINATION

Mathematics Extension

General Instructions

- Working time – 90 minutes.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Only Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Start each section in a **SEPARATE** answer booklet.
- Marks may not be awarded for messy or badly arranged work

Total Marks - 82

- Attempt Sections A – D
- All sections are **NOT** of equal value.

Examiner: *C. Kourtesis*

SECTION A
[20 marks]

1. Convert (a) $\frac{5\pi}{2}$ radians to degrees 1

(b) 135^{o} to radians 1

2. The point $(9, k)$ lies on the line $x + 3y = 6$ 2
Find the value of k

3. If $m(x) = 3^{x} + 3^{-x}$ find $m(-3)$ 1

4. Solve the inequalities

(a) $-4x + 1 \leq 12x$ 2

(b) $|x| < 4$ 1

5. Find the point of intersection of the straight lines

$x + 2y = 6$ and $x - 3y = 10$ 2

6. On separate diagrams sketch the graphs of:

(a) $xy = 4$ 1

(b) $y = 2^{-x}$ 1

(c) $y = \log_{10} x$ 1

7. Find the roots of the equation $\theta^2 - \theta - 2 = 0$ 2

8. Sketch the region in the number plane indicated by $y \geq x^2$ 2

9. The equation of a parabola is given by $x^2 = 8y$
Find the (a) equation of the directrix 1
(b) coordinates of the focus 2

SECTION B (Start a NEW Booklet)
[18 marks]

10. Explain why $f(x) = \sin x$ represents an odd function 2

11. For the function $f(x) = \dfrac{1}{(x+1)(1-3x)}$ write down the

(a) equations of the vertical asymptotes 2

(b) y intercept 1

12. The equation of a circle is given by

$$x^2 + y^2 + 4x - 8y = 0$$ 4

Find the (a) coordinates of the centre

(b) length of the radius

13. Find the acute angle between the lines, answer to the nearest minute.

$$y = 2x - 1 \text{ and } y = -\frac{1}{3}x$$ 3

14. Given the quadratic equation $2x^2 - mx + 8 = 0$

Find the (a) discriminant 1

(b) value(s) of m for which the above equation has two distinct real roots 2

15. Find the coordinates of the point that divides the interval joining $A(1,4)$ and $B(5,10)$ externally in the ratio 3:2 3

SECTION C (Start a NEW Booklet)
[23 marks]

16. Solve the inequalities

(a) $(x+4)(x-2)(x-3)>0$ 2

(b) $\frac{x-3}{1-x}>2$ 4

17. Find the equation of the locus of a point $P(x,y)$ which moves so that its distance from $x-y+2=0$ is equal to its distance from the point $(1,-1)$ 3

18. Solve the inequality $\frac{1}{|4-3x|}<4$ 3

19. (a) Write down the algebraic definition of $|x|$ 1

(b) Sketch the graph of $y=x|x|$ 3

20. Find the value(s) of k if the equation

$$x^2-3kx+(k+3)=0$$

has:

(a) one root that is double the other 2

(b) one root that is the reciprocal of the other 2

21. Solve the equation

$$1-\cos x-2\sin^2 x=0 \quad \text{for} \quad 0\le x\le 2\pi$$ 3

SECTION D (Start a NEW Booklet)
[21 marks]

22. Solve $x^6 - 7x^3 - 8 = 0$ 3

23. Solve the inequality $(x+\frac{1}{x})^2 - (x+\frac{1}{x}) \le 6$ 4

24. Prove that

$$\frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} = \tan\theta$$ 4

25. (a) Show that $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ 2

(b) Hence show that

$$\tan 15° = 2 - \sqrt{3}$$ 3

26. A man travelling along a straight flat road passes three points at intervals of 200m. From those points he observes the angle of elevation of the top of a hill to the left of the road to be respectively $30°$, $45°$ and again $45°$.

(a) Draw a neat diagram to represent this information 1

(b) Find the height of the hill 4

THIS IS THE END OF THE EXAMINATION

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Sample Solutions

Section	Marker
A	Mr Choy
B	Mr Parker
C	Mr Hespe
D	Mr Bigelow

Section A.

① (a) $\pi^c = 180$

[1] $\frac{5\pi}{2} = \overset{90}{\cancel{180}} \times \frac{5}{\cancel{2}_1} = 450$

[1] (b) $180 = \pi^c$

$\frac{135}{180} \times 180 = \pi \times \frac{135}{180} = \frac{3\pi}{4}^c$

$2{\cdot}36^c$

② $9 + 3k = 6$.

[2] $3k = -3$

$k = -1$.

③ $m(-3) = 3^{-3} + 3^3$

[1]

$= 27\frac{1}{27}$

$(27 \cdot 04)$.

④ (a) $-4x + 1 \leq 12x$

[2] $16x \geq 1$

$x \geq \frac{1}{16}$.

(b) $|x| < 4$

[1] $-4 < x < 4$

⑤ $x + 2y = 6$ —①

$x - 3y = 10$ —②

① − ② $5y = -4$

$\therefore y = -4/5$

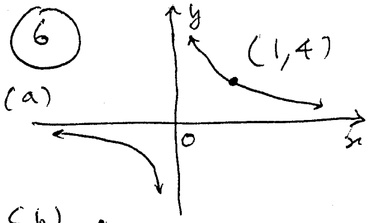
$x - 8/5 = 6$

$\therefore x = 7\,3/5$ $(38/5)$,

$\therefore$ pt of intersection

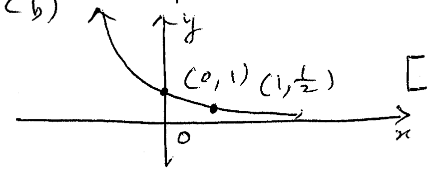
$(7\,3/5, -4/5)$ [2]

⑥ (a)



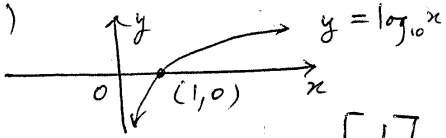
[1]

(b)



[1]

(c)



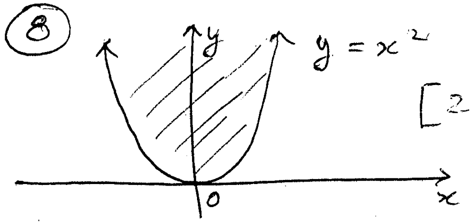
[1].

⑦ $\theta^2 - \theta - 2 = 0$

$(\theta - 2)(\theta + 1) = 0$ [2]

$\therefore \theta = -1, 2$.

⑧



[2]

[1]

⑨ $x^2 = 8y$, $\boxed{\begin{array}{c}4a = 8\\ a = 2\end{array}}$

(a) $y = -2$ equation of directrix

(b) $(0, 2)$ focus

[2].

SECTION B [18 marks]

10. An odd function satisfies $f(-x) = -f(x)$

$\therefore f(-x) = \sin(-x) = -\sin x = -f(x)$

11. $f(x) = \dfrac{1}{(x+1)(1-3x)}$

(a) The equations of the vertical asymptotes are $x = -1$ and $x = \dfrac{1}{3}$.

(b) y intercept is when $x = 0 \Rightarrow f(0) = \dfrac{1}{(0+1)(1-0)} = 1$.

So the $y-$intercept is $(0,1)$.

12. $x^2 + y^2 + 4x - 8y = 0$

$\therefore \left(x^2 + 4x\right) + \left(y^2 - 8y\right) = 0$

$\Rightarrow \left(x^2 + 4x + \left(\dfrac{4}{2}\right)^2\right) + \left(y^2 - 8y + \left(\dfrac{-8}{2}\right)^2\right) = 0 + 4 + 16$

$\Rightarrow (x^2 + 4x + 4) + (y^2 - 8y + 16) = 20$

$\Rightarrow (x+2)^2 + (y-4)^2 = 20$

(a) coordinates of the centre are $(-2,4)$

(b) length of the radius is $\sqrt{20} = 2\sqrt{5}$

13. The acute angle between the lines is given by $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$.

$y = 2x - 1 \Rightarrow m_1 = 2$ and $y = -\dfrac{1}{3}x \Rightarrow m_2 = -\dfrac{1}{3}$

$\tan\theta = \left|\dfrac{2-(-1/3)}{1+(2)(-1/3)}\right| = 7 \Rightarrow \theta = 81°52'$

14. $2x^2 - mx + 8 = 0$

(a) The discriminant is $\Delta = b^2 - 4ac = (-m)^2 - 4 \times 2 \times 8 = m^2 - 64$

(b) Two distinct real roots $\Rightarrow \Delta > 0$ ie $m^2 - 64 > 0$

$\Rightarrow (m-8)(m+8) > 0$

$\therefore m < -8, m > 8$

15. $P\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$ where $A(1,4) = (x_1, y_1)$, $B(5,10) = (x_2, y_2)$ and $m : n = 3 : -2$.

$P\left(\dfrac{3\times5+(-2)\times1}{3-2}, \dfrac{3\times10+(-2)\times4}{3-2}\right) = (13,22)$

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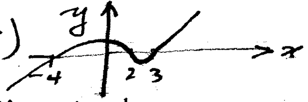
Part C

16:(a) 

$-4 < x < 2, \quad x > 3$

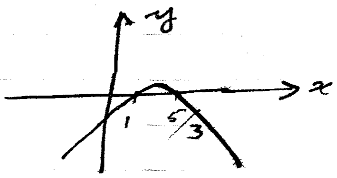
(b) Method 1:

$(x-3)(1-x) > 2(1-x)^2$

$(1-x)(x-3-2[1-x]) > 0$

$(1-x)(3x-5) > 0$

$1 < x < 5/3$



Method 2:

If $x > 1$, $x-3 < 2(1-x)$

$3x < 5$

$x < 5/3$.

If $x < 1$, $x-3 > 2(1-x)$

$3x > 5$

$x > 5/3$

Contradiction.

$\therefore \quad 1 < x < 5/3$.

17 $\frac{|x-y+2|}{\sqrt{2}} = \sqrt{(x-1)^2+(y+1)^2}$

$\frac{x^2+y^2+4-2xy+4x-4y}{2} = x^2-2x+1+y^2+2y+1$

$x^2+y^2+4-2xy+4x-4y = 2x^2-4x+4+2y^2+4y$

$x^2+y^2+2xy-8x+8y = 0$

18 $1 < 4|4-3x|$

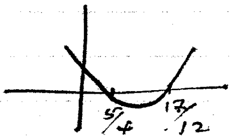
Method 1:

If $x < 4/3$	If $x > 4/3$
$1 < 16-12x$	$1 < 12x-16$
$12x < 15$	$12x > 17$
$x < 5/4$	$x > 17/12$

as $5/4 < 4/3$ and $17/12 > 4/3$
then $x < 5/4$ and $x > 17/12$

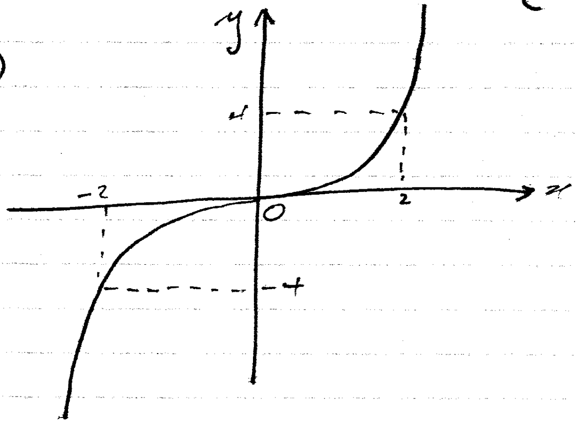
Method 2:

$$1 < 16(16-24x+9x^2)$$
$$1 < 256-384x+144x^2$$
$$0 < 255-384x+144x^2$$
$$0 < 85-128x+48x^2$$
$$0 < (12x-17)(4x-5)$$
$$\therefore \quad x < 5/4 \text{ and } x > 17/12 .$$



19 (a) $|x| = \sqrt{x^2}$ or $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

(b)



$y = x^2$ when $x \geq 0$
$= -x^2$ when $x < 0$

20 Not counted (a) $k = -1, 3/2$
(b) $k = -2$

21 $1-\cos x - 2(1-\cos^2 x) = 0$
$2\cos^2 x - \cos x - 1 = 0$
$(2\cos x + 1)(\cos x - 1) = 0$
$\cos x = -1/2, 1$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}, 0, 2\pi.$

SECTION D

22. $x^6 - 7x^3 - 8 = 0$

$(x^3 - 8)(x^3 + 1) = 0$

$x^3 = 8, -1$

$\boxed{x = 2, -1}$

23. $\left(x + \frac{1}{x}\right)^2 - \left(x + \frac{1}{x}\right) \leq 6.$

$\left(x + \frac{1}{x}\right)^2 - \left(x + \frac{1}{x}\right) - 6 \leq 0$

$\left(x + \frac{1}{x} - 3\right)\left(x + \frac{1}{x} + 2\right) \leq 0$

$\therefore \quad -2 \leq x + \frac{1}{x} \leq 3.$

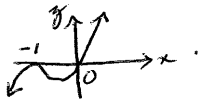
Consider $x + \frac{1}{x} \geq -2.$

$x + \frac{1}{x} + 2 \geqslant 0$

$\frac{x^2 + 1 + 2x}{x} \geqslant 0$ (MULTIPLY BOTH SIDES by x^2)

$x(x^2 + 2x + 1) \geqslant 0$

$x(x+1)^2 \geqslant 0$



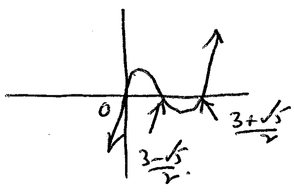
$\therefore \quad x = -1, \ x \geqslant 0. \qquad NB \ x \neq 0$

$\therefore \boxed{x = -1, \ x > 0.}$ Ⓐ

Consider $x + \frac{1}{x} \leq 3$

$\frac{x^2 - 3x + 1}{x} \leq 0$ (MULTIPLY BOTH SIDES by x^2)

$x(x^2 - 3x + 1) \leq 0$ (NOW solving $x^2 - 3x + 1 = 0$
$x = \frac{3 \pm \sqrt{5}}{2}$).



$\therefore \boxed{\frac{3-\sqrt{5}}{2} \leq x \leq \frac{3+\sqrt{5}}{2}, \ x < 0}$ Ⓑ

Taking the intersection of sets Ⓐ u Ⓑ

$\boxed{x = -1, \ \frac{3-\sqrt{5}}{2} \leq x \leq \frac{3+\sqrt{5}}{2}}$

24. $LHS = \frac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1}$

$= \frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta - 1 + \cos\theta + 1}$

$= \frac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta + \cos\theta}$

$= \frac{\sin\theta(2\cos\theta+1)}{\cos\theta(2\cos\theta+1)}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$= RHS.$

25(a) Using

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}.$

Let $\alpha = \beta = \theta$

$\therefore \tan(\theta+\theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$

$\therefore \boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$ Ⓐ

(b) (NB Question states HENCE!!)

From Ⓐ

$\tan 30^\circ = \frac{2\tan 15^\circ}{1-\tan^2 15^\circ}$

Let $t = \tan 15^\circ$.

$\therefore \frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$

$1 - t^2 = 2\sqrt{3}t$.

$t^2 + 2\sqrt{3}t - 1 = 0$

$t = \frac{-2\sqrt{3} \pm \sqrt{12+4}}{2}$

$= \frac{-2\sqrt{3} \pm 4}{2}$

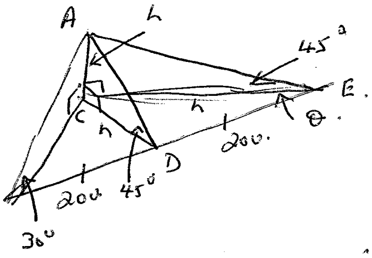
$= -\sqrt{3} \pm 2.$

now $t = \tan 15^\circ > 0$.

$\therefore \boxed{\tan 15^\circ = 2 - \sqrt{3}.}$

26.

(a)



NB △'s ACD and ACE are isosceles & right angled

Let $AC = CD = CE = h$.

In △ACB, $BC = \sqrt{3}h$.

Let $\angle CEB = \theta$.

Using Cosine Rule twice.

$\cos\theta = \frac{400^2 + x^2 - 3x^2}{2 \times 400 \times x}$ Ⓐ

& $\cos\theta = \frac{200^2 + x^2 - x^2}{2 \times 200 \times x}.$ Ⓑ

$\nearrow$

From Ⓐ & Ⓑ

$\frac{400^2 - 2x^2}{800x} = \frac{200^2}{400x}$

$\therefore 400^2 - 2.200^2 = 2x^2.$

$80000 \qquad = 2x^2$

$x^2 = 40000$

$\boxed{x = 200m}$