SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2005
YEAR 11 ACCELERATED

ASSESSMENT TASK #1

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each section is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 96

- Attempt questions 1 – 8
- All questions are of equal value.

Examiner: *AM Gainford*

Section A

(Start a new booklet.)

Question 1. (12 Marks)

(a) Calculate $\sqrt{\dfrac{185}{5\cdot4\times3\cdot7}}$ correct to two decimal places. **1**

(b) Simplify $a-3(2-a)$. **1**

(c) Solve the equation $\dfrac{x+2}{3}-1=\dfrac{x}{4}$. **2**

(d) Simplify $\sqrt{243}-\sqrt{27}$. **2**

(e) Find x if $\log_2 x=5$. **1**

(f) Find θ to the nearest minute if $0^0\le\theta\le90^0$ and $\cos\theta=0\cdot147$. **1**

(g) Solve the equation $4x^2=12x$. **2**

(h) Graph on a number line the solution of the inequality $\dfrac{x-1}{2}<3$. **2**

Question 2. (12 Marks)

(a) Simplify $\dfrac{\left(x^2y\right)^3}{\left(xy^3\right)^2}$. **1**

(b) Find the exact value of $\sin 315^0 - \tan 150^0$. **2**
Express your answer as a single fraction with rational denominator.

(c) Given that $f\left(x\right) = \dfrac{4x^2}{\sqrt{9-x^2}}$: **2**

(i) Find $f(2)$.

(ii) Show that $f\left(x\right)$ is an even function.

(d) Simplify $\dfrac{x^2-y^2}{\left(x+y\right)^2}$. **1**

(e) Calculate $(2\cdot 4371\times 10^{23}) \div (7\cdot 148\times 10^{-12})$, expressing your answer in scientific notation. **1**

(f) Sketch the graphs of the following, showing their principal features: **4**

(i) $y = 1 - x^2$

(ii) $y = -\sqrt{x}$

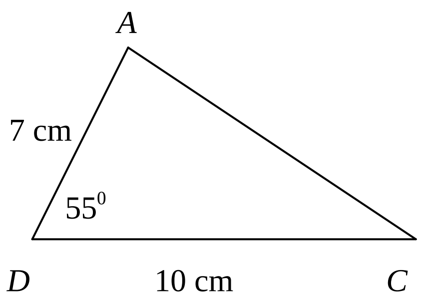
(g) Express the recurring decimal $0\cdot 27777\ldots$ as a common fraction. **1**

Section B

(Start a new booklet.)

Question 3. (12 Marks)

(a) Given the triangle *ACD*: **4**



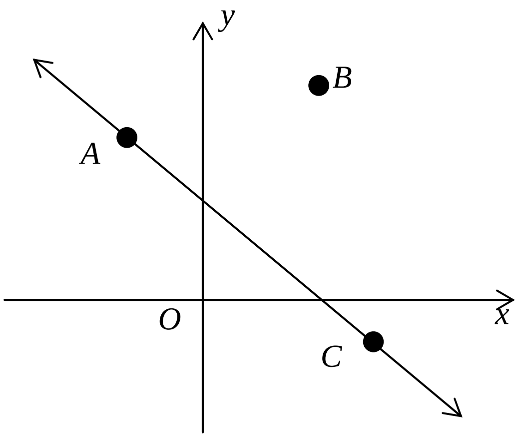
(i) Find the length of the side *AC*, correct to 2 decimal places.

(ii) Calculate the measure of $\angle ACD$ correct to the nearest minute.

(b) State the natural domain and range of the function $f(x) = \dfrac{x}{\sqrt{4-x^2}}$. **2**

(c) Solve $|3-2x| < 4$ and graph the solution on the number line. **2**

(d) **4**



The diagram above shows the points *A*(-2,4), *B*(3,5), and *C*(4, -1).

Copy the diagram to your answer booklet.

(i) Find the equation of the line through the points *A* and *C*.

(ii) Write the equation of the line through *B* perpendicular to *AC*.

(iii) Find the distance from *B* to *AC*.

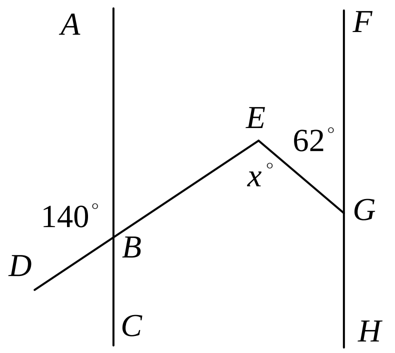
Question 4. (12 Marks)

(a) Given the line $y = 4 - x$ and the parabola $y = x^2 - 2$ **3**

(i) Find the points of intersection of the line and the parabola.

(ii) Hence sketch the region where $y \le 4 - x$ and $y > x^2 - 2$ hold simultaneously.

(b) **1**

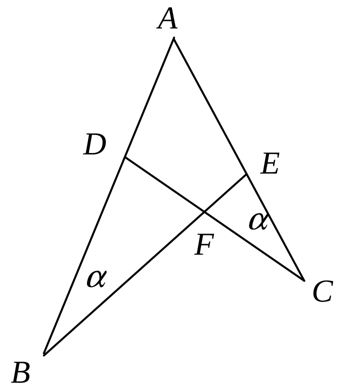


$AC \parallel FH$

Find the measure of x.

(c) In the diagram AB = 10 cm, AC = 8 cm, and BE = 6 cm. $\angle DBF = \angle ECF$. **2**

Find the length of DC, giving brief reasons.



(d) Solve the following equations: **4**

(i) $x^4 - 13x^2 + 36 = 0$

(ii) $4^x - 9\left(2^x\right) + 8 = 0$

(e) Find the centre and radius of the circle $x^2 + y^2 - 4x + 6y = 3$. **2**

Section C

(Start a new booklet.)

Question 5 (12 Marks)

(a) Differentiate the following with respect to x: **3**

(i) $x^3 - 3x^2 + 7$

(ii) $4\sqrt{x-1}$

(iii) $\dfrac{1}{2x^3}$

(b) **6**

(i) Use the product rule to find $\dfrac{dy}{dx}$ if $y = 3x(x-1)^9$.

(ii) Differentiate $y = \dfrac{x+1}{1-2x}$ by using the quotient rule.

(iii) If $f(x) = x + \dfrac{1}{x}$, find

(α) $f'(2)$

(β) $f'(-3)$

(c) The fourth term of a geometric sequence is $-\dfrac{27}{8}$, and the seventh term is $\dfrac{729}{64}$. **3**

(i) Find the values of the first term and the common ratio.

(ii) Find the sum of the first 10 terms.

Question 6 (12 Marks)

(a) For the curve $y = x - x^3$, find the gradient of the tangent to the curve at the point $(2, -6)$. Also find the gradient of the normal to the curve at this point. **2**

(b) Given that $f(x)$ is defined as below **2**

$$f(x) = \begin{cases} -5 & \text{for } x \le -3 \\ 2x & \text{for } -3 < x < 0 \\ x^2 & \text{for } x \ge 0 \end{cases}$$

(i) Find the value of $f(-3) + f(4) + f(-1)$.

(ii) Find $f(a^2)$.

(c) For the parabola $x^2 - 4x - 8y - 4 = 0$ write down the **4**

(i) equation of the axis of symmetry

(ii) coordinates of the vertex

(iii) equation of the directrix and coordinates of the focus.

(d) Two cadets on a compass march proceed from their campsite at *A* a distance of 1300 m on a bearing of $275^\circ T$ to a point *B*, then travel 2100 m on a bearing of $170^\circ T$ to a point *C*. **4**

(i) Draw a neat diagram to represent this situation.

(ii) Determine the bearing and distance for their final leg from *C* back to camp at *A*.

Section D

(Start a new booklet)

Question 7 (12 Marks)

(a) Let α and β be the roots of the equation $x^2 - 7x + 2 = 0$. Find the values of: **4**

(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $(\alpha + 1)(\beta + 1)$

(b) Solve the following equations simultaneously: **2**

$$2a - 3b = -21$$
$$4a + 2b = -2$$

(c) It is given that the series $\log_2 64 + \log_2 32 + \log_2 16 + \ldots$ is either geometric or arithmetic. **4**

(i) Determine whether the series is geometric or arithmetic, and state its common ratio, or difference.

(ii) Find the value of the seventh term.

(iii) Determine how many terms must be taken to produce a sum of zero.

(d) Prove the trigonometric identity **2**

$$\frac{1}{\sec A - \tan A} = \sec A + \tan A$$

Question 8 (12 Marks)

(a) (i) On a number plane diagram sketch the locus of all points equidistant from the co-ordinate axes. **2**

(ii) Write down an equation to describe this locus.

(b) If one root of the quadratic equation $x^2 + bx + c = 0$ is twice the other, show that $2b^2 = 9c$. **3**

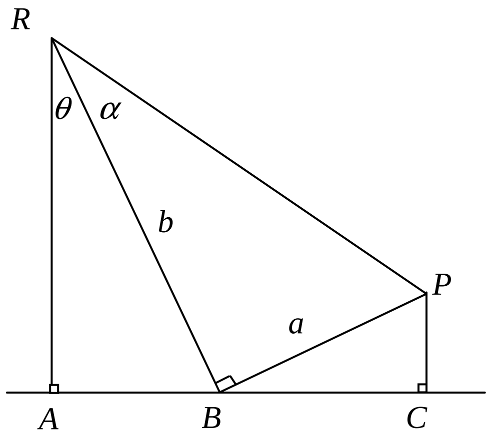
(c) In the diagram prove that **3**

(i) $AC = a\cos\theta + b\sin\theta$

(ii) $AC = \sqrt{a^2 + b^2}\sin(\theta + \alpha)$

(iii) Hence deduce that

$\sin(\theta + \alpha) = \cos\theta\sin\alpha + \sin\theta\cos\alpha$



(d) (i) Show that if x and y are positive and unequal then **4**

$$x^2 + y^2 > 2xy.$$

(ii) Hence or otherwise show that if a and b are positive and unequal then

$$a + b > 2\sqrt{ab}.$$

This is the end of the paper.

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ASSESSMENT TASK #1

Mathematics

Sample Solutions

Question	Marker
1	DM
2	FN
3	JD
4	AF
5	PRB
6	DMH
7	CK
8	PSP

Question 1

a) 3.04.

b) $4a - 6$

c) $\frac{x+2-3}{3} = \frac{x}{4}$

$4x - 4 = 3x$

$x = 4$

d) $3\sqrt{27} - 3\sqrt{3} = 9\sqrt{3} - 3\sqrt{3}$

$= 6\sqrt{3}$

e) $x = 32$

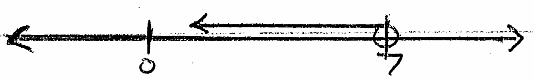
f) $81°33'$

g) $4x^2 - 12x = 0$

$x^2 - 3x = 0$

$x = 0, 3.$

h) $x < 7$



QUESTION 2

(a) $\frac{x^6 y^3}{x^2 y^6} = \frac{x^4}{y^3}$ ①

(b) $\sin 315° - \tan 150°$

$= -\sin 45 - - \tan 30°$

$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$ ②

(c)(i) $f(2) = \frac{4 \times 2^2}{\sqrt{9-(2)^2}} = \frac{16}{\sqrt{5}}$ ①

(ii) $f(-x) = \frac{4 \times (-x)^2}{\sqrt{9-(-x)^2}}$

$= \frac{4x^2}{\sqrt{9-x^2}} = f(x)$

∴ even function ①

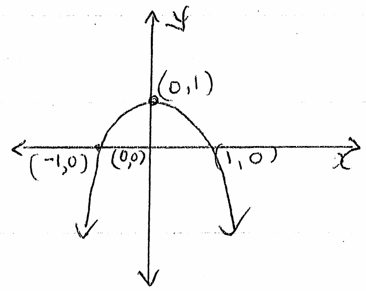
(d) $= \frac{(x+y)(x-y)}{(x+y)^2}$

$= \frac{x-y}{x+y}$ ①

(e) 3.40.9485 × 10^{34}

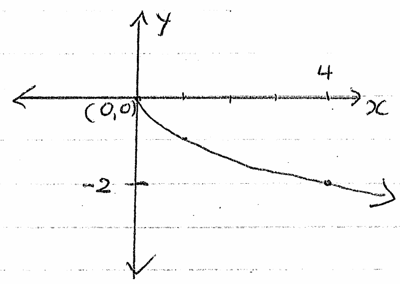
①

(f) $y = 1 - x^2$



②

$y = -\sqrt{x}$



②

(g) 0.2 + 0.07 + 0.007

$= \frac{1}{5} + S_\infty$ geo. series $a = \frac{7}{10}$, $r = \frac{1}{10}$

$S_\infty = \frac{a}{1-r} = \frac{7}{10} \div \frac{9}{10}$

$\frac{1}{5} + \frac{7}{90} = \frac{5}{18}$

or $10x = 2.777\ldots$

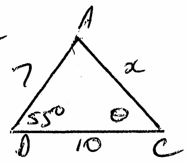
$100x = 27.777\ldots$

$90x = 25$

$x = \frac{25}{90}$

$= \frac{5}{18}$ ①

Q3a



i) $x^2 = 7^2 + 10^2 - 2\times7\times10\cos 55$

$= 68.69$

$x = 8.29$ (2dp)

ii) $49 = 10^2 + 8.29^2 - 2\times10\times8.29\cos\theta$

$49 = 168.7241 - 165.8\cos\theta$

$\cos\theta = .7220$

$\theta = 43°46'$

Or use sine rule.

b) $f(x) = \frac{x}{\sqrt{4-x^2}}$

Domain $-2 < x < 2$

Range y = Reals

c) $|3-2x| < 4$

$3 - 2x < 4$

$-2x < 1$

$x > -\frac{1}{2}$

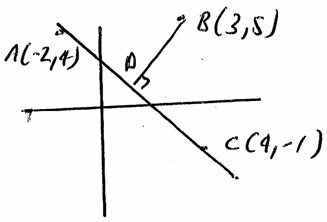
$2x - 3 < 4$

$2x < 7$

$x < 3\frac{1}{2}$

$-\frac{1}{2}$ ———— $3\frac{1}{2}$

d)



Equation of AC

i) $\frac{y-4}{x+2} = \frac{4+1}{-2-4}$

$5x + 10 = -6y + 24$

$5x + 6y - 14 = 0$

OR $y = -\frac{5}{6}x + \frac{7}{3}$

ii) Gradient of AC is $-\frac{5}{6}$

Gradient of BD = $\frac{6}{5}$

Eqn of BD

$y - 5 = \frac{6}{5}(x-3)$

$5y - 25 = 6x - 18$

$6x - 5y + 7 = 0$

OR $y = \frac{6}{5}x + \frac{7}{5}$

iii) length of BD

$BD = \left|\frac{5\times3 + 6\times5 - 14}{\sqrt{36+25}}\right|$

$= \frac{31}{\sqrt{61}}$

$= \frac{31\sqrt{61}}{61}$

≈ 3.969

4.a.i. $y = 4 - x$ ——— ①

$y = x^2 - 2$ ——— ②

sub ① into ②

$4 - x = x^2 - 2$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$\therefore x = -3$ or $x = 2$

sub into ①

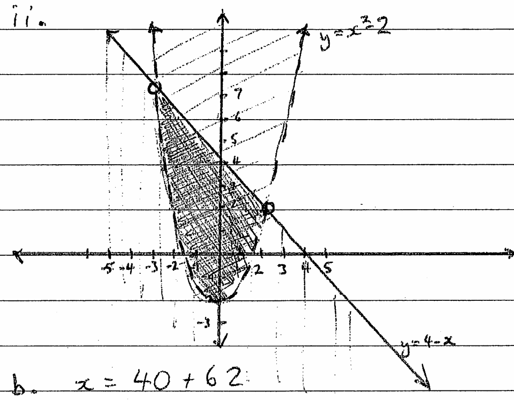
$y = 4 - (-3)$ $\quad y = 4 - 2$

$y = 7$ $\quad y = 2$

$\therefore$ points of intersection are

$(-3, 7)$ and $(2, 2)$

ii.



b. $x = 40 + 62$

$x = 102°$

In $\triangle$'s ABE & ADC

c. $\angle DBF = \angle ECF$ (given)

$\angle BAC$ is common

$\therefore \triangle ABE \,|||\, \triangle ADC$

$\therefore$ corresponding sides in same ratio

$\frac{DC}{BE} = \frac{AC}{AB}$

$\frac{DC}{6} = \frac{8}{10}$

$\therefore DC = 4.8\text{cm}$

d.i. $x^4 - 13x^2 + 36 = 0$

let $m = x^2$

$m^2 - 13m + 36 = 0$

$(m-9)(m-4) = 0$

$m = 9$ or $m = 4$

$x^2 = 9$ $\quad x^2 = 4$

$x = \pm 3$ $\quad x = \pm 2$

$\therefore x = -3, -2, 2, 3$

ii. $4^x - 9(2^x) + 8 = 0$

$2^{2x} - 9(2^x) + 8 = 0$

let $m = 2^x$

$m^2 - 9m + 8 = 0$

$(m-8)(m-1) = 0$

$m = 8$ or $m = 1$

$2^x = 8$ $\quad 2^x = 1$

$2^x = 2^3$ $\quad 2^x = 2^0$

$\therefore x = 3$ $\quad \therefore x = 0$

$x = 0, 3$

e. $x^2 + y^2 - 4x + 6y = 3$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 3 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 16$

circle with centre $(2, -3)$

and radius 4 units.

QUESTION 5

(a) (i) $3x^2 - 6x$

(ii) $\frac{d}{dx}\left[4(x-1)^{\frac{1}{2}}\right] = 2(x-1)^{-\frac{1}{2}} = \frac{2}{\sqrt{x-1}}$

(iii) $\frac{d}{dx}\left(\frac{1}{2x^3}\right) = \frac{d}{dx}\left(\frac{1}{2}x^{-3}\right) = -\frac{3}{2}x^{-4} = \frac{-3}{2x^4}$

(b) (i) $y = 3x(x-1)^9$

$y' = 3x \cdot 9(x-1)^8 + 3(x-1)^9$

$= 3(x-1)^8\left[9x + (x-1)\right]$

$= 3(x-1)^8(10x-1)$

(ii) $f(x) = \frac{x+1}{1-2x}$

$f'(x) = \frac{(1-2x) \cdot 1 - (x+1) \cdot -2}{(1-2x)^2}$

$= \frac{1-2x+2x+2}{(1-2x)^2}$

$= \frac{3}{(1-2x)^2}$

(iii) $f(x) = x + x^{-1}$

$f'(x) = 1 - \frac{1}{x^2}$

(α) $f'(2) = 1 - \frac{1}{4} = \frac{3}{4}$

(β) $f'(-3) = 1 - \frac{1}{9} = \frac{8}{9}$

(c)

(i) $ar^3 = -\frac{27}{8}$ — (1)

$ar^6 = \frac{729}{64}$ — (2)

$r^3 = -\frac{27}{8}$

$\therefore r = -\frac{3}{2}$

Sub in (1)

$a \times -\frac{27}{8} = -\frac{27}{8}$

$\therefore a = 1$

(ii) $S_{10} = \frac{1\left(\left(-\frac{3}{2}\right)^{10} - 1\right)}{-\frac{3}{2} - 1}$

$= \frac{2}{5}\left(1 - \left(\frac{3}{2}\right)^{10}\right)$

$= -22\frac{341}{512}$

6. (a) For the curve $y = x - x^3$, find the gradient of the tangent to the curve at the point $(2,\ 6)$. Also find the gradient of the normal to the curve at this point.

> **Solution:** $\frac{dy}{dx} = 1 - 3x^2$. When $x = 2$, gradient of tangent is -11,
> gradient of normal is $\frac{1}{11}$.

(b) Given that $f(x)$ is defined as below

$$f(x) = \begin{cases} -5 & \text{for} \quad x \leq -3 \\ 2x & \text{for} \quad -3 < x < 0 \\ x^2 & \text{for} \quad x \geq 0 \end{cases}$$

(i) Find the value of $f(-3) + f(4) + f(-1)$.

> **Solution:** $-5 + 16 - 2 = 9$.

(ii) Find $f(a^2)$.

> **Solution:** As $a^2 \geq 0$, $f(a^2) = a^4$.

(c) For the parabola $x^2 - 4x - 8y - 4 = 0$ write down the

(i) equation of the axis of symmetry,

> **Solution:**
> $$\begin{aligned} x^2 - 4x + 4 &= 8y + 4 + 4, \\ (x-2)^2 &= 4 \times 2 \times (y+1). \end{aligned}$$
> So axis of symmetry is $x = 2$.

(ii) coordinates of the vertex,

> **Solution:** Vertex $(2,\ -1)$.

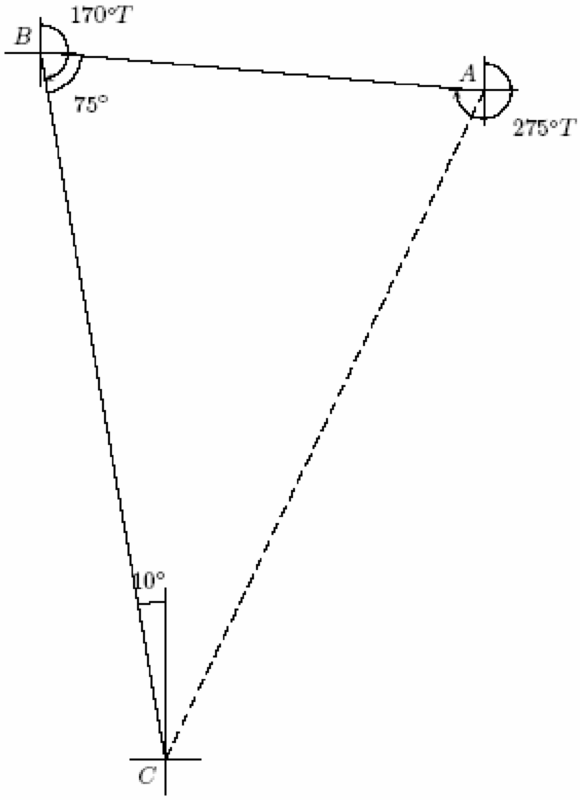
(iii) equation of the directrix and coordinates of the focus.

> **Solution:** Directrix is $y = -3$,
> Focus is $(2,\ 1)$.

(d) Two cadets on a compass march proceed from their camp at A a distance of 1300 m on a bearing of $275°T$ to a point B, then travel 2100 m on a bearing of $170°T$ to a point C.

(i) Draw a neat diagram to represent this situation.

Solution:



(ii) Determine the bearing and distance for their final leg from C back to camp at A.

Solution:

$$\begin{aligned} b^2 &= 1300^2 + 2100^2 - 2 \times 1300 \times 2100 \times \cos 75°, \\ &= 4\,686\,848. \\ b &= 2164{\cdot}912934 \text{ (calculator)}. \\ \frac{\sin C}{1300} &\approx \frac{\sin 75°}{2164{\cdot}9}. \\ \sin C &= 0{\cdot}580024976 \text{ (calculator)}, \\ C &\approx 35°27'. \end{aligned}$$

$\therefore$ Distance is 2165 m and bearing is $025°T$.

QUESTION 7 (SECTION D)

(a)

(i) $\alpha + \beta = -\frac{(-7)}{1} = 7$ (1)

(ii) $\alpha\beta = \frac{2}{1} = 2$ (1)

(iii) $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1$

$= 2 + 7 + 1$ (2)

$= 10$

(b)

$2a - 3b = -21$ —— (A)

$4a + 2b = -2$ —— (B)

$2 \times (A) \Rightarrow 4a - 6b = -42$ —— (C)

$(B) - (C) \Rightarrow 8b = 40$ (2)

$\boxed{b = 5}$

$\boxed{a = -3}$

(c)

(i) $\log_2 64 + \log_2 32 + \log_2 16 + \ldots$

ie $6\log_2 2 + 5\log_2 2 + 4\log_2 2 + \ldots$

∴ The series is <u>arithmetic</u> (1)

$6 + 5 + 4 + \ldots$

<u>Common Difference</u> is -1 (1)

(ii) $T_7 = 0$ (1)

(iii)

$6+5+4+3+2+1+0-1-2-3-4-5-6=0$

∴ 13 terms (1)

(d) (2)

$$\frac{1}{\sec A - \tan A} = \sec A + \tan A$$

$$LHS = \frac{1}{\sec A - \tan A}$$

$$= \frac{1}{\sec A - \tan A} \cdot \frac{\sec A + \tan A}{\sec A + \tan A}$$

$$= \frac{\sec A + \tan A}{\sec^2 A - \tan^2 A}$$

$$= \frac{\sec A + \tan A}{1}$$

$= RHS$

[using $\tan^2 A + 1 = \sec^2 A$]

<u>OR</u>

$$LHS = \frac{1}{\frac{1}{c} - \frac{s}{c}}$$

$$= \frac{c}{1 - s}$$

$$= \frac{c}{1-s} \times \frac{1+s}{1+s}$$

$$= \frac{c + cs}{1 - s^2}$$

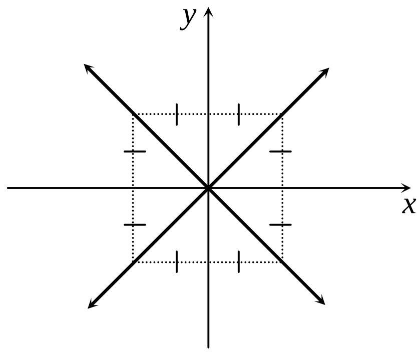
$$= \frac{c(1+s)}{c^2}$$

$$= \frac{1}{c} + \frac{s}{c}$$

$= \sec A + \tan A = RHS$

Question 8

(a) (i)



(ii) $y^2 = x^2$

$\therefore y = \pm x$

(b) Let α, β be the roots of $x^2 + bx + c = 0$

$\therefore \alpha + \beta = -b$ and $\alpha\beta = c$

Let $\alpha = 2\beta$

$\therefore \alpha + \beta = -b \Rightarrow 3\beta = -b$

$\therefore \beta = -\frac{b}{3}$

$\alpha\beta = c \Rightarrow 2\beta^2 = c$

$\therefore 2\left(-\frac{b}{3}\right)^2 = c$

$\therefore \frac{2b^2}{9} = c$

$\therefore 2b^2 = 9c$

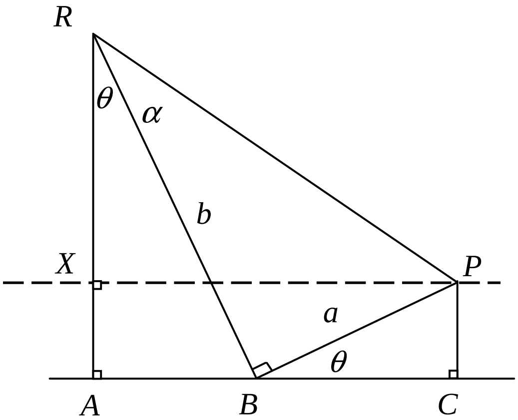
(c) (i) In ΔABR, $AB = b\sin\theta$

In ΔBPC, $\angle PBC = \theta$

$\therefore BC = a\cos\theta$

$AC = AB + BC \Rightarrow AC = a\cos\theta + b\sin\theta$

(ii) Draw in a line through P parallel to AC. So $XP = AC$.



$RP = \sqrt{a^2 + b^2}$ [Pythagoras' Theorem]

In ΔRXP: $XP = RP \times \sin(\theta + \alpha)$

$\therefore XP = \sqrt{a^2 + b^2}\sin(\theta + \alpha)$

$\because XP = AC \Rightarrow AC = \sqrt{a^2 + b^2}\sin(\theta + \alpha)$

(iii) $\therefore \sqrt{a^2 + b^2}\sin(\theta + \alpha) = a\cos\theta + b\sin\theta$

$\therefore \sin(\theta + \alpha) = \dfrac{a}{\sqrt{a^2 + b^2}}\cos\theta + \dfrac{b}{\sqrt{a^2 + b^2}}\sin\theta$

In ΔRBP: $\cos\alpha = \dfrac{b}{\sqrt{a^2 + b^2}}, \sin\alpha = \dfrac{a}{\sqrt{a^2 + b^2}}$

$\therefore \sin(\theta + \alpha) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$

(d) (i) $x \neq y \Rightarrow (x-y)^2 > 0$

$\therefore x^2 + y^2 - 2xy > 0$

$\therefore x^2 + y^2 > 2xy$

(ii) Let $a = x^2$ and $b = y^2$ in (i)

Clearly $a, b > 0$ and $a \neq b$

$x = \sqrt{a}, y = \sqrt{b}$

$\therefore x^2 + y^2 > 2xy \Rightarrow a + b > 2 \times \sqrt{a} \times \sqrt{b}$

$\therefore a + b > 2\sqrt{ab}$

Alternatively consider $\left(\sqrt{a} - \sqrt{b}\right)^2$

$\because a \neq b \Rightarrow \left(\sqrt{a} - \sqrt{b}\right)^2 > 0$

$\therefore a + b - 2\sqrt{a}\sqrt{b} > 0$

$\therefore a + b > 2\sqrt{ab}$

Alternatively (Proof by Contradiction)

Assume $a + b \leq 2\sqrt{ab}$

$\therefore a + b - 2\sqrt{ab} \leq 0$

$\therefore \left(\sqrt{a} - \sqrt{b}\right)^2 \leq 0$

$\therefore \left(\sqrt{a} - \sqrt{b}\right)^2 = 0$ $\left[\because x^2 \geq 0, x \in \square\right]$

$\therefore \sqrt{a} - \sqrt{b} = 0$

$\therefore a = b$

This contradicts the fact that $a \neq b$

$\therefore a + b > 2\sqrt{ab}$