SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2006
YEAR 11 MATHEMATICS EXTENTION
HALF YEARLY EXAM

Mathematics Continuers

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 81

- Attempt questions 1-8
- Start each new section in a separate answer booklet

Examiner: *D.McQuillan*

Total marks – 81
All questions are NOT of equal value

Answer each SECTION in a SEPARATE writing booklet

Section A

Question 1 (10 Marks) Marks

(a) Evaluate $8^{2.1}$ to 4 significant figures. 2

(b) Write $0.\dot{1}\dot{3}$ as a simplified fraction. 1

(c) Simplify

(i) $3x-(4-x)$ 1

(ii) $\dfrac{x+1}{3}+\dfrac{2x}{5}$ 1

(d) Convert $270°$ to radians in exact form. 1

(e) Factorise

(i) x^2-9 1

(ii) $64+x^3$ 1

(f) Given that

$$f(x)=\begin{cases}6-x^2 & \text{if } x\ge 0\\ |x| & \text{if } x<0\end{cases},$$

evaluate

(i) $f(-2)$ 1

(ii) $f(0)$ 1

Question 2 (11 Marks)

(a) Solve $|2x+6|=10$. 2

(b) Simplify $\frac{1}{\sec^2\theta}+\frac{1}{\text{cosec}^2\theta}$. 2

(c) (i) Solve the inequation $|x+3|<2$. 2

(ii) Hence graph the solution on a number line. 1

(d) Find equation of the circle with centre (–4, 6) and radius $\sqrt{5}$. 2

(e) Find the exact value of $\tan\frac{3\pi}{4}$. 2

Question 3 (9 Marks)

(a) (i) Write down the expansion for $\sin(A+B)$. 1

(ii) Hence find the exact value of $\sin 75°$. 2

(b) If α and β are the roots of $2x^2+3x+4=0$, find the value of:

(i) $\alpha\beta$ 1

(ii) $\frac{1}{\alpha}+\frac{1}{\beta}$ 2

(c) Sketch the intersection of the regions $y>x^2-1$ and $y\le x$. 3

Question 4 (11 Marks)

(a) State the domain and range of $g(x) = \sqrt{x+4}$ 2

(b) Show that $f(x) = x^3 + 3x$ is an odd function. 2

(c) Find correct to the nearest minute the acute angle between $y = 6x - 7$ and $y = x + 3$. 3

(d) Find the values of A and B if $2(x-1)^2 \equiv A(x^2+1) + Bx$. 2

(e) For A(5, 1) and B(-3, 7) find the coordinates of the point that divides the interval AB internally in the ratio 3:1. 2

End of Section A

START A NEW WRITING BOOKLET

Section B

Question 5 (9 Marks) Marks

(a) The graph of the equation $x^2 - 6x + y^2 + 10y + 9 = 0$ is a circle.

(i) Explain why the graph of the equation does **not** represent a function. 1

(ii) Find the radius and the coordinates of the centre of the circle. 2

(b) Find the equation of a parabola with vertex (0, –2) and focus (0, 1). 2

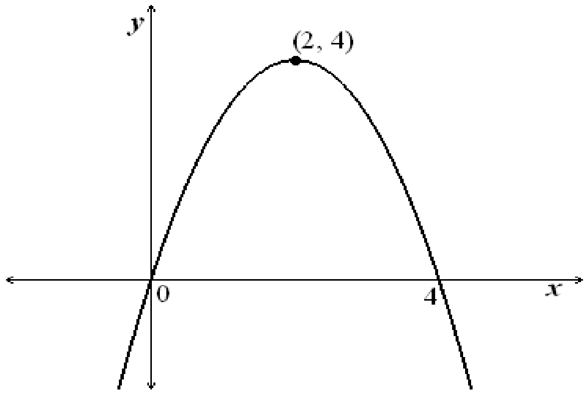
(c) Solve the inequality $\frac{x-3}{x} < 0$. 2

(d) Solve the equation $x^4 - 10x^2 + 9 = 0$. 2

Question 6 (9 Marks)

(a) Sketch the graph of $y = |x + 3|$. 2

(b) Find the equation of the following parabolic graph. 2



(c) Find the values of p for which the equation $x^2 + px + 16 = 0$, has real roots. 2

(d) If $\tan\theta = \sqrt{3}$ and $0 < \theta < \frac{\pi}{2}$ evaluate $\cos 2\theta$ in exact form. 3

Question 7 (11 Marks)

(a) Let P be the point (–2, 3) and *l* be the line $4x - 3y = 8$.
Find the shortest distance from P to *l*. 2

(b) Find the equation of the straight line which passes through the point of intersection of the lines $2x - y - 3 = 0$ and $y - 3x - 2 = 0$ and the point (3, -1). 3

(c) From a point B which is 80 metres due east of a flagpole, F, the angle of elevation of its top is 15 degrees. Point A is 60 metres due south of the flagpole.

(i) Draw a neat diagram of the above. 1

(ii) Find the angle of elevation from point A to the top of the flag pole. 2

(d) Solve the following simultaneous equations, 3

$$\begin{aligned} 2x + y - z &= -3 \\ -x + 3y - 2z &= 1 \\ x - y + 5z &= 12 \end{aligned}$$

Question 8 (11 Marks)

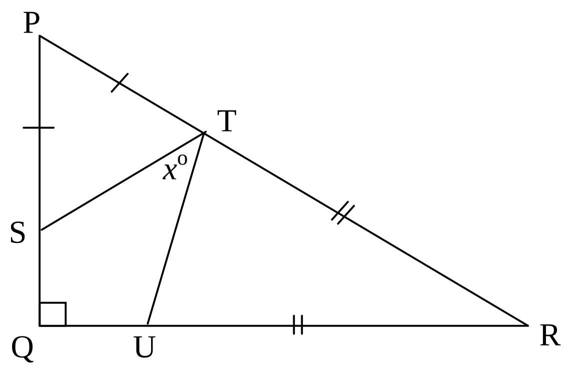
(a) The quadratic equation $x^2 + Lx + M = 0$ has one root which is twice the other. Prove that

(i) $2L^2 = 9M$ 3

(ii) The roots are rational whenever L is rational. 2

(b) Find the equation of the locus of the point P(x, y) such that it is equidistant from (3, 1) and $y = -1$. 3

(c) The triangle PQR is right angled at Q and triangles PST and RTU are isosceles as shown. If $\angle STU = x°$ find the value of x. No reasons need to be given. 3



End of Section B

End of Paper

Question 1 (10 Marks)

(a) Evaluate $8^{2\cdot 1}$ to 4 significant figures, [2]

> **Solution:** By calculator, $78\cdot 79324245 \approx 78\cdot 79$ to 4 sig. fig.

(b) Write $0\cdot\dot{1}\dot{3}$ as a simplified fraction. [1]

> **Solution:**
> $$\begin{aligned} \text{Let } x &= 0\cdot\dot{1}\dot{3}, \\ 100x &= 13\cdot\dot{1}\dot{3}, \\ 99x &= 13, \\ \therefore\ x &= \frac{13}{99}. \end{aligned}$$

(c) Simplify

(i) $3x - (4 - x)$ [1]

> **Solution:** $3x - 4 + x = 4x - 4$

(ii) $\dfrac{x+1}{3} + \dfrac{2x}{5}$ [1]

> **Solution:** $\dfrac{5x + 5 + 6x}{15} = \dfrac{11x + 5}{15}$

(d) Convert 270° to radians in exact form. [1]

> **Solution:** $270° \times \dfrac{\pi}{180°} = \dfrac{3\pi}{2}$

(e) Factorise

(i) $x^2 - 9$ [1]

> **Solution:** $(x + 3)(x - 3)$

(ii) $64 + x^3$ [1]

> **Solution:** $4^3 + x^3 = (4 + x)(16 - 4x + x^2)$

(f) Given that

$$f(x) = \begin{cases} 6 - x^2 & \text{if } x \geq 0, \\ |x| & \text{if } x < 0, \end{cases}$$

evaluate

(i) $f(-2)$ [1]

Solution: $|-2| = 2$

(ii) $f(0)$ [1]

Solution: $6 - 0^2 = 6$

Question 2 (11 Marks)

(a) Solve $|2x + 6| = 10$. [2]

Solution:
$$\begin{aligned} 2x + 6 &= 10, & \text{or} \quad -2x - 6 &= 10, \\ 2x &= 4, & -2x &= 16, \\ \therefore\ x &= 2. & x &= -8. \end{aligned}$$

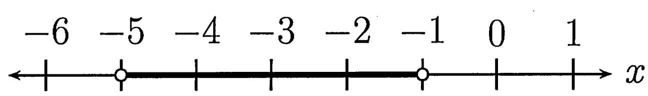
(b) Simplify $\dfrac{1}{\sec^2\theta} + \dfrac{1}{\operatorname{cosec}^2\theta}$ [2]

Solution: $\cos^2\theta + \sin^2\theta = 1$

(c) (i) Solve the inequation $|x + 3| < 2$. [2]

Solution:
$$\begin{aligned} -2 < \ & x + 3 < 2, \\ -5 < \ & \ \ x \ \ < -1. \end{aligned}$$

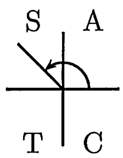
(ii) Hence graph the solution on a number line. [1]

Solution: 

(d) Find the equation of the circle with centre $(-4, 6)$ and radius $\sqrt{5}$. [2]

Solution: $(x + 4)^2 + (y - 6)^2 = 5$

(e) Find the exact value of $\tan \frac{3\pi}{4}$. [2]

Solution: 

$$-\tan \frac{\pi}{4} = -1$$

Question 3 (9 Marks)

(a) (i) Write down the expansion for $\sin(A + B)$. [1]

Solution: $\sin A \cos B + \cos A \sin B$

(ii) Hence find the exact value of $\sin 75°$. [2]

Solution:
$$\begin{aligned}\sin(45° + 30°) &= \sin 45° \cos 30° + \cos 45° \sin 30°, \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}, \\ &= \frac{1+\sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}+\sqrt{6}}{4}.\end{aligned}$$

(b) If α and β are the roots of $2x^2 + 3x + 4 = 0$, find the value of

(i) $\alpha\beta$ [1]

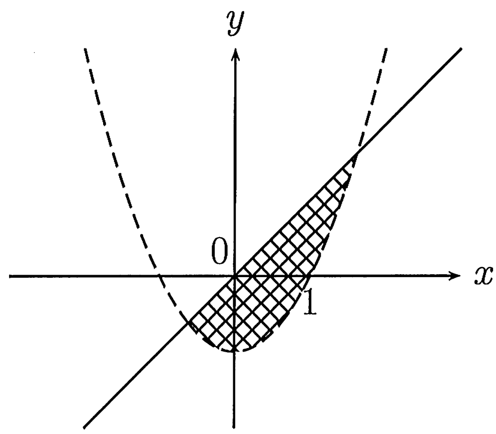
Solution: $\frac{4}{2} = 2$

(ii) $\frac{1}{\alpha} + \frac{1}{\beta}$ [2]

Solution:
$$\begin{aligned}\frac{\alpha+\beta}{\alpha\beta} &= \frac{-3}{2} \times \frac{1}{2}, \\ &= -\frac{3}{4}.\end{aligned}$$

(c) Sketch the intersection of the regions $y > x^2 - 1$ and $y \leq x$. [3]

Solution:



Question 4 (11 Marks)

(a) State the domain and range of $g(x) = \sqrt{x+4}$. [2]

Solution: Domain: $x \geq -4$,
Range: $g(x) \geq 0$.

(b) Show that $f(x) = x^3 + 3x$ is an odd function. [2]

Solution:
$$\begin{aligned} f(-x) &= (-x)^3 + 3(-x), \\ &= -x^3 - 3x, \\ &= -(x^3 + 3x), \\ &= -f(x). \end{aligned}$$
$\therefore\ f(x)$ is odd.

(c) Find correct to the nearest minute the angle between $y = 6x - 7$ and $y = x + 3$. [3]

Solution: $m_1 = 6$, $m_2 = 1$.
$$\begin{aligned} \tan\alpha &= \left|\frac{6-1}{1+6\times 1}\right|, \\ &= \frac{5}{7}. \end{aligned}$$
$\therefore\ \alpha = 35°32'$.

(d) Find the values of A and B if $2(x-1)^2 \equiv A(x^2+1)+Bx$. [2]

Solution: Let $x = 0$, $2 = A$.
Let $x = 1$, $0 = 4 + B$,
$B = -4$.

(e) For $A(5,\ 1)$ and $B(-3,\ 7)$ find the coördinates of the point that divides the interval AB internally in the ratio $3:1$. [2]

Solution: $\left(\dfrac{3\times(-3)+1\times 5}{3+1},\ \dfrac{3\times 7+1\times 1}{4}\right) = (-1,\ 5\tfrac{1}{2})$

Y11 CONTINUERS

SECTION B

5 (a) $x^2-6x+y^2+10y+9=0$

$(x-3)^2+(y+5)^2 = -9+9+25$

$= 25$

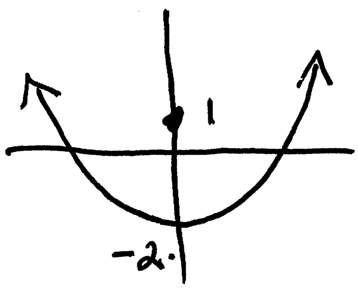
(i) The circle doesn't pass the vertical line test for a function.

OR. There are points (ie two) where the x-value is the same.

OR. It is not a 1-1 correspondence or mapping

(ii) **Centre (3, −5) radius 5.**

(b)



NB $a=3$.

$\therefore (x-0)^2 = 4\times3(y+2)$

$x^2 = 12(y+2)$

(c) $\frac{x-3}{x} < 0$

$x(x-3) < 0$ (ie multiply both sides by x^2)

$\therefore$ $0 < x < 3$

(d) $x^4-10x^2+9=0$ let $u=x^2$

$u^2-10u+9=0$

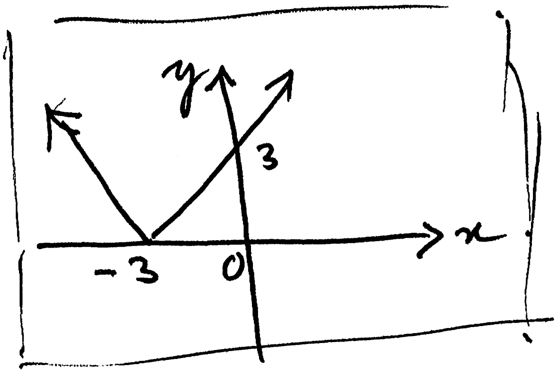
$(u-9)(u-1)=0$

$u = 9, 1$

ie $x^2 = 9, 1$

$x = \pm3, \pm1$

6 (a)



✓✓

(b) $y = -a(x-0)(x-4)$

$\therefore y = -ax(x-4)$

now $(2,+4)$ lies on it.

$\therefore +4 = -2a \cdot -2$

$\therefore \underline{a = 1}$

$\therefore \boxed{y = -x(x-4)}$ ✓✓

(c) For real roots $\Delta \geq 0$

$\therefore p^2 - 64 \geq 0$

$(p-8)(p+8) \geq 0$

$\therefore \boxed{p \geq 8,\ p \leq -8}$ ✓✓

(d) $\tan\theta = \sqrt{3}$, $0 < \theta < \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore \boxed{\cos 2\theta = \cos\frac{2\pi}{3} = -\frac{1}{2}}$ ✓✓✓

Q7

(a) $4x-3y-8=0$ and point $(-2,3)$

$$d = \left| \frac{4\times -2 - 3\times 3 - 8}{\sqrt{4^2+(-3)^2}} \right|$$

$$= \left| \frac{-8-9-8}{5} \right|$$

$$= \left| \frac{-25}{5} \right|$$

$$= 5$$

✓ ✓

(b) Find the intersection of

$2x - y = 3$ — ①

$3x - y = -2$ — ②

② − ①

$x = -5$

Sub in ①

$-10 - y = 3$

$y = -13$

$\therefore (-5, -13)$

∴ Equation of line passing through $(3,-1)$ and $(-5,-13)$

$$\frac{y+1}{x-3} = \frac{-13+1}{-5-3} = \frac{-12}{-8}$$

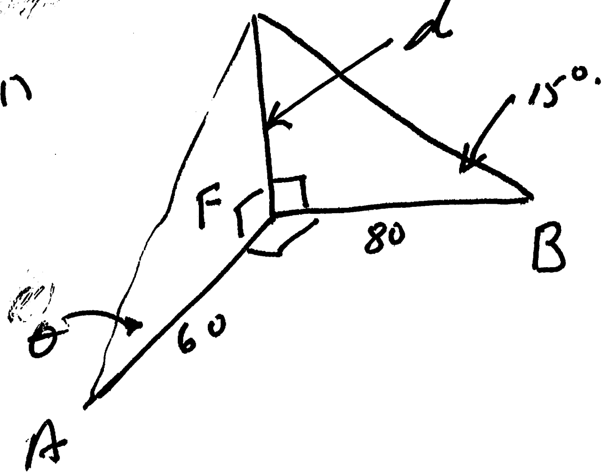
$$= \frac{3}{2}$$

$\therefore 2y+2 = 3x-9$

$3x-2y-11=0$

✓ ✓ ✓

(c) (i)



(ii) $\frac{d}{80} = \tan 15°$

$\therefore d = 80 \tan 15°$

now $\tan\theta = \frac{d}{60}$

$= \frac{80\tan 15°}{60}$

$= 0.3573$

$\therefore \theta = 19°40'$ (OR 20°)

(d) $2x + y - z = -3$ —— ①

$-x + 3y - 2z = 1$ —— ②

$x - y + 5z = 12$ —— ③

① + ③

$3x + 4z = 9$ —— ④

③ × 3

$3x - 3y + 15z = 36$ (3a)

(3a) + ②

$2x + 13z = 37$ ⑤

④ × 2

$6x + 8z = 18$ —(4a)

⑤ × −3

$-6x - 39z = -111$ (5a)

(4a) + (5a)

$-31z = -93$

$z = 3$

Sub in ④

$3x + 12 = 9$

$3x = -3$

$x = -1$

Sub in ①

$-2 + y - 3 = -3$

$y = 2$

$\therefore (-1, 2, 3)$

Q8

(a) $x^2 + Lx + M = 0$ has roots $\alpha, 2\alpha$.

(i) now $\alpha + 2\alpha = -L$ & $\alpha \times 2\alpha = M$

$\therefore 3\alpha = -L$ — ① $2\alpha^2 = M$ — ②

$\alpha = -\frac{L}{3}$

From ① & ②

$2 \times \left(-\frac{L}{3}\right)^2 = M$

$\frac{2L^2}{9} = M$

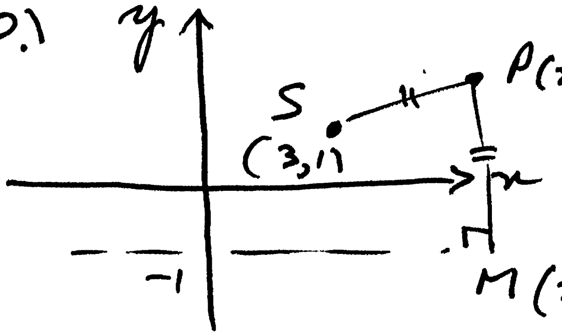
$\boxed{2\alpha^2 = 9M}$

✓✓✓

(ii) If L is rational then $-\frac{L}{3} = \alpha$ is rational ✓ ✓

& hence $2\alpha = -\frac{2L}{3}$ is also rational

(b)



now $SP = PM$

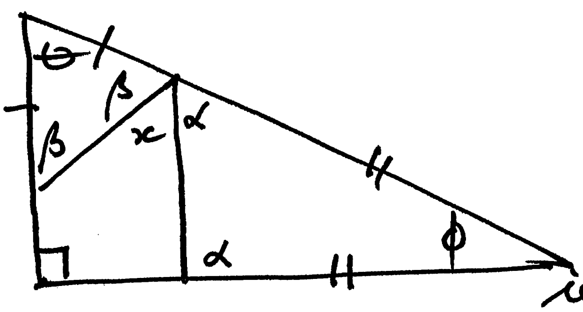
ie $\sqrt{(x-3)^2 + (y-1)^2} = y + 1$

$(x-3)^2 + (y-1)^2 = (y+1)^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = y^2 + 2y + 1$

$\boxed{(x-3)^2 = 4y}$ ✓✓✓

(c)



now

$\alpha + \beta + x = 180°$ — ①

$\theta + \phi = 90°$

ie $(180 - 2\beta) + (180 - 2\alpha) = 90°$ ✓ ✓

ie $2\alpha + 2\beta = 270°$ ✓

$\alpha + \beta = 135°$ — ②

From ① & ② $\boxed{x = 45°}$